A Comparison of Traditional and Conceptual Instruction on Students' Algorithmic Performance and Understanding of Area

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A COMPARISON OF TRADITIONAL AND CONCEPTUAL INSTRUCTION ON STUDENTS' ALGORITHMIC PERFORMANCE AND UNDERSTANDING OF AREA

by

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A thesis submitted to the Division of Curriculum and Instruction in partial fulfillment of the requirements for the degree of Master of Education

UNIVERSITY OF NORTH FLORIDA

COLLEGE OF EDUCATION AND HUMAN SERVICES

May, 1991
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Accepted for the University:
Acknowledgements

Achieving my personal goal of attaining a Masters degree could not have been possible without the love, support and encouragement of my husband, Michael, and our two sons, Brian and Jason. I thank them for the sacrifices they made, their patience and their belief in me.

I would like to thank Dr. Jan Bosnick for her continued support and guidance. She shared with me not only her knowledge and experience, but her enthusiasm for teaching. I would also like to thank Dr. Mary Grimes for her editing assistance and Dr. Robert Drummond for his statistical expertise. Last, but certainly not least, I would like to thank Joe Capitanio for his technical assistance and patience throughout the project.
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Abstract

This study investigated the effects of conceptual instruction on conceptual understanding and algorithmic performance as well as the student's ability to relate the two. The sample consisted of 83 fifth grade students, divided into four classes. A total of 44 were in the experimental group and 39 served as the control group. Both groups were taught the concept of area. The experimental group received conceptual instruction and the control group received traditional instruction. Two regular classroom teachers implemented the experiment, each taught one experimental group and one control group. A pretest/posttest design was used to collect the data. Analysis of covariance was the statistical analysis used to test the three null hypotheses with a significance level at <.05. Results indicated that conceptual instruction did improve the student's conceptual understanding but did not improve algorithmic performance. Also, no significant difference was found regarding the student's ability to relate the concept and the algorithm.
Chapter I

Introduction

By junior high school, students should understand elementary mathematical concepts sufficiently to apply them in new problem solving situations. However, more often than not, junior high teachers find themselves reteaching, not just reviewing, many of those basic elementary concepts. The time involved reteaching those concepts reduces the time available to teach that particular grade level's curriculum requirements. Thus the junior high teacher must reduce the time spent teaching the concepts for that grade level, thereby giving those students a weaker foundation for future study. As those students proceed to high school, the high school teachers often find themselves reteaching, not just reviewing, concepts from the junior high curriculum.

The cycle, unfortunately, can continue beyond the school setting. The business community is frustrated by graduates who are unprepared for the job market. Thus, the business community must spend time and money preparing these young people before they can become productive workers.

At the same time state and school administrators are pushing for increased achievement test scores. Despite emphasis on basic skills in the 1970's, Scholastic Aptitude
Test (SAT) scores declined steadily from 1964 through the early 80's (Schoenfeld, 1987). In addition, both the education and business communities are calling for students to develop more critical-thinking skills and problem-solving abilities.

In light of these needs and of the rapidly advancing technological society, several reports have surfaced detailing future directions for mathematics education. In the summer of 1987 twenty-six mathematics educators working on a project for the National Council of Teachers of Mathematics (NCTM), drafted a set of Curriculum and Evaluation Standards for School Mathematics (Standards). For all grade levels, the Standards call for improving problem solving and reasoning abilities as well as ways to represent mathematical ideas. Most importantly, the Standards stress placing greater emphasis on conceptual development (Thompson & Rathmell, 1988).

In January 1989 the National Research Council (NRC) released its report, Everybody Counts, on the future of mathematics education in the United States. It called for major changes in philosophy for the teaching of mathematics and for changes in the perception of mathematics by the public. The Council argued that changing curriculum content and instructional style by focusing on exploring patterns, searching for solutions and formulating conjectures would help students view mathematics as an evolving discipline
that involves patterns and relationships and not just numbers.

In order to initiate the proposals made by the NCTM and NRC, current instructional methods must be revised. A common belief of mathematics students is that mathematics is very mechanical: merely a set of rules, formulas and procedures to follow. Consequently this attitude does not encourage independent thinking or reasoning about mathematics (Garofalo, 1989). If students have developed this mechanical view of mathematics, then teachers must assume some of the responsibility for creating such thinking.

The proper classroom environment is necessary to develop these reasoning and thinking skills as well as healthy beliefs about mathematics (Garofalo, 1989). Quite often the current teaching strategy is to name a concept, explain a procedure, give a few examples, and provide a set of exercises for practice by the student. The lessons seem fragmented and may have no relevance to the student or have no relationship to prior knowledge. In addition, rote learning makes the application and retention of concepts difficult at best.

Concepts are basic subject matter. Through the attainment of a concept a student is able to deduce, classify, generalize, extend knowledge and communicate with others (Cooney, Davis & Henderson, 1983). A student must
understand "what it is" before that concept can be applied. In order for this to occur a greater amount of time must be devoted to developing a conceptual base, particularly for those concepts that are central to each curricular level.

As stated in the Standards of the NCTM, conceptual development must be emphasized. Although teachers will wholeheartedly agree, they argue that time does not allow for such in-depth presentations. Suppose, however, that extra time was devoted toward developing concepts. Could time then be saved doing application problems or reviewing or even in presenting subsequent lessons? More importantly, though, can the student be more successful doing algorithmic skills and application problems if a stronger conceptual foundation is provided?

Problem Statement

The question, then is whether instruction in and knowledge about concepts affect the development of algorithmic skills. Specifically, can a student with a stronger conceptual base proceed more competently and efficiently through algorithmic skills?

Purpose

The purpose of this study was to investigate the effects of conceptual instruction versus traditional instruction on learning concepts and the related algorithmic
skills. In particular, the study looked at whether students receiving conceptual instruction, which develops a particular concept in greater detail, can perform the related algorithmic skills better than students receiving traditional instruction. The study also investigated whether conceptual instruction can improve a student's understanding of the concept as well as understanding the relationship between the concept and the related algorithm.

Thus the null hypotheses tested were stated as follows:

**hypothesis 1:** There will be no significant difference between conceptually based instruction and traditional instruction on the student's ability to use related algorithmic skills.

**hypothesis 2:** There will be no significant difference between conceptually based instruction and traditional instruction on the student's understanding of the concept.

**hypothesis 3:** There will be no significant difference between conceptually based instruction and traditional instruction on the student's understanding of the relationship or connection between the concept and the algorithm.
Definition of Terms

**Algorithm**: A systematic sequenced procedure in mathematics used to calculate the answer to a problem.

**Bugs**: Consistent error patterns in procedure when computing the solution to a problem.

**Concept**: A fundamental idea of mathematics that is the basis for solving problems.

**Conceptual Instruction**: Instruction where the objective is to develop the concept prior to introducing any algorithm or procedure for calculating solutions. Instructional methods emphasize visual representation, exploring attributes of examples and non-examples, student participation and guided discovery techniques.

**Conceptual Understanding**: An ability to apply the concept in a problem solving situation as well as an ability to generalize beyond that concept and develop more ideas utilizing the prior concept.

