Predator-Prey Model with Herding Behavior and Hunting Quota

RANDY LEE; DR. MAHBUBUR RAHMAN, PH.D. • UNIVERSITY OF NORTH FLORIDA, COLLEGE OF ARTS AND SCIENCE, JACKSONVILLE, FLORIDA

INTRODUCTION

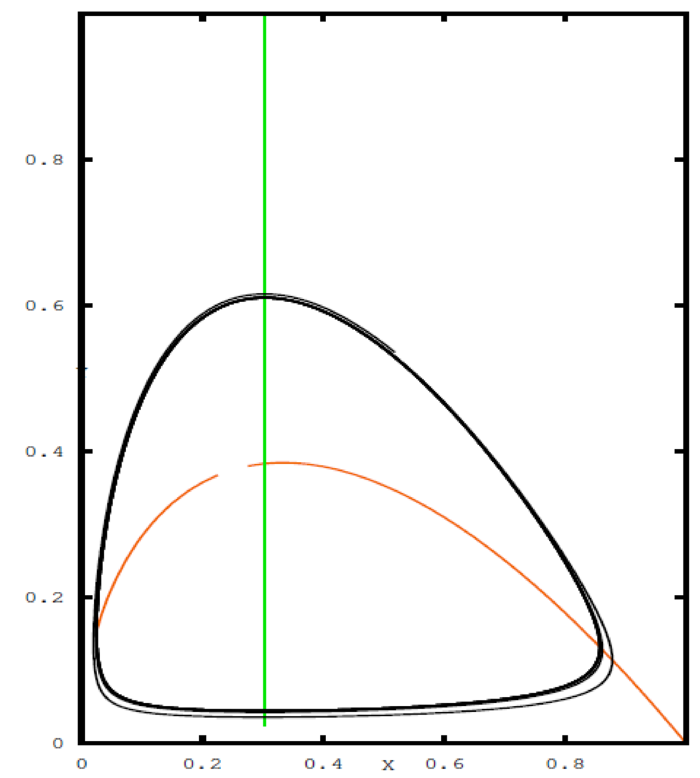
It is self-evident that the populations of predator and prey species affect each other. The Lotka-Volterra system is the first to model these changing dynamics by analyzing how one predator species and one prey species affect each other. The standard Lotka-Volterra system of equations is:

$$\frac{dx}{dt} = (a - by)x$$

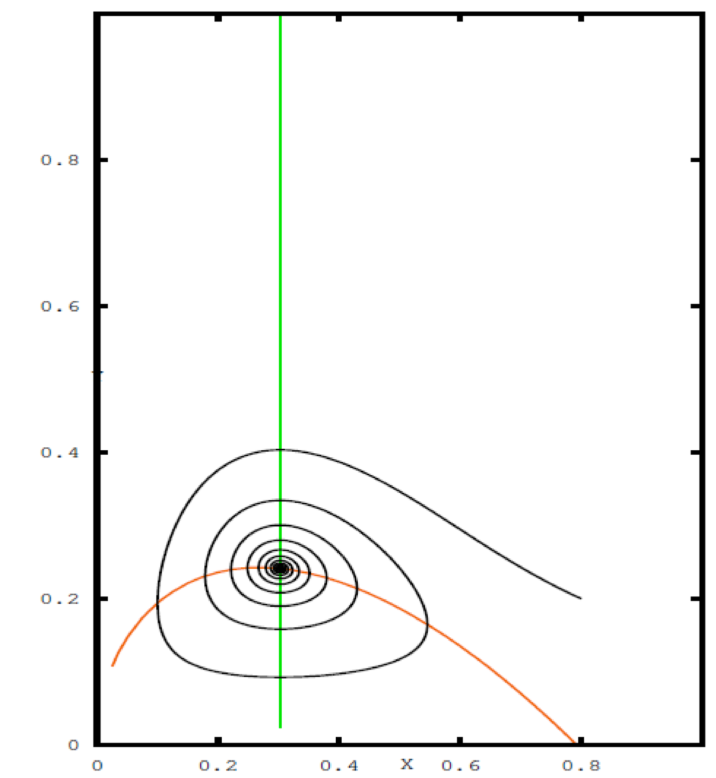
$$\frac{dy}{dt} = (cx - d)y$$

The first equation represents the change in population of the prey species. The second equation represents the change in population of the predator species. A variation of this model assumes the stronger members of the herd will surround and protect the weaker members, causing less of the prey population to be killed by the predator. We will review the analysis of this model to include examples of the system at certain parameters, time graphs of predator and prey populations at certain parameters, the Jacobian matrix, and bifurcation diagrams. We will then be adding a hunting quota to the model and studying the changes it causes.

