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An Investigation of the Effects of Compressed Heuristics Instruction on Problem Solving in Mathematics

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AN INVESTIGATION OF THE EFFECTS OF COMPRESSED HEURISTICS INSTRUCTION ON PROBLEM SOLVING IN MATHEMATICS

by

James Murray Dunlop

A thesis submitted to the Department of Curriculum and Instruction in partial fulfillment of the requirement for the degree of Master of Education

UNIVERSITY OF NORTH FLORIDA

COLLEGE OF EDUCATION AND HUMAN SERVICES

April, 1988

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An Investigation of the Effects of Compressed Heuristics Instruction on Problem Solving in Mathematics

James M. Dunlop

University of North Florida
Problem Solving

Abstract

The purpose of this study was to investigate the effect that heuristics instruction for certain strategies and skills used in the solution of non-routine mathematical problems would have on problem solving behavior. It was conjectured that subjects given compressed but explicit instruction in problem solving strategies would exhibit higher achievement than subjects who did not receive such explicit instruction. Subjects were elementary education student volunteers from the University of North Florida. They were randomly assigned to experimental and control groups for instruction. A pretest and a posttest were administered to collect the data to evaluate this experimental design. The null hypothesis that there would be no difference in the mean gain scores between the experimental and control groups could not be rejected at the .05 level of significance. The results of this study indicate that successful generalization of complex concepts should not be expected following such a short instructional period.
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An Investigation of the Effects of Compressed Instruction on Problem Solving Strategies in Mathematics

Introduction

Throughout the 1980s, one goal in the secondary school mathematics curriculum has been directed toward helping students acquire power in problem solving, an ability Polya (1980) considered to be the specific manifestation of intelligence. "If education fails to contribute to the development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems..." (p. 2).

Problem solving occurs in many different professions and disciplines. The term is all-encompassing, interpreted differently by different people, and carries different connotations for the same people according to the circumstances in which it is used. This study will, however, define problem solving as the application of previously acquired knowledge and skills to new and unfamiliar situations through the use of reasoning, comprehension, logic, and the procedures, strategies, and heuristic methods that are essential in finding a way to a desired end. Heuristics is here defined as general suggestions or techniques which
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encourage individuals to discover solutions by themselves.

A review of nationally marketed secondary school (grades 7-12) mathematics textbooks used locally (Dressler, 1981; Price, Rath & Leschensky, 1982; Travers, Dalton & Brunner, 1978) as well as discussions with secondary school mathematics teachers indicate the importance of both problem solving and the efforts made to teach it. The review and discussions led to the following assumption: A student's problem solving ability is positively related to instruction in both the processes and skills of problem solving.

Problem solving is referred to in many institutional goal statements. As indicated earlier, a basic problem solving model is included as a process in almost all secondary school mathematics texts. However, subsequent to publication of the report from the National Commission on Excellence in Education (1983), A Nation at Risk, numerous reports on the status of education indicated that problem solving -- both its processes and skills -- was not being adequately addressed in the classroom. Increasingly, the glare of media spotlights has centered on the steadily falling scores on the Scholastic Aptitude Tests (SAT) and the Graduate Record Examinations (GRE) across the nation,
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emphasizing students' weaknesses in critical reading, comprehension, analogous thinking, and problem solving. This evidence lends credence to the notion that, without appropriate supporting instruction, the problem solving process described in the textbooks will be generally ineffective. This unfortunate deficiency continues to cause many anxious moments for both students and teachers in preparing for the increasing number of competency and aptitude tests required for students to advance through our formal education system.

Yet there is little question that teachers and their supervisors want to improve the problem solving abilities of their students. Although the topic of teaching problem solving has received new attention in recent years, none of the educators or psychologists most closely identified with the effort would contend that all the ideas being put forward are new (Kilpatrick, 1987). However, there is a recognizable redirection in the literature, from a theoretical discussion of ideas about teaching problem solving to the more practical application of how to use these ideas effectively in the classroom.

Much of the research on problem solving has focused on the characteristics of the problem and the characteristics of the problem solvers or learners.
In addition, there has been a trend toward answering the question of just how an individual solves problems. Much of this research has been conducted at the elementary school level: Suydam and Weaver (1977) provide a list of findings that focus on "...the strategies that children use in solving problems, the process of problem solving" (p. 40). Their findings also include teaching strategies applicable to the improvement of problem solving instruction at the elementary level.