**Experimental Treatment**: Conceptual instruction implemented to test the hypotheses.

**Guided Discovery**: An instructional approach that assists students through questions and activities to developing concepts, ideas or algorithms on their own.

**Procedural Understanding**: An ability to compute, follow a given formula or sequenced steps in order to calculate a solution to a problem.
**Traditional Instruction:** Instruction that begins by defining the concepts and then focuses on developing an understanding of the algorithm through the use of the definition and examples.

**Transfer of Knowledge:** Applying what has been learned to a new or unrelated situation.
Chapter II
Review of Related Literature

There is a consensus among educators that concepts are basic subject matter (Cooney, Davis & Henderson, 1983). In particular, understanding key concepts furthers the learning process. Through conceptual knowledge a foundation is built that leads to an ability to generalize and assimilate new knowledge. While a concept cannot be seen, examples of a concept can be visualized or physically represented (Cooney, Davis & Henderson, 1983; Hiebert & Lefevre, 1986; Skemp, 1973).

For a concept to be formed, the examples must have something in common (Skemp, 1973). Tennyson, Chao, and Youngers (1981) say that concept learning depends in part on the ability to generalize and discriminate the presence or absence of relevant attributes in examples and nonexamples. Concept learning is a twofold process which first involves acquiring some "prototype" or representation of the concept. The second part of this process requires contrasting the prototype with new examples to determine if the example parallels the prototype and thus supports the concept. Another strategy for concept learning, according to Mayer (1984), has the learner focus on the distinctive features
which characterize the concept. Depending on the learner and the situation, both strategies are effective (Mayer, 1984).

Clearly, conceptual knowledge or understanding involves knowledge based on relationships. When existing knowledge is linked with new knowledge the concept becomes broader. The conceptual base grows as new bits of knowledge are connected to earlier ideas (Hiebert & Wearne, 1986; Van de Walle, 1988). According to Putnam (1987) it is the linking of earlier concepts to new concepts that is important for developing the higher levels of mathematical literacy needed by our advancing technological society. Davis (1984) agrees that strong conceptual knowledge is key to understanding today's challenging problems.

Traditional classroom instruction in mathematics emphasizes the procedures or algorithms, with minimal time spent on conceptual development (Blais, 1988; Davis, 1984; Garofalo, 1989; Hiebert, 1984; Resnick, 1989). Thus, most students learn mathematics as a routine skill without developing understanding and reasoning abilities (Resnick, 1989; Blais, 1988). While procedural knowledge is important, students must see the limits of procedures and use the related concepts to understand, analyze and solve the problem (Carpenter, 1986; Nesher, 1986; Silver, 1986).

According to Hiebert and Lefevre (1986), conceptual knowledge and meaningful learning are interrelated. Facts
and rules are learned by rote and stored as bits of data in the memory. Procedures are composed of symbols and rules, or algorithms, used to solve mathematical tasks. Thus, procedures can be learned by rote and concepts require careful development. Procedural knowledge requires familiarity with symbols in mathematical expressions and the ability to use sequenced steps of a rule to arrive at an answer. Procedural knowledge, therefore, does not necessarily imply knowledge of meaning. On the other hand, "procedures that are learned with meaning are procedures that are linked to conceptual knowledge" (Hiebert & Lefevre, 1986, p.8). Hiebert and Lefevre (1986) argue that linking conceptual and procedural knowledge will enhance memory for procedures and improve and simplify the use of procedures. Silver (1986), Resnick (1989), Resnick and Omanson (1986), Blais (1988), and Hiebert and Wearne (1986) are just a few of the educators who agree upon the benefits of linking conceptual and procedural knowledge.

Silver (1986) examines factors which contribute to difficulty in analyzing word problems. Different variations of a single problem plus two additional, related problems were administered to a large sample of eighth-graders. The variations included some problems with the arithmetic worked out in detail and some problems without the work. Other variations contained explicit information about the problem while still others contained implicit information, that is,
information that is assumed without being stated. While the data could not account for all factors contributing to problem difficulty, they did suggest that lack of procedural knowledge may be due to a lack of understanding of the relationships among the symbolic expressions and the semantics of a given problem.

Zukor (cited in Nesher, 1986) tested whether students who had a better understanding in decimals would also perform the algorithm better. The subjects were average and honor students in grades 7, 8, and 9. They were given two tests one examining understanding and the other algorithmic performance. Zukor found a slight positive correlation between conceptual understanding and algorithmic performance for high-level students, but basically no correlation was found between understanding and algorithmic performance for the students in general.

Resnick and Omanson (1987) studied eighteen students in the fourth, fifth and sixth grades who had difficulty with the subtraction algorithm. The study was designed to show a relationship between understanding and performance on the subtraction algorithm. All of the students were given a pretest. Half of the group received mapping instruction which involved representing the problem with place-value blocks. The blocks were manipulated to represent each step of the algorithm and each step was recorded with paper and pencil. The student was slowly weaned from physical
representation to mental representation. The other half of the group was given prohibition instruction. The instructor pretended to be a robot who recorded each step of the algorithm as the student directed. When the student erred, the robot said it was not programmed to do that. The robot could only do correct procedures. If, after several attempts, the student did not know the correct procedure, the robot did the next step without assistance. Following the instruction students were given an immediate posttest and a delayed posttest.

Pretest and second posttest comparisons revealed that neither group showed any significant improvement in correcting "bugs", i.e., consistent error patterns in procedures. There was some temporary improvement on the immediate posttest, but the "bugs" reappeared or new ones were invented by the second posttest. Resnick and Omanson (1987) concluded that mapping instruction was somewhat more effective than prohibition in teaching understanding, however, not to the extent they had predicted.

To further investigate the potential of mapping instruction, Resnick and Omanson (1987) conducted a second study. The methodology and subjects were similar to the previous study, but some changes were made in instruction. Verbalization by the student was increased and students worked with the blocks, outside the context of subtraction, to emphasize the principle of conservation. While students
improved in their understanding of the underlying principles, their written performance did not improve. Thus, although mapping could not cure "buggy" procedures it did help conceptual understanding. Resnick and Omanson (1987) concluded that the design of the mapping instruction did not allow for students to reflect on the principles that justify the calculation procedures. Rather, students recorded the steps of their procedure without thinking about why the block representation and the writing were related.

Putnam (1987) was interested in students' conceptual understanding of the principle of sign-change rules before and after instruction in algebra. He studied 30 students each in grades 5, 7, and 9. Each student participated in a 40-minute interview where he or she judged the equivalence of story situations and equivalence of pairs of arithmetic expressions (e.g., a-(b+c) & a-b-c). Each student was asked to choose expressions to fit the story situation. Students at all grade levels were successful in judging the equivalence of story situations as well as justifying their decisions in informal terms. However, they were not successful when judging the equivalence of formal mathematical expressions (e.g., 16-(8+3) & 16-8-3). Even ninth-graders, who had received instruction in algebra, performed poorly. On the other hand, most students were able to map the formal expressions to the story situations as well as justify their decisions.
Putnam (1987) concluded that students do have intuitive understanding about how quantities behave in similar situations, but they do not draw on this informal knowledge when dealing with symbolic expression and manipulations. He suggests developing instructional methods to help students connect the formal and informal knowledge bases.