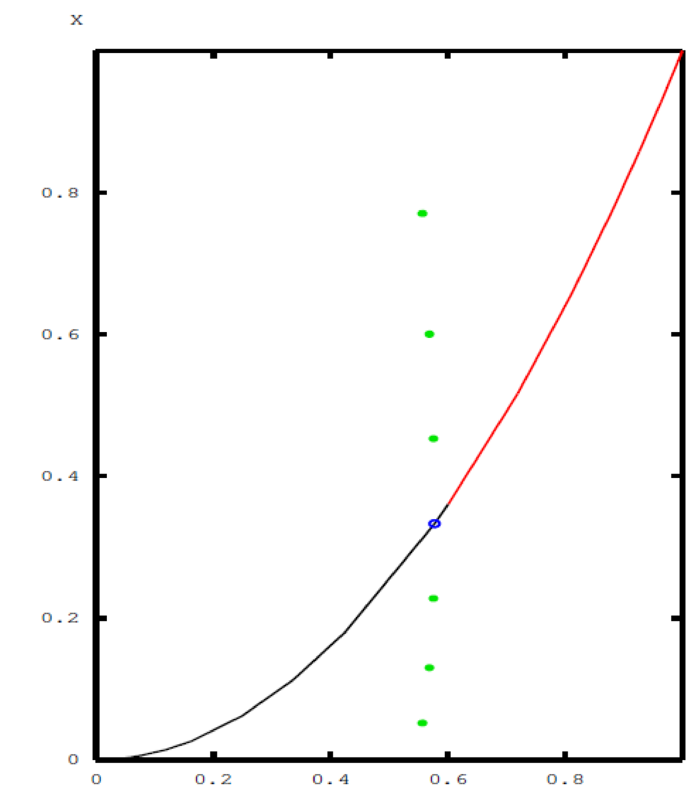
RESULTS



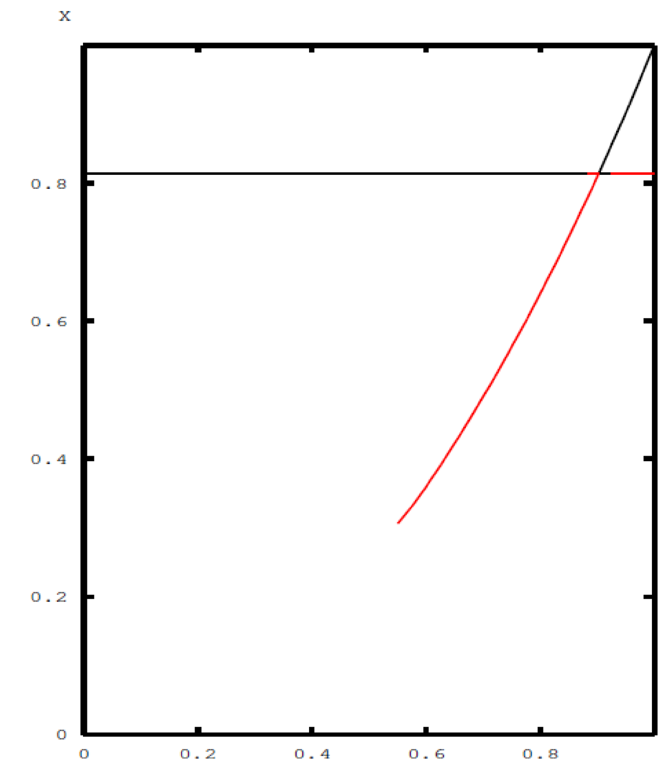
This is a phase diagram of Model 1 when s=0.55 and c=1. The green line is the x-nullcline while the orange line is the y-nullcline. This diagram portrays a limit cycle.



This is a phase diagram of Model 2 when s=0.55, c=1, b=0.3, and d=1. This diagram portrays a stable steady state.



This is the bifurcation diagram of the parameter s plotted with the prey population x for Model 1. The black line is the unstable steady state while the red line is the stable steady state. The green dots represent the stable periodic solution branch's minimum and maximum amplitudes.



This is the bifurcation diagram of the parameter s plotted with the prey population x for Model 2.