Research confirms the common sense notion that if problem solving performance is to improve, then problem solving strategies need to be taught (Polya, 1957; Steinberg, 1985; Thornton, Jones & Tooker, 1983). These researchers support the contention that a positive relationship exists between specific instruction in problem solving processes and skills and subsequent problem solving ability.

Much of the recent research has identified many of the processes underlying effective problem solving. Several of these studies have isolated the more basic strategies applicable to mathematics. All have shown that the problem solving ability of subjects who were given specific instruction in these processes was significantly improved. The specific strategies
identified as common to these studies were simulation, pattern search, simplification, trial and error or guess-and-check, and working backwards. From this, it can be reasonably adduced that instruction and practice in using the strategies or processes just identified can enhance problem solving performance.

The research also indicates that results from instructional syllabi based on the processes and skills associated with problem solving are measurable -- that these processes and skills can enhance students' success with test instruments.

However, very little research has been conducted with compressed instructional time. For most research surveyed, the instruction periods were seldom less than a single academic grading period of six to nine weeks. Consequently, it can be shown from the literature that the problem solving process can be taught but it is not clear how much time, as a minimum, is required.

This leads to an academically important question: Can such instruction in problem solving skills and processes be compressed and still provide a measurable enhancement of those skills and processes? Using the strategies identified in the paragraphs above, this study, a pretest-posttest experimental design, addresses whether one group of students, given compressed,
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process-oriented instruction in problem solving, could perform significantly better than could a control group given only generalized instruction on solving problems.

Review of Related Literature

Data from both national and state assessment tests indicate that scores for the application of computational and other skills in problem solving are consistently lower than are scores achieved for straight computation (Suydam, 1980). For this to be significant, we need to understand the difference between computational exercises and problem solving. An exercise, by definition, is specific to a particular computational process and is used for practice in that process. Problem solving, on the other hand, involves activities which Robinson (1972) describes as "requiring creativity, or originality ... situations for which no specific routine [computational] process has been previously learned" (p. 22).

Measurement of Effect

Research indicates that students instructed in the processes and skills of problem solving perform predictably better, when applying previously acquired knowledge to new and unfamiliar situations, than
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students who did not receive such instruction. In an analysis of 33 studies, Marcucci (1980) concludes that students who know a number of ways to tackle a problem are more likely to be better problem solvers than students without such knowledge.

Schoenfeld (1980) reports results of a study he conducted with a small group of lower division liberal arts college students enrolled in a problem solving course. Even though there were only a few subjects in this study the results were convincing. Given very specific instruction in problem solving heuristics, the experimental group showed a strong pretest-to-posttest gain, while the control group, which received no heuristics instruction, did not.

Kraus (1980) concludes that problem solving ability also generalizes. He found that eighth grade students who played games (variants of Nim) against an Algorithmic Player (AP) computer program, and who viewed the games as problems, used a variety of problem solving heuristics. Among the most common were working forward or backward, systematic trial and error, pattern search, and the use of subgoals. Those students not viewing the games as problems used random trial and error against the AP almost exclusively. This difference in the application of heuristics accounted for a significant
difference in the problem solving performance of the two groups.

The same generalization or projection of problem solving processes and skills to general tests of aptitude is demonstrated in a study by Charles and Lester (1984). Testing for a subject's ability to understand problems, plan solution strategies, and arrive at correct answers, they found that subjects in a process-oriented problem solving program scored significantly higher than did those in the control group. Beach (1985) also administered a problem solving test to high ability eighth and ninth grade students to determine the effects of two methods of instruction in heuristic processes. The results indicate that students given specific instruction organized around heuristics scored significantly higher on the test than did students given an intuitive, global approach to problem solving.

Further, there is evidence that complex problem solving strategies can be reduced to constituent parts and that these individual components can be explicitly taught. Swinton and Powers (1983), in a study of analytic problem solving required for the Graduate Record Examination (GRE), showed success in score improvement through the direct instruction of component
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processes of analytical reasoning, such as diagramming and sequential analysis.