Some of the studies previously discussed were designed to correct existing "buggy" procedures by developing the related concept. These studies hypothesized that increasing conceptual understanding would correct the procedural errors and improve algorithmic skills. However, none of these studies were able to support such a hypothesis. Yet, educators intuitively feel increased conceptual understanding will, in fact, improve algorithmic skills. (Maurer, 1987; Nesher, 1986; Skemp, 1973). Perhaps consideration should be given to the timing of conceptual development. Thus, rather than trying to correct an already "buggy" procedure, the emphasis on concepts should come before the procedures are taught and the "bugs" established (Resnick & Omanson, 1987).

Because a student learns the procedures, it cannot be assumed that the related conceptual knowledge has also been learned (Carpenter, 1986). Students can proceed routinely through skills without relating them to some conceptual base (VanLehn, 1986). On the other hand, Resnick and Omanson (1987) found that understanding concepts did not necessarily
improve procedures. Likewise, Zukor's study (cited in Nesher, 1986) did not show a correlation between conceptual understanding and algorithmic performance. However, Zukor's study (cited in Nesher, 1986) merely tried to correlate test scores without examining the effects instruction may have had on students' abilities to relate concepts and algorithms. While Resnick and Omanson (1987) used two types of instruction to develop conceptual understanding, they did so in an attempt to remediate procedural difficulties the students already had acquired.

As Putnam (1987) recommends, perhaps the instructional methods used when concepts are introduced should be examined more closely. The goal of instruction should be to establish connections between conceptual and procedural knowledge. Establishing connections is important for transferring understanding to new learning situations as well as for problem solving tasks (Putnam, 1987). But designing instruction to develop these connections is not an easy task (Carpenter, 1986). According to Lovell (1968) concepts do not develop suddenly. Rather, they evolve over time through reflections and experience. Thus the process of concept formation is different from that for learning facts and details (Lovell, 1968). Therefore the type of instruction and the classroom environment are two important factors to consider when trying to develop concepts learned for the first time (Carpenter, 1986; Davis, 1984; Garofalo,
The constructivist theory argues that, to understand the "essence" of a topic, learners must develop and discover the knowledge for themselves (Blais, 1988). Knowledge cannot simply be transmitted. Students must construct ideas and restructure thinking based on their experiences and their environment (Blais, 1988; Dewey, 1964; Piaget, 1973).

This constructivist theory has support from two very notable educators, John Dewey and Jean Piaget (Dewey, 1964; Donaldson, 1978; Piaget, 1973). John Dewey (1964) believed that learning is an active process. Knowledge cannot be given but must be experienced. If always provided with logically structured material, a child loses the opportunity to think. For Piaget, knowledge develops through discovery, manipulating things or acting on objects (Donaldson, 1978). It is through assimilating and accommodating experiences that students construct, restructure, modify and interpret ideas and concepts, thereby building greater cognitive understanding (Donaldson, 1978; Lovell, 1968; National Research Council [NRC], 1989). Furthermore, according to the NRC (1989) "students retain best the mathematics that they learn by processes of internal construction" (p.59).

In addition to Dewey and Piaget, Jerome Bruner (1966) says instruction designed to encourage independent thought and action is critical to learning. He agrees, further, that it is necessary to have the learner participate in the
process of gathering knowledge in order to develop problem solving abilities (Bruner, 1966; NRC, 1989). Therefore the classroom environment should be structured so that presentations encourage reflection, discussion, discovery and critical reasoning. In addition, the mathematics teacher should be the facilitator and discussions leader rather than the dispenser of information (Ausubel, cited in Joyce & Weil, 1972; Garofalo, 1989; Resnick, 1989). All confirm that the student should be an active participant (Bruner, 1966; Dewey, 1964; Garofalo, 1989; Putman, 1987; Resnick, 1989).

Research indicates the importance of several components to include when designing instruction to develop concepts. A student's previous knowledge (Davis, 1984; Gagne, 1977; Howard, 1987); the use of representations (Davis, 1984; Heibert, 1984; Putnam, 1987); a student's ability to verbalize during instruction (Blais, 1988; Resnick & Omanson, 1987); and the teacher's knowledge of students' potential error patterns (Glaser, 1979; Maurer, 1987; Schoenfeld, 1987; Silver, 1986) are four components to consider when designing instruction for concept development. The student's previously acquired information or existing conceptual framework must be considered when introducing new material (Davis, 1984; Gagne, 1977; Howard, 1987; Trafton, 1984). An individual's existing framework will affect how the new material is interpreted and comprehended (Putnam,
Learning new information is easier and more meaningful when it can be linked to prior knowledge (Gagne, 1977; Howard, 1987; Martinez, 1988; Trafton, 1984). Then as a new concept is acquired it becomes an aid to learning yet another concept. Thus the learner continues to structure a conceptual foundation that can be used later to analyze and solve problems and to broaden the existing conceptual framework (Fehr, 1968; Howard, 1987). Since mathematics is a highly structured cohesive discipline based on logic and rich in relations and patterns, instruction that connects and relates new ideas and concepts to already learned ones will enable students to view mathematics as a unified body of knowledge rather than a set of fragmented ideas (Trafton, 1984).

The use of a variety of concrete representations or models helps in providing meaning for the many abstract mathematical concepts (Davis, 1984; Hiebert, 1984; Putnam, 1987; Trafton, 1984). From these concrete representations it is hoped the student will construct mental or cognitive representations to transfer to new learning situations (Putnam, 1987). Resnick and Omanson (1987) showed that the use of place value blocks increased the understanding of subtraction concepts. The Montessori method of using concrete materials to teach number concepts to preschool children has been effective in accelerating the development of their seriation and classification skills (Bauch and Hsu,
Davis (1984) says that a specific problem will be easier to solve with representations appropriate for the problem. A wide variety of situations in which to practice a skill or visualize a concept helps students transfer that learning to new situations (Gagne, 1977) as well as helps them retain the material (Howard, 1987).

Encouraging students to verbalize their thought processes develops their reasoning skills and provides the teacher a way to determine whether the concept has been mastered. Resnick and Omanson (1987) determined that verbalization by the student was important for transferring understanding from concrete models to written work. Martinez (1988) says that verbalizing mental processes helps establish parallels between the basic and more complex applications of concepts. Kamii (1982) believes that students would develop more quickly in mathematics and in their ability to think more logically if they were encouraged to exchange ideas and have small group discussions. Further, by responding to a question with a question, students are given the opportunity to reason for themselves and think mathematically (Blais, 1988).