Model 1 Equilibrium Points

Real World Interpretation	Equilibrium Points	Stability
Extinct Populations	x=0, y=0	Variable interpretations: Saddle, extinction, or in-between
Prey-Only	x=1, y=0	Stable when s > c; Unstable when s < c
Co-existence	$x=\frac{s^2}{c^2}$, $y=\frac{s(c^2-s^2)}{c^3}$	Exists only when s < c; Stable when $s > \frac{c}{\sqrt{3}}$; Unstable when $s < \frac{c}{\sqrt{3}}$

Model 2 Equilibrium Points

Real World Interpretation	Equilibrium Points	Stability
Extinct Populations	x=0, y=0	Variable interpretations: Saddle, extinction, or in-between
Prey-Only	x=z, y=0 where z comes from $-be^{-dz}+z^2+b-z=0$	Stable when s > c and $-bd(1-e^{-dz}) < 1$; Unstable otherwise
Co-existence	$x=\frac{s^2}{c^2}$, $y=\frac{bc^4e^{-\frac{ds^2}{c^2}}-bc^4+s^2c^2-s^4}{c^3s}$	Stable when $-bc^4e^{\frac{ds^2}{c}}+bc^4+s^2c^2-3s^4-2bds^2c^2(1-e^{\frac{-ds^2}{c^2}}) < 0$; Unstable when $-bc^4e^{\frac{ds^2}{c}}+bc^4+s^2c^2-3s^4-2bds^2c^2(1-e^{\frac{-ds^2}{c^2}}) > 0$;

MODELS AND STABILITY ANALYSIS

We will be comparing the results of the model with only the herding behavior with the model that has both herding behavior and a hunting quota. Given are the respective models:

Model 1:

$$\frac{dX}{dt} = rX(1-\frac{X}{N}) - \frac{\alpha\sqrt{X}Y}{1+t_h\alpha\sqrt{X}}$$

$$\frac{dY}{dt} = -sY + \frac{c\alpha\sqrt{X}Y}{1+t_h\alpha\sqrt{X}}$$

Model 2:

$$\frac{dX}{dt} = rX(1-\frac{X}{N}) - \frac{\alpha\sqrt{X}Y}{1+t_h\alpha\sqrt{X}} - B(1-e^{Dx})$$

$$\frac{dY}{dt} = -sY + \frac{c\alpha\sqrt{X}Y}{1+t_h\alpha\sqrt{X}}$$

These are the nondimensionalized forms of the respective models, after applying a handling rate of zero:

Model 1:

$$\frac{dx}{dt} = x(1-x) - \sqrt{x}y$$

$$\frac{dy}{dt} = -sy + c\sqrt{x}y$$

Model 2:

$$\frac{dx}{dt} = x(1-x) - \sqrt{x}y - b(1-e^{-dx})$$

$$\frac{dy}{dt} = -sy + c\sqrt{x}y$$

We analyze the stability of the steady states using the following Jacobian matrices:

Model 1:

$$\begin{bmatrix} 1-2x-\frac{y}{2\sqrt{x}} & -\sqrt{x} \\ \frac{cy}{2\sqrt{x}} & -s+c\sqrt{x} \end{bmatrix}$$

Model 2:

$$\begin{bmatrix} 1-2x-\frac{y}{2\sqrt{x}}-bd(1-e^{-dx}) & -\sqrt{x} \\ \frac{cy}{2\sqrt{x}} & -s+c\sqrt{x} \end{bmatrix}$$

We plug the equilibrium points into the Jacobian matrices to find the eigenvalues trace and determinant. For the prey-only steady states, we determine the range of parameters that would make the eigenvalues negative, as this is when the steady state would be stable. For the co-existence steady states, we determine the range of parameters that would make the trace negative, as this is when the steady state is stable. This is fine because we have already determined a transcritical bifurcation for Model 2.

CONCLUSION

The inclusion of a hunting quota has changed some of the behavior of Model 1. The existence and behavior of the extinct population steady state remains the same. The specific value of the prey-only steady state in Model 2 depends on the hunting quota parameter. This represents the fact a portion of the prey population will be hunted until a certain point set by the hunting quota. The behavior of the coexistence steady state of Model 2 remains to be studied, though the behavior is likely similar to the co-existence steady state of Model 1.

ACKNOWLEDGEMENTS

I would like to thank the University of North Florida Math Department for their support and Dr. Rahman for his academic guidance during this project.