Process Identification

The literature indicates that specific strategies of problem solving can be successfully learned through heuristics. The question then becomes which of the many problem solving strategies to employ in order to achieve consistent success. Of the many strategies that could be adapted (Polya, 1957; Wickelgren, 1974), there are five which Musser and Shaughnessy (1980) consider basic to school mathematics: trial and error, pattern search, simplification, simulation, and working backwards.

Musser and Shaughnessy describe trial and error as the most direct method of problem solving. It involves the systematic application of allowable operations to the information given. They further refine the concept by differentiating between the systematic trial and error just described and inferential trial and error, which uses relevant knowledge to narrow the search for solution. The pattern process is described as looking at selected, and possibly sequential, instances of the problem and then generalizing a solution from these several specific solutions. Simplification, on the other hand, involves solving a special case, or
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shortened version, of a problem; it is often accompanied by a pattern heuristic. Simulation can be substituted for experimenting, collecting data and making decisions when carrying out a solution to the original problem is unrealistic. Finally, Musser and Shaughnessy describe working backwards as beginning with the goal and seeking to find a statement, or a series of statements, that will imply a solution.

In a study of inductive pattern search with middle school students, Vissa (1985) found that students showed greater flexibility in solving problems when instructed in the use of heuristics. This included specific skills such as making tables and organizing data, and the use of processes or strategies such as diagramming, simplifying, and guess-and-check or trial and error. A similar investigation was conducted by Ghunaym (1986) on the structure of problem solving strategies and the effect of instruction in these strategies on test performance. He concluded that advanced mathematics students who were encouraged to use problem solving heuristics such as diagrams, substitution, working backwards, contradiction, pattern discovery, and guess-and-check consistently produced better problem solving scores than did students who received no explicit instruction in problem solving strategies.
Subjects

Subjects for this study totaled 15 upper-level, female volunteers from the College of Education and Human Services, University of North Florida. Selection was restricted to those students who had had at least one term of college algebra. The fifteen subjects were randomly assigned to either the group receiving specific instruction in strategies for solving problems or to the group receiving general problem solving instruction.

Instrumentation

Both groups were given identical pretests and posttests. Each test consisted of ten problems, two questions from each of the five basic strategies identified in the research: simulation, pattern search, trial and error, simplification, and working backwards. Copies of the pretest and posttest are found in Appendix A. Items in both pretest and posttest were scored using an analytic scoring scale developed by Charles and Lester (1987). The scale is described in Appendix B.

Problem shall here be defined as a problem of either the "to find" or the "to prove" character which
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may be composed of information concerning given expressions, information regarding constraints in transforming expressions, information regarding operations for transforming expressions, and information concerning a single, terminal goal. The problem may be one of a practical nature, or a puzzle, and its solution may be found without resort to mathematical knowledge beyond that required of a first algebra course.

Instruction

Over a period of five hours, the two groups received essentially the same instruction in problem solving. However, instruction for the experimental group also included additional, explicit instruction in the five basic strategies as described by Musser and Shaughnessy (1980) and identified in the research on page eight. Due to scheduling complications subjects in the experimental group received instruction three days after the pretest and 10 days prior to the posttest. Subjects in the control group, on the other hand, received their instruction 10 days after the pretest and three days prior to the posttest. The researcher conducted all sessions.

Instruction was controlled in both content and scope through the use of overhead transparencies. The
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content of the instruction consisted of the pretest problems and a set of five additional problems reflecting each of the five strategies. The method of instruction was to demonstrate the solution to the first test problem in the strategy set, coach the subjects through the solution to the second test problem in the set, and simply record the subjects' solution to the unfamiliar third problem. The experimental group, in addition, received with each problem an explanation of the strategy as it applied to that example. They were then provided with hints and instructed in those skills that facilitated use of the strategy in a solution of the problem.

Each group had the same amount of time, approximately 20 minutes per problem, for problem solving and seeing the solutions. When working problems, all subjects were reminded periodically to review carefully what they were doing, with special reminders to the experimental group to look over the list of strategies.

Analysis of Data

At the end of the experiment, two hypotheses were examined. The first hypothesis to be tested was that any observed difference in the mean pretest scores of
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the experimental group and the control group would be
due to chance. The first null hypothesis then stated
there would be no significant difference in the means of
the pretest scores of the experimental group and the
control group, as tested by a t-test at the 0.50 level
of significance.