Another factor to consider in the instructional design is understanding students' error patterns (Maurer, 1987). As was seen in Resnick and Omanson's study (1987), trying to correct existing "buggy" procedures is difficult. Silver (1986) suggests that when dealing with procedural "bugs" it
may be necessary to look not only at the conceptual basis for the error but also at the fact that the instructional design may be reinforcing the error. Maurer (1987) argues that teachers need to be familiar beforehand with potential error patterns, particularly with the most common types. Examples and non-examples of a concept can be carefully chosen to help students avoid errors. Being aware of the nature of student errors can make it easier for teachers to prevent them. In addition, the ability to analyze errors will enable teachers to provide proper remediation for the different types of errors (Glaser, 1979; Schoenfeld, 1987).

Summary

Research has been mixed regarding the effects of conceptual development on algorithmic performance. While some studies tried to establish a correlation between a student's procedural skills and conceptual understanding, other studies used conceptual development as a means to correct poor procedural performance. Still, none of the studies specifically demonstrated a positive relationship between increased conceptual understanding and improved algorithmic skills. However, educators continue to agree that conceptual development is a necessary part of learning mathematics. Consequently, mathematics instruction should be designed to build a strong conceptual foundation in order to make procedural learning more meaningful.
Research showed that instruction designed to teach concepts needs to contain some specific components. To teach concepts more effectively, manipulatives and visual representations should be part of the instructional design. More student verbalization, which helps students reason better and transfer knowledge from one form to another, is considered another important component for the instructional design. Relating the current lesson to the student's past knowledge base and anticipating future error patterns are the two other necessary components research indicated should be considered.

Since there is such a strong feeling among educators to direct mathematics education toward more conceptual knowledge, researchers should continue to examine the effects of specific concept instruction. In particular, researchers should look further at the effects of using conceptual instruction, not only for remedial purposes, but when the concept is introduced into the curriculum.
Chapter III
Procedures and Methodology

The purpose of this study was to investigate the effects of conceptual instruction versus traditional instruction on learning a concept and its related algorithmic skills. The concept used for the purpose of this study was area. Area is a concept that is confusing for many students, particularly in relation to that of perimeter and volume. Students seem unsure when to apply the appropriate concept. Although area is typically introduced at the fifth-grade level, the concept of area is one that reappears in the curriculum of subsequent grade levels all the way through high school. Because it is applied in increasingly complex situations throughout the mathematics curriculum, an early, thorough understanding of the concept is necessary and important.

Sample

The subjects were eighty-three fifth graders enrolled in one of the four mathematics classes used for this study. The subjects were students at Alimacani Elementary School, a new, urban public school in Jacksonville, Duval County, Florida, with a total enrollment of almost 1100. The students live in the neighboring suburban areas representing
low to high socio-economic groups, including a large military population. The fifth grade level was chosen since the concept, area, is introduced at that level and there should be few, if any, prior misconceptions regarding the concept that might affect the study. Students represented all ability levels, with the only prerequisite being the ability to add, subtract, multiply and divide with whole numbers (the set of numbers 0,1,2,3,...). The use of whole numbers kept the concept pure and simple, free from complications brought on by the use of fraction and decimal numbers. Students' attainment of prerequisite skills was determined by their teachers.

Two fifth grade teachers from Alimacani Elementary School implemented the experiment. Regular classroom teachers were used to minimize the effect of an experimenter. Two teachers were used to help control the validity of the experimental treatment. The instructional sessions were conducted in the regular classroom setting on the school campus. Each teacher taught two of their regular classes with one class being the experimental group and the other class serving as the control group. The teachers decided which of their two classes would receive the treatment and which one would be the control group. The two cooperating teachers received no specific training, but they were given specific lesson plans. Brief discussions and review of the purpose of the project and lesson plans
occurred three times before school. A copy of the lesson plans was given to each of them at the first meeting. The pretests were delivered at the second meeting and further discussions were held. The day before the treatment began another meeting was held to deliver and explain the materials to be used for the treatment. This gave them some time to familiarize themselves with all the materials and the sequence in which to use them.

Instrumentation

A pretest/posttest design was used to evaluate the study. The pretest and posttest were constructed by the researcher (See Appendix A). The items on both tests were similar in nature, as was the length of both tests. Items were a combination of short answers, true/false, simple computation and word problems. Test items were developed based on specific questions the researcher asked in relation to each null hypothesis being tested. These questions more clearly define the parameters of the hypotheses. The hypotheses and questions are stated as follows:

Hypothesis 1: There will be no significant difference between conceptually based instruction and traditional instruction on the students's ability to use related algorithmic skills.

1. Can the student state the formula necessary to perform the related algorithm?

2. Can the student perform the related algorithmic skills?
**Hypothesis 2:** There will be no significant difference between conceptually based instruction and traditional instruction on the student's understanding of the concept.

3. Can the student recognize examples and/or non-examples of the concept?

4. Can the student generate an example of the concept?

5. Can the student explain the concept in their own words?

**Hypothesis 3:** There will be no significant difference between conceptually based instruction and traditional instruction on the student's understanding of the relationship or connection between the concept and the algorithm.

6. Can the student recognize the concept and apply the algorithm to solve a problem?

**Treatment**

Two weeks prior to implementation of the experiment, all subjects were administered the pretest with no explanation other than to do their best to answer all questions. To implement the study both teachers were given detailed lesson plans to follow for each group: control and experimental (See Appendix B). All manipulatives, visual representations, worksheets and teaching aids were also provided, except for rulers which the classroom teachers provided.

The day following the final instruction the same posttest was administered to all subjects. All groups did
not receive the posttest on the same day because instruction for the experimental groups lasted one day longer than the control group.

The sessions were voice-recorded to allow the researcher the opportunity to analyze teacher/student interactions at a later time. In addition, the researcher observed classroom instruction in both the experimental and control groups of one instructor. Time and scheduling did not allow for observations to be made in the other instructor's groups.

The two control groups received traditional instruction. The lesson was expository in nature rather than developmental. The traditional instruction utilized Mathematics Today, the county adopted text currently used in the schools, and paralleled closely the sequence in the text. While concrete representations were incorporated in the instruction, the purpose was to support the information given, rather than develop the concept. Although the concept was presented before the algorithm, the time spent developing and emphasizing an understanding of the concept was less than that spent in the experimental treatment. However, more time was available to practice the algorithm. The control group received two separate assignments that involved working with the algorithm and computing solutions. One assignment was from the text and the other was a worksheet designed by the researcher (See Appendix D).
The two experimental treatment groups received conceptual instruction. The conceptual lesson plan was divided into two lessons (See Appendix B). The first lesson developed the concept and the second lesson developed the algorithm. Instructional methods for both lessons incorporated visual representations, attributes of examples and non-examples of the concept, student participation and guided discovery techniques. In addition, student's verbalization was encouraged for two reasons. First, verbalization allowed the student to be a participant in the learning process rather than just a recipient of data; second, verbalization provided a means to help assess the student's understanding of the concept and/or algorithm. In lesson one the teachers began by displaying and describing examples and non-examples of the concept and asked students to list attributes of the item. As the lesson proceeded the common attributes of the examples of area were evident and could then be stated as a framework for the definition. Once the framework was established the teachers then told the students the concept they defined was that of area. The lesson continued with the students deciding whether an item described and/or displayed by the teacher was an example or non-example and why it was. Students were assigned a worksheet (See Appendix C) describing particular problem situations. They had to determine whether or not the situation was an area problem.
Lesson two reviewed the attributes of area and proceeded to develop the algorithm through the use of visual aides. Students used graph paper, polygon shapes, and rulers to copy or draw shapes and then counted the number of square units in the interior of those shapes. From this exercise they then developed the algorithm for area, i.e. area = length \times width. One assignment was given that involved practicing the use of the algorithm.