The second hypothesis to be tested was that any
observed difference in the mean gains on the posttest
scores of the experimental group and the control group
would be due to the specific differences in the problem
solving instruction provided the two groups. The second
null hypothesis then stated there would be no
significant difference between the mean of the gains on
posttest scores of the group receiving specific
instruction in problem solving strategies and the mean
of the gains on the posttest scores of the control
group, as assessed by a t-test at the 0.05 level of
significance.

Results

Table 1 below shows the key descriptive statistics
for both the experimental and control groups. A
complete table of raw scores and descriptive statistics
for individual categories of the scoring scale and the
cumulative results is included in Appendix C.

16
Table 1

**Descriptive Statistics for Experimental and Control Groups**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>34.4</td>
<td>32.7</td>
</tr>
<tr>
<td>SD</td>
<td>9.5</td>
<td>12.0</td>
</tr>
<tr>
<td>Posttest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>31.9</td>
<td>36.3</td>
</tr>
<tr>
<td>SD</td>
<td>10.1</td>
<td>10.9</td>
</tr>
<tr>
<td>Gain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>-2.6</td>
<td>3.6</td>
</tr>
<tr>
<td>SD</td>
<td>7.4</td>
<td>3.4</td>
</tr>
</tbody>
</table>

*Note:* Maximum Pretest/Posttest score = 60.

M = mean, SD = standard deviation

To determine if the difference in pretest mean scores was attributable only to random sampling fluctuation, a Student's t-ratio of 0.30 was computed on the difference between the pretest mean scores for the two groups. With a critical value of ± 0.694 at the .50
level of confidence and 13 degrees of freedom, the test was not significant and the null hypothesis could not be rejected. That is, the chances are greater than even that a difference of 1.7 in pretest means would appear by chance if the population means were equal.

The significance of the instruction on skills and strategies for the experimental group was also investigated using Student's t-ratio. A ratio of $-1.68$ was computed on the mean difference in gains between the experimental and control groups. The 13 degrees of freedom and .05 level of confidence gave a critical value on the t-distribution of ± 2.16, which indicates that this test was also not significant. Therefore the hypothesis could not be rejected.

Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Difference</th>
<th>Standard Error of Difference</th>
<th>Student t-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1.7</td>
<td>5.6</td>
<td>0.30</td>
</tr>
<tr>
<td>Gain</td>
<td>$-6.2$</td>
<td>3.7</td>
<td>$-1.68$</td>
</tr>
</tbody>
</table>

Table 2 provides the ratios for both hypotheses as well
as the mean differences between groups and the standard errors of the differences.

Discussion

The results of this study, fortunately, do not discredit the practice of problem solving instruction. The cited studies by Beach (1985), Ghunaym (1986), Kraus (1980), Marcucci (1980), and Schoenfeld (1980) are generally encouraging and would suggest that reasons can be found for the results. Several factors may have affected the outcome of this experiment.

First, there may have been too many strategies and associated skills to expect successful generalization from the experimental group after such a short instructional period. Schoenfeld (1980), in discussing the explicit instruction given to his experimental group, suggested two points that made a difference. There were a limited number of strategies to consider and the test problems were clearly amenable to those given in the instruction.

Secondly, it is probable that the strategies are more complex than their descriptions would indicate. The fact that the posttest scores of the experimental group were not enhanced by the special instruction does not mean that they would not have responded to some
other form of instruction. However, the fact that each item on the posttest presented a novel problem may mean that transfer is less likely through familiarity with sample problems than it is for logical instruction. That is, performance cannot be improved as well simply through familiarity with a fixed response as it can with an analysis of explanations.

Third, through scheduling conflicts and drop outs, the study experienced a 32% experimental mortality. The small size of the experimental and control groups resulting from this attrition, eight subjects and seven subjects respectively, may have had an effect larger than anticipated on the mean scores and the variance.

Finally, the voluntary aspect of subject selection vitiated control of the time factor between instruction and posttest and is possibly a major contributor to the inconclusive results. The experimental group received the posttest ten days after receiving instruction while the control group was given the posttest only three days after instruction. Intervening activities may have been detrimental to scores from the experimental group. By the same token, the scores of the control group may have benefited from the short period between instruction and posttest.
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Recommendations

The subjects in both groups of the experiment were enthusiastic about the experiment and were genuinely interested in the solution of these problems. This observation, in light of the study results, has two implications: enthusiasm is routinely the mark of the volunteer, and the prudent course with volunteers would be to assume nothing beyond their enthusiasm.

A replication of this study should be conducted with two recommended changes in procedure. First, the number of strategies should be reduced to two, or possibly three, which would provide more time for a careful analysis of their application. Secondly, instruction for both groups should be on the same day -- or at worst, consecutive days -- to ensure that approximately equal time elapses between instruction and posttest for both groups.

Each step in the problem solving model, each skill used in the employment of a strategy, and the strategy itself should be elicited from the learner during the problem solving process on each problem. This will ensure that the concepts are understood and are being applied appropriately to the problem. Such a procedure should be self-imposed by the researcher or instructor during demonstration problems as well, and it should be
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done with an elaborate thoroughness.

Repeated suggestions for problem resolution, through both word and example as just described, will not suffice beyond the near term. They should be accompanied by the problem solver's thoughtful analysis of explanations. The logical rationale for the selection of a particular strategy or skill may not be apparent to the learner until its use has been demonstrated, and its selection carefully explained, with several novel but related problems. Once the concept has been accepted, it should be reinforced through identical application procedures.

Although this study focused on a single instruction period, no precedent was intended. Problem solving cannot be taught in such a fashion. Problem solving should be presented as an integral part of the content and, beginning with the basic skills, should be slowly and carefully infused in a practical way that neither disheartens the learner nor detracts from the course content.

Finally, time should not be considered a precious commodity when teaching problem solving. Good problem solving behavior cannot be rushed. Upon presentation of a problem the learner should be given ample time to think about the problem's conditions, constraints, and
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other relevant data before being asked to begin the process of solving it. It is absolutely necessary that the learner fully understand the problem. The learner also should be coached to reflect on the problem solving process while proceeding through the final steps of the model and be required to spend more time during the last step of the problem solving model actively looking back. The researcher should ensure that the learner understands the process used in arriving at a solution and has endeavored to look for alternate ways of solving the problem.
References


Kilpatrick, J. (1985). A retrospective account of the


mathematics of the 1980s. Reston, VA: NCTM.


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Footnotes

1 For example, in its position paper on basic skills in mathematics, the National Council of Supervisors of Mathematics (NCSM, 1977) considers the development of problem solving abilities to be one of the most important goals of mathematics education. This was followed by An Agenda for Action, published by The National Council of Teachers of Mathematics (NCTM, 1980) in which it is recommended -- as a first priority -- that problem solving be the focus of mathematics instruction and that basic skills in mathematics be defined to encompass more than computational facility. Similar statements were put forth by the Conference Board of the Mathematical Sciences (CBMS, 1982), and in 1986, NCTM reaffirmed its position regarding a concentration of effort on the problem solving process rather than calculations associated with the problems.

2 The National Commission on Excellence in Education (NCEE) was appointed by Secretary of Education T. H. Bell to make practical recommendations regarding reform in the schools. Actions recommended were to be taken by educators, public officials, and others having a vital interest in American education. The eighteen
member commission, under the chairmanship of David P. Gardner (now President, University of California), released its report, "A Nation at Risk: The Imperative for Educational Reform", in April 1983 after eighteen months of deliberation.

Nim is a two-person game played with any number of counters placed in any number of rows or piles. Believed to be of Chinese origin, the game was given its name in 1901 by C. L. Bouton, a professor of mathematics at Harvard University. It is an obsolete English word meaning to steal. The game has been completely analyzed using the binary system of mathematics. In one of its most simple variants a supply of counters is arranged arbitrarily in three rows. For example: three rows with 2, 3, and 4 counters respectively. Each player in turn may remove as many counters as desired from one of the rows. At least one counter must be taken at each turn. The person who is forced to draw the last counter is the loser.

Simulation can save time, effort, and money when applied to a certain class of problems -- problems involving probabilities. For example: In 1986 the Kellog Company included self-inking rubber stamps in its boxes of Frosty Flakes. In order to collect all six of
these clever little stamps, how many boxes of Frosty Flakes would have to be purchased? Buying them is impractical. Such transactions can be simulated, however, by using a die and assigning a stamp to each numbered face on the die. The average number of boxes needed to collect all six stamps may be calculated by counting the number of rolls of the die required to have each of the numbers occur face up once.