Data Collection and Analysis Procedures

A pretest/posttest design was used to evaluate this study. The same pretest was administered to all subjects in order to establish some degree of equivalence among them. Although the subjects were not ability grouped, they were intact classes, so pretesting helped check for similarities among the groups. Following the treatment, the same posttest was administered to all subjects to determine the level of concept acquisition and performance on algorithmic tasks. The pretest and posttest scores of 83 subjects were analyzed, 44 of them from the experimental group and 39 from the control group. Pretest and posttest scores of only those subjects who were present each day of the treatment were considered for the analysis.

Each pretest and posttest was scored in the same manner. Test items were first categorized by hypothesis (See Tables 4.1a, 4.2a, 4.3a in Chapter IV). The correct number of items out of the total number for the particular
hypothesis was used as the raw data for the analysis. Since random assignment of students to classes was not possible, the statistical analysis used was analysis of covariance (ANCOVA). When randomization is not done analysis of covariance checks for possible effects of differing academic abilities of the groups that may affect the outcome. If a difference appeared between the experimental and control groups it might be due to different academic abilities rather than the treatment. Analysis of covariance adjusts the dependent variable scores, in this case the posttest scores, based on the correlation of those scores with some other variable called the covariate, in this case the pretest scores. Adjustments are made so that the posttest scores are independent of the influence of any confounding variable. The means of the posttest scores are equated with the means of the pretest scores. A ratio of two variances is computed and the sampling distribution used is the F-distribution. Conclusions and inferences then were made to the adjusted posttest scores (Wiersma, 1986).
Chapter IV  
Presentation of Data

Each hypothesis is restated below and the results of the data gathering procedures presented. The pretest, posttest and adjusted means for each null hypothesis are reported in Tables 4.1, 4.2, 4.3, respectively, where M represents the mean score and SD the standard deviation. Each hypothesis corresponded to particular test items in both the pretest and posttest. Tables 4.1a, 4.2a, 4.3a list the numbers of the pretest/posttest items that were used to test each hypothesis, respectively.

Hypothesis 1: There will be no significant difference between conceptually based instruction and traditional instruction on the student's ability to use related algorithmic skills.

Table 4.1 Analysis Results for Hypothesis 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Control</td>
<td>.79</td>
<td>1.00</td>
<td>9.26</td>
</tr>
<tr>
<td>Experimental</td>
<td>.50</td>
<td>0.84</td>
<td>5.50</td>
</tr>
</tbody>
</table>
The experimental group had an adjusted mean score of 5.45 and the control group had an adjusted mean score of 9.32. An F of 33.63 was computed and with 1 and 80 degrees of freedom was found to be significant at the .001 level. Thus, a significant difference was found between conceptual instruction versus traditional instruction on a student's ability to use related algorithmic skills. However, it was the control group that had the higher adjusted mean score and consequently performed better than the experimental group on those test items relating to the first hypothesis. Therefore, the data revealed the first null hypothesis was not supported.

Table 4.1a Test Item Numbers for Hypothesis 1

<table>
<thead>
<tr>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.1,2</td>
<td>II.3</td>
</tr>
<tr>
<td>II.1,2,3</td>
<td>III.1,2,3</td>
</tr>
<tr>
<td>III.2</td>
<td>IV.1,2,3</td>
</tr>
<tr>
<td>IV.1,2,3</td>
<td>V.1,2,3,4</td>
</tr>
</tbody>
</table>

Hypothesis 2: There will be no significant difference between conceptually based instruction and traditional instructional on the student's understanding of the concept.
Table 4.2 ANCOVA Results for Hypothesis 2

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Control</td>
<td>2.51</td>
<td>1.23</td>
<td>4.36</td>
</tr>
<tr>
<td>Experimental</td>
<td>2.73</td>
<td>1.37</td>
<td>5.80</td>
</tr>
</tbody>
</table>

The experimental group had an adjusted mean score of 5.77 and the control group had an adjusted mean score of 4.39. An F of 30.19 was computed and with 1 and 80 degrees of freedom was found to be significant at the .001 level. The experimental group had a significantly higher adjusted mean than the control group on those test items relating to the second hypothesis. That is, a significant difference was found between the two instructional groups on the student’s understanding of the concept. Consequently, the data indicated the second null hypothesis was not supported.

Table 4.2a Test Item Numbers for Hypothesis 2

<table>
<thead>
<tr>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>III.1,2</td>
<td>I.1,2,3,4,5</td>
</tr>
<tr>
<td>V.1,2,3,4</td>
<td>II.1,2</td>
</tr>
</tbody>
</table>

Hypothesis 3: There will be no significant difference between conceptually based instruction and traditional instruction on the students understanding of the relationship or connection between the concept and the
algorithm.

Table 4.3 ANCOVA Results for Hypothesis 3

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Control</td>
<td>0.59</td>
<td>1.60</td>
<td>2.08</td>
</tr>
<tr>
<td>Experimental</td>
<td>0.31</td>
<td>0.64</td>
<td>1.65</td>
</tr>
</tbody>
</table>

The experimental group had a adjusted mean score of 1.68 and the control group had an adjusted mean score of 2.06. An F of 2.6 was computed. With 1 and 80 degrees of freedom, the significance level of the F was .11. To be significant the level must be .05 or less, therefore this F was not significant. Thus, the third null hypothesis was supported and no significant difference was found between the two instructional groups in their understanding of the relationship between the concept and the algorithm.

Table 4.3a Test Item Numbers for Hypothesis 3

<table>
<thead>
<tr>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI.1,2,3,4</td>
<td>VI.1,2,3,4</td>
</tr>
</tbody>
</table>
Chapter V
Conclusions

Summary
As was discussed in the review of related literature, previous research and reports indicate the importance of developing mathematical concepts prior to teaching algorithms and/or procedures. Although some studies did not demonstrate directly the effects of conceptual knowledge, educators such as Maurer (1987), Nesher (1986), Resnick and Omanson (1987) and Skemp (1973) still feel that increased conceptual understanding will improve algorithmic performance. The research also indicates that instructional methods should be developed to foster concept formation and establish links between concepts and procedures.

The purpose of this study was to see whether conceptually based instruction would improve a student's understanding of the concept as well as improve the ability to perform the related algorithmic skills. It was also hypothesized that the relationship between the concept and algorithm would be more clearly understood by the student receiving conceptual instruction rather than by those receiving traditional instruction.
Discussion of Results

The results of this study are varied. The first null hypothesis was not supported, but the results were contrary to what had been anticipated. Students in the control group performed better on the related algorithmic skills than the students in the experimental group. These results raise some questions about the treatment. The lesson used in the treatment followed a developmental approach, whereas the lesson for the control group was a directed method. Consequently, the control group's lesson took less classroom time and focused on one objective--learning to use the algorithm. To account for the available time, an additional assignment (See Appendix D) was given to the control group. This assignment involved repetitive practice in using the algorithm, thereby giving a possible unintended advantage to the control group. Since this assignment was not given to the experimental group, it may have been a factor in the final results. If the study were repeated, this assignment should either be removed from the traditional lesson or placed in the conceptual lesson for a more accurate assessment of the effects of the conceptual lesson.