This definition may be clarified with a simple example. A ball, made of a special compound, is dropped from a platform sixteen feet high (given). Each time the ball hits the floor it rebounds only half as high as the distance it fell (constraint). The ball is caught when it bounces back to a maximum height of one foot (goal). How many times does the ball hit the floor (operation)?
Appendix A

Pretest and Postest Forms

DIAGNOSTIC PRETEST

INSTRUCTIONS

This diagnostic test consists of ten problems. There will be no discussion once the test begins. If you complete the test before the allotted time has elapsed, check your work carefully. Do your best to answer all the problems in the allotted time.

However, read each problem carefully. Work carefully also, but do not spend too much time on a problem that seems difficult for you. Do all work for each problem, including any scribbles or doodling, only on the sheet containing that problem. Should you solve a problem without having to work it out, be certain to explain how you arrived at your answer.

If you have any questions, please ask your monitor to answer them now. Otherwise, wait until your monitor asks you to turn the page, then begin.

TOTAL TIME ALLOTTED: 60 minutes
1. How many angles are formed by ten rays with a common endpoint, as shown below?

2. You are given a strip of paper about 30 centimeters long and two centimeters wide. You fold it in half, then in half again. When you unfold it, you see that three creases have been made.

If you folded the strip in half eight times, how many creases would there be?
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3. Every year of a dog's life is equivalent to seven years of a human's life. If the dog were a human, it would be twice the age of its owner. If the owner were a dog, he would be six years younger than the dog. How old is the dog?

4. How many squares are there on a standard checkerboard? (Hint: they can be different sizes.)
5. Mary Jane played a card game in which each loss meant she had to give the other player half the cards she had left in her hand. She just lost four straight times, and is now holding three cards. How many cards did she have at the start of her losing streak?

6. A man has to take a fox, a goose, and a sack of seed corn across a river. His rowboat has enough room for the man plus either the fox or the goose or the corn. If he takes the corn with him, the fox will surely eat the goose. If he takes the fox with him, the goose will just as surely eat the corn. Only when the man is present are the goose and the corn safe from their enemies. All the same, the man safely carries the fox, the goose, and the seed corn across the river. How does he manage this?
7. The Chairperson of the Beaches Racquet Club asked you, a member in good standing -- and a volunteer, to order the scoring cards for the upcoming Ladies Single Elimination Tournament. One card per match will suffice. Player's registration indicates that 72 women have signed up to play, which means there will be thirty-six matches in the first round. The thirty-six winners will then be paired for the second round and so on. How many scoring cards will you have to order?

8. The "Tri Delt" girls intramural basketball team scored 50 points in its last game. They made twice as many field goals as free throws. With two points for a field goal and one point for a free throw, how many field goals did they make?
9. A work train of the Norfolk and Southern Railway, made up of a locomotive and five cars, has stopped at Yulee for lunch. The AMTRAK express passenger train is due. The station has a small spur siding but it can only hold an engine and two cars. How do they let the AMTRAK express through?

10. Professor R. E. Bound developed a unique energy damping compound. To demonstrate its amazing qualities, he moulded a fist-sized ball of the material and proceeded to let it drop from a platform sixteen feet high. Upon impact, this new material bounces to a height just one-half the distance from which it was dropped. If you caught the ball at the peak of a bounce, and it was just one foot from the floor, how many bounces had the ball made?
INSTRUCTIONS

This posttest consists of ten problems. There will be no discussion once the test begins. If you complete the test before the allotted time has elapsed, check your work carefully. Do your best to answer all the problems in the allotted time.

However, read each problem carefully. Work carefully also, but do not spend too much time on a problem that seems difficult for you. Do all work for each problem, including any scribbles or doodling, only on the sheet containing that problem.

If you have any questions, please ask your monitor to answer them now. Otherwise, wait until your monitor asks you to turn the page, then begin.

TOTAL TIME ALLOTTED: 60 minutes
1. Jeff opened his mathematics book and, musing over the operations to be studied, saw that the product of the facing pages was 702. To what pages had Jeff opened his book?

2. Place the numbers 1 through 17 into the 17 circles of the diagram below so that any three numbers in a row will give the same sum.
3. A patch of lily pads doubles in size each day, once it gets started. If a certain pond is completely covered on the twentieth day, on what day was one-fourth of the pond covered with lily pads?

4. At a hunting lodge on the edge of Ocala National Forest, the gamekeeper has a very large "eco-pen" in which to keep his "targets". Currently it holds both rabbits and pheasants. They have, between them, 35 heads and 98 feet. How many rabbits and pheasants are in the pen?
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5. The 4x4 "magic" square shown below is two thousand years old and comes from India. Place the missing numbers in the square so that:

a. the sum of each row, column, and diagonal is the same, and

b. the sum of each 2x2 corner square and the 2x2 center square is the same.

\[
\begin{array}{cc}
1 & 15 \\
7 & 6 & 9 \\
8 & 11 & 10 \\
2 & 3 \\
\end{array}
\]

6. A small mouse slipped and fell into a well, hitting the water 20 feet below with a resounding splash (for a mouse)! Frightened beyond words, he frantically clawed his way eight feet up the side of the well, got a good grip, and rested. As he dozed, he relaxed his grip on the mossy stones and slipped down five feet. Upon awakening the next morning he again assailed the well's mossy sides and climbed another eight feet, only to slide down five feet again as he slept. If he repeats this effort daily, how many days will it take this persistent mouse to get out of the well?
7. Two hikers come to a fork in the trail and decide that each will take a different trail. They need to divide their water supply evenly between them. Their total water supply completely fills an unmarked eight liter jug. They also have two smaller jugs that are empty -- and unmarked -- one holds five liters and the other holds three liters. How can they divide the water evenly so that each goes down his separate trail with four liters of water in his jug?

8. What is the sum of the first fifty odd counting numbers?
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9. At the last debate of the Democratic presidential candidates in Des Moines, Iowa, all seven of the candidates shook hands with each other on stage just before taking their seats. How many handshakes did the audience observe?

10. This design was made by sticking matchsticks into a piece of cardboard. The top level (level 1) takes three matchsticks.

To generate a second level (level 2) requires adding six more matches and so on. How many matchsticks will be need to complete the design through level 10?
## Appendix B

### Analytic Scoring Scale for Problem Solving Evaluation

<table>
<thead>
<tr>
<th>Understanding the Problem</th>
<th>0: Complete misunderstanding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1: Part of the problem mis-understood/misinterpreted</td>
</tr>
<tr>
<td></td>
<td>2: Complete understanding of the problem</td>
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<table>
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<tr>
<th>Planning a Solution</th>
<th>0: No attempt, or totally inappropriate plan</th>
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<tbody>
<tr>
<td></td>
<td>1: Partially correct plan based on part of the problem being interpreted correctly</td>
</tr>
<tr>
<td></td>
<td>2: Plan could have led to a correct solution if implemented properly</td>
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<table>
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<tr>
<th>Getting an Answer</th>
<th>0: No answer, or wrong answer based on an inappropriate plan</th>
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<tbody>
<tr>
<td></td>
<td>1: Copying error; computational error; partial answer for a problem with multiple answers</td>
</tr>
<tr>
<td></td>
<td>2: Correct answer and correct label for the answer</td>
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Charles, Lester, & O'Daffer (1987)
Problem Solving

Appendix C

Raw Scores and Descriptive Statistics

for Experimental Group Pretest

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Note: U = Understanding, P = Planning, A = Answer, C = Cumulative Score, SD = Standard Deviation, VAR = Variance.
Problem Solving

**Raw Scores and Descriptive Statistics**

**for Control Group Pretest**

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**Note:** U = Understanding, P = Planning, A = Answer, C = Cumulative Score, SD = Standard Deviation, VAR = Variance.
Problem Solving

**Raw Scores and Descriptive Statistics**

for Experimental Group Posttest

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| SUM 2 | 115 | 84  | 56   | 255 |
| SUM   | 1703| 1004| 484  | 8847|
| MEAN  | 14.38| 10.50| 7.0  | 31.88|
| SD    | 2.67 | 4.18 | 3.63 | 10.13|
| VAR   | 7.13 | 17.43| 13.14| 102.70|

**Note:** U = Understanding, P = Planning, A = Answer, C = Cumulative Score, SD = Standard Deviation, VAR = Variance.
## Raw Scores and Descriptive Statistics

for Control Group Posttest

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</table>

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C = Cumulative Score, SD = Standard Deviation, 
VAR = Variance.

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