The second null hypothesis was rejected, and test results showed conceptual instruction does indeed improve a student's understanding of the concept. The treatment in this case had a positive effect on the student's conceptual understanding. Although the control group was better able
to perform the algorithm, as was shown in the test results of the first hypothesis, procedural knowledge did not imply conceptual understanding in this case. This result supports the research of noted educators who state that procedural knowledge does not necessarily involve understanding (Hiebert & Lefevre, 1986; Resnick, 1986; Silver, 1986). The results of the second null hypothesis demonstrate that conceptual instruction is a necessary ingredient for developing conceptual understanding.

The third null hypothesis was supported, since test results showed no significant difference existed between the two groups. The treatment did not improve a student's understanding of the relationship between the concept and the related algorithm. Results of the first hypothesis may have had some residual effects on these results. Since the experimental group did not perform as well on the algorithmic skills, their ability to relate the concept to the algorithm also may have been impaired.

Limitations

Lack of control over external factors was a problem. In all classes there were continuous interruptions consisting of: students entering late, leaving early or leaving to retrieve a book, other teachers breaking in with messages or deliveries, or the class leaving for lunch before the lesson was completed. These interruptions seemed
more of a problem for the experimental groups than the control groups. Since conceptual instruction was a new process for both students and teachers, the disruptions were more frustrating than normal, particularly for the teachers.

Overall, teachers as well as students were unfamiliar and seemed uncomfortable with the conceptual instruction process. Teachers had difficulty trying to elicit the necessary attributes of the concept. For this study, the teachers' schedule did not allow for specific training. In the future, teachers who are unfamiliar with teaching conceptually should be given some instruction on how to present such a lesson. At times, the teachers were unsure of how to phrase questions without telling the student the concept, and consequently the students were unsure of how to respond. One observation was the students' readiness to supply a numerical value when asked to describe the surface of an item. In mathematics classes, students are used to computing answers and therefore are unfamiliar with expressing ideas that do not involve numerical values.

Time was another limitation to the study. The conceptual lesson is slightly longer by its developmental nature. But because the teachers and students were unfamiliar with the process the lesson took even longer than expected. A decrease in time spent teaching would be another advantage to having the teachers better prepared beforehand.
Conclusions and Implications

The results of the first null hypothesis show that conceptual instruction did not improve algorithmic skills. Again, these results may be skewed due to the additional worksheet on the algorithmic process given to the control group and not to the experimental group. To conclude that conceptual instruction cannot improve algorithmic skills would not be prudent. Further study is necessary in this case.

The results of the second null hypothesis support the positive effects of conceptual instruction. Specific instruction in concepts is necessary to develop an understanding of the concept. These results indicate it is equally important to teach the concept as well as the algorithm. If a goal of educators is to increase conceptual understanding, then instruction in concepts must be stressed.

Results of the third null hypothesis show that no difference existed between conceptual instruction and traditional instruction. Conceptual instruction did not make a difference in a student's understanding of the relationship between concept and algorithm. Therefore no conclusion can be drawn. However, this is not to say that conceptual instruction is without merit. Continued emphasis must be placed on linking conceptual knowledge and algorithmic skills.
While it was clear that more time is needed to present a lesson conceptually, this should not be a discouraging factor for future implementation. As instructors and students become more comfortable with teaching and learning conceptually, time will not be as critical a factor. Also, extra time spent on the front end developing the concept may have long term positive effects. This could be a possibility for further study.
Appendix A
PRETEST-AREA

I. Find the area. Write your answer on the space below the figure. Each little square is one centimeter square.

1. ________

2. ________

II. Find the area. Write your answer on the space below the figure.

1. ________

2. ________

3. ________

III. 1. In your own words write the definition for AREA.

2. Write the formula for finding the AREA of a rectangle or square.

3. Give an example of a problem where you would need to find the AREA.

IV. Find the area of:

1. A rectangle with:
   length = 6 ft. and width = 5 ft. ____________
2. A square with:
   length = 12 in. and width = 12 in. ____________

3. A rectangle with:
   length = 15 cm. and width = 8 cm. ____________

V. True or False. Write T for true if the sentence describes an example of a problem that can be solved by finding the AREA, and F for false if it does not.

1. The boy scout troop found a small clearing in the woods to set up camp. The tent floor covers 80 square feet. They need to know if there is enough ground space to set up the tent.

2. The scouts also want to hang up a rope between two trees for a clothesline. They need to know the distance between the two trees.

3. For the troop project this year, the scouts are building a playhouse for a local day care center. They want to know how much wood is needed to make the frame of the playhouse.

4. When the playhouse is finished they plan to carpet the floor so it will be comfortable for the children. They want to know how much carpet is needed to cover the floor of the playhouse.

VI. Solve these problems. Show all of your work.

1. Mrs. Kirk want to cover the top of her ugly desk with a pretty Contact Paper. Her desk measures 40 inches long and 28 inches wide. How many square inches of Contact Paper does Mrs. Kirk need to cover her desk?
2. Jake wants to paint the floor of his treehouse with one quart of blue paint. One quart of paint is enough to paint 50 square feet. The floor of the treehouse measures 6 feet long and 5 feet wide. Does Jake have enough paint? Explain your answer.

3. The Smith family built a new family room and they want to carpet the floor. The floor measures 6 yards long and 5 yards wide. How many square yards of carpet should they buy?

4. Find the area of the figure. Show all of your work.

```
23 ft.
11 ft.   8 ft.
       3 ft
15 ft.
```
I. True or False. Write True if the sentence describes an example of a problem that can be solved by finding the area and False if it does not.

1. Rose wants to tie a yellow ribbon around a tree in her front yard to honor the soldiers in the Middle East? She needs to know how big around the tree is in order to buy enough yellow ribbon.

2. Jeffrey and his grandfather will be planting a garden. They need to know how much space the garden will cover in the back yard because someday they also want to put in a swimming pool.

3. To warm up for the big game, the soccer team has to run around the track for a distance of one mile. They want to determine the number of laps they would have to run.

4. The neighborhood kids want to hang a rope from the treehouse to use as a quick escape. They need to know the distance from the treehouse to the ground.

5. The floor of the treehouse is very uncomfortable to sit on and play, so the kids want to carpet it. They need to know how much carpet to buy to cover the floor.

II. Name the concept.

1. In your own words write a definition for AREA.

2. Give an example of a problem where you would need to find the AREA.
3. Write the formula for finding the AREA of a rectangle or square.

III. Find the area. Write your answer on the space below the figure. Each little square is one centimeter square.

1. 2. 3.

IV. Find the area. Calculate the area of the figures below and write your answer on the space provided.

1. \[ \text{14 in.} \times \text{6 in.} \]

2. \[ \text{12 yd.} \times \text{12 yd.} \]

3. \[ \text{16 cm.} \times \text{2 cm.} \]

V. Find the area. Calculate the area and write your answer on the space provided.

1. A rectangle with:
   \[ L = 18 \text{ in.} \] and \[ W = 5 \text{ in.} \]

2. A rectangle with:
   \[ L = 12 \text{ cm.} \] and \[ W = 11 \text{ cm.} \]

3. A square with:
   \[ L = 14 \text{ ft.} \] and \[ W = 14 \text{ ft.} \]
4. A square with:
   \( L = 20 \text{ m.} \) and \( W = 20 \text{ m.} \)

VI. Solve these problems. Show all of your work.

1. The art class is going to make a picture of the school using one inch square tiles. But first they must buy the tiles. The picture is going to be 42 inches long and 25 inches wide. How many square inches of tile do they need to buy?

2. You get to wallpaper one wall of your room. The wall measures 8 feet wide and 11 feet long. How many square feet of wallpaper do you need to cover the wall?

3. Joey received a large train set for his birthday. The directions say that a space of 21 square feet is needed to set up the whole train set. Joey found a board in his garage that measures 4 feet wide and 6 feet long. Is there enough space on the board for Joey to set up his new train set? Explain your answer.

4. Find the area of the figure. Show all of your work please.

```
13 \text{ ft.}

7 \text{ ft.}

\[ \begin{array}{c}
5 \text{ ft.} \\
3 \text{ ft.}
\end{array} \]

\[ \begin{array}{c}
5 \text{ ft.} \\
3 \text{ ft.}
\end{array} \]

7 \text{ ft.}

3 \text{ ft.}
```
Appendix B
Traditional instruction is defined, for this study, to be instruction which begins by stating the definition of the concept, in this case, area. Through the use of some basic examples and visuals, instruction moves directly into computing the area by counting squares and then develops computing the area by using the appropriate formula. The instruction follows the procedure of the classroom text.

**LESSON ONE--AREA OF RECTANGLES**

**Step 1. Presenting the Concept by Definition**

Give definition as written in text book (p.250), and write it on the board. "The number of square units that cover a surface is the area of the surface."

Explain: 1. that a unit can be any measurement such as inches, meters, centimeters, feet, yards, etc.
2. Area has 2 dimensions, length (how long the figure is) and width (how wide the figure is).

**Step 2. Activity**

Give students a sheet of graph paper and ruler. Ask students to draw a rectangle 5 units across (length) and 4 units down (width). Have students begin at the top left of the paper since they will be drawing several figures. Now ask students to count the number of squares inside the rectangle. WHILE STUDENTS ARE DOING THE
ACTIVITY, TEACHER SHOULD ALSO DO THE ACTIVITY ON THE OVERHEAD TRANSPARENCY.

Step 3. Questions and Explanations
T. "How many squares did you count inside the rectangle?" Elicit student response of 20 squares.
T. Repeat the answer to the class. Explain: Because there are 20 squares we say the rectangle has an area of 20 square units. Ask students to place the answer with the proper label below the rectangle. Explain: if each little square is one foot on all four sides then we say the rectangle has an area of 20 square feet. Repeat explanation using centimeters and inches. Ask for questions from students at this point.

Step 4. Activity
On the same graph paper ask students to draw a square that is 6 units long and 6 units wide. Now ask students to count the number of squares inside the 6x6 square. (Again, teacher should do this activity on the overhead transparency)

Step 5. Questions and Explanations
T. "How many little squares did you count inside the larger square?"
Elicit student response of 36 squares.
T. Repeat the answer to the class. Explain: We say the
large square has an area of 36 square units. Ask students to place the answer with proper label below the square. If the little squares are one inch on all four sides we say the area is 36 square inches instead of square units. ("Units" is a generic term used when we don't know what the measurement unit is).

Step 6. Activity
On the same graph paper ask students to draw a rectangle 3 units long and 8 units wide. Now ask students to count the number of squares inside the 3x8 rectangle. (Again, teacher should do this activity on the overhead transparency).

Step 7. Questions and Explanations
T. "How many squares did you count inside the rectangle?"
Elicit student response of 24 squares.
T. Repeat the answer to the class. Explain: the rectangle has an area of 24 square units. Ask students to place the answer with proper label below the rectangle. If the little squares are one foot on all four side we say the area is 24 square feet. The word "units" is a generic term used when we don't know what the measurement unit is."
T. Tell students there is an easier or faster way to find the area instead of counting. Ask if anyone has an
idea for a faster way of finding the area of rectangles and squares?"

(wait for student response--If they need a hint ask them to look for a relationship between the dimensions and the answer)

Elicit student response of: multiply length times the width.

T. Put the formula, \( A=L \times W \), on the board and explain what the letters represent.

Step 8. **Activity**

Ask students to turn to page 250 in their book and find the area of #1 by counting squares. Have volunteers respond orally and then repeat the answer to the class. Repeat with #2 and #3.

T. "For #4 since there are no squares to count we will have to use the formula on the board, \( A=L \times W \), to find the answer." Ask for volunteers to supply the answer and remind them to label their answers properly i.e. "square _____"

Step 9. **Explanation**

This would be a good time to show the students how to abbreviate "square _____" as "sq.cm." or "sq.in." for example. Also ask for any questions by the students at this time.

Step 10. **Activity**

Refer to #17 on p. 251. Since there is no figure drawn
for that problem ask students to draw a rectangle with those dimensions on that same piece of graph paper. The little squares are one centimeter on all four sides--find the area of the rectangle.

Ask a student for the answer. Repeat the answer and have students place the answer below the figure.

Step 11. Explanation

Discuss real experiences where it would be important to know the area. Examples: carpeting a floor that is 10' x 8'; sodding a yard that is 25' x 63'; wallpapering a wall that is 8' x 12';

Step 12. Activity

Have students find the area of the classroom floor, the chalkboard, and the door. This could be done by allowing small groups to work together or as individuals, whichever works best for the teacher. Teacher should discuss the dimensions as well as the answer with the entire class.

Homework assignment: Text book, Mathematics Today, p.251 #7-11, #18-21, #27-28

**DAY 2:** Go over homework, answer questions, review, and give the practice worksheet to do in class.

**DAY 3:** Correct worksheet. Administer Posttest.
Conceptual instruction is defined, for this study, to be instruction that begins by developing the concept through a guided discovery method. Exploring attributes of examples and non-examples and the use of visual representations are techniques that will be used to develop the concept. There is no discussion of how to find the area only discussion of what the area is. Once students have discovered the concept, can formulate a definition, recognize the concept and be able to generate an example of the concept, instruction will begin on how to calculate the area.

LESSON ONE--DEVELOPING THE CONCEPT OF AREA

Step 1. Presenting Examples and Non-Examples

Tell students that today they will be detectives. You intend to show them some examples and non-examples of today's "mystery topic". As each example and non-example is shown, students are to describe it and you will list the attributes (clues) on the board under the appropriate column, "Examples" and "Non-Examples". Then, as detectives they will try to figure out from the clues what the "mystery topic" is all about.

Use the list in the order given. Tell them whether
the item is an example or non-example before beginning. Recite the statement and show the associated visual. Point to what you are describing on the visual. As students describe the item, write on the board the attribute they use in the appropriate category. Do not rewrite an attribute that is already listed.

Attributes of Area:

1. flat surface, if students are familiar with a plane one can also say a surface that lies in a plane.
2. shape doesn't matter, i.e. the shape can be circle, triangle, square or even an irregular shaped figure.
3. looking for the amount of space that covers the inside of the flat surface as opposed to the distance around the surface or the amount of space within a 3-dimensional shape.

[Some possible attributes the students may suggest are: color, shape, size, texture, flat, not-flat, etc. You may need to prompt them to get them started. If the words: "amount", "number", "quantity", or something along that line does not come up, remind them that each item begins with "how much" or "how far", etc.. Ask them "what do the words "how much" imply
or what are we looking for when we ask "how much" etc.?

List of examples and non-examples  (E)=example  (N)=non-example  (*)=indicates there is an associated visual

(E) * 1. The amount of surface space on this RECTANGLE.

(N) * 2. The number of jelly beans that can fill this jar.

(N) * 3. How much water is needed to fill this balloon to make a water balloon. (inflate partially to demonstrate it is not flat).

(E) * 4. How many slices of cheese are needed to cover a 12" pizza.

(E) * 5. The amount of surface space on this TRIANGLE.

(E) * 6. The amount of surface space on this SQUARE or CIRCLE.

(E) 7. How much space there is on the floor of your clubhouse.

(N) 8. How much water is needed to fill the inside of a swimming pool.

(E) * 9. The amount of surface space on the odd-shaped cut-outs.

(N) * 10. How much fencing is needed to close in the yard for your dog.

(E) * 11. How much sod is needed for a new soccer field.

(E) * 12. How much wallpaper is needed to cover the
surface of one classroom wall.

(N) * 13. How long of a piece of tape is needed to go around a box. (extend the tape from the roll and demonstrate placing it "around" the box).

(E) *14. How much tile is needed to cover the floor of the classroom. (display the square foot tile).

(N) 15. How much water is needed to fill an aquarium.

(N) 16. The distance you have run if you run across a field like this: (draw on the board a diagonal direction).

Step 2. Discovering Common Attributes

Once the list is complete, the class needs to determine the common attributes of the examples. Go down the list of example attributes and cross off the unnecessary ones, that is, attributes that the examples and non-examples have in common, like color and shape. What remains should be attributes that belong ONLY to the examples, specifically the attributes of AREA.

Step 3. Developing a Definition or Hypothesis About the Concept

Ask students to give ideas of what the "mystery topic" might be. Write down all of their ideas. Respect everyones contribution and if someone wants
to amend a definition ask the original author if that meets with his/her approval.

[At this point they should be making statements like: an amount of space on a flat surface and the shape doesn't matter, or the amount of space inside a flat surface.---if the word AREA is mentioned by a student at this time do not discourage its use--]

T. "Does everyone agree that from the clues the "mystery topic" has to do with a flat surface and the amount of space inside that surface and also that the shape doesn't really matter?" Write these attributes on the board in a list format like this:

1. has to do with a flat surface
2. the amount of space inside the flat surface
3. the shape of the flat surface doesn't really matter

Step 4. Refining the definition

T. Give some positive feedback about their good detective work of uncovering the "mystery topic". Then tell them the three clues on the board describe today's mystery topic and that the name of the "mystery topic" is AREA. Then proceed to have students write a definition for area using all the clues that have been uncovered.

Ask for volunteer detectives to help put together
the final picture of what mystery topic, area, really is. You begin by writing on the board "Area is" and let them complete the sentence. Write all their suggestions on the board. The goal is to get them to say that "area is the amount of space inside any flat surface." Direct your questions so that they can edit the suggestions that are given. For example: "Since it's hard to remember long sentences or definitions can someone rephrase this one?

Ask if there are any questions at this time. If not go on to Step 5.

Step 5. Recognizing the Concept
Tell the students to identify examples and non-examples of area as you call them out. Have the students give reasons why they classify it as such. Repeat the correct answer. Use the following list. Before you begin remind students to check the item with the definition and see if the item has the right attributes (is it a flat surface, are we looking for how much space covers the inside of the flat surface, and that any shape is fine as long as it is flat).

(E) * 1. The floor of your treehouse is looking pretty
bad and you want to cover it with a rug. How much rug will you buy?

(E) * 2. Before spring season your baseball league wants to put new sod on the field. How much sod is needed to cover the baseball field?

(N) 3. How far do you run if you make a home run?

(N) 4. How much air is inside of a beachball?

(E) * 5. We are going to decorate the top of your desk by covering it with Contact Paper. How much paper do we need?

(N) 6. What is the distance you have skated when you skate around the rink 3 times.

(N) 7. How far have you traveled if you ride your bike around the block.

(N) * 8. How much wood do you need to make a frame for a picture you drew.

(E) * 9. How much glass do you need to put inside of that frame you are building for your picture.

(N) * 10. How many ping-pong balls does it take to fill up a Volkswagen Beetle.

Step 6. **Students Provide Examples**

Ask 1/2 of the students to write down an example of the concept, and 1/2 to write a non-example. Call on students to give their choice (alternate example with non-example). Discuss reasons why the choice
is an example or a non-example by referring to the definition. Write the choices on the board under the Example and Non-example columns. If they repeat one that has been used, that is fine because it still reinforces the concept.

Homework: Area worksheet with a list of examples and non-examples that they are to classify.
LESSON TWO--DEVELOPING THE ALGORITHM FOR AREA

Step 1. REVIEW PREVIOUS DAY'S LESSON

T. "Yesterday, as detectives, we discovered the mystery topic was AREA. You developed a definition for it. Can anyone remember and tell me what area is? Please begin your sentence with "Area is..."

Elicit responses from students that state "Area is the amount of space covering the inside a flat surface, and the shape doesn't matter."

Positively reinforce proper responses. Then write on the board the response that "area is the amount of space that covers the inside of any flat surface.

AT THIS POINT REVIEW THE HOMEWORK WORKSHEET AND SEE IF ANYONE HAS ANY QUESTIONS. ERRORS SHOULD BE CORRECTED BY REFERRING TO THE LIST OF AREA ATTRIBUTES AND HAVE STUDENTS CHECK TO SEE IF THE EXAMPLE CONTAINS THOSE ATTRIBUTES OR NOT.

Step 2. QUESTION TO INTRODUCE ALGORITHM
T. Today students will continue to be detectives because there are more interesting things to discover about area.

QUESTION: "What exactly are we counting when we try to figure out the amount of space in any flat surface?"

Discuss the following facts:
When we weigh ourselves we measure or count pounds.
When we measure our height we count feet and inches.
When we measure how long something is we count feet, inches, yards, centimeters or meters for example.
So how do you know what the amount of floor space there actually is in the treehouse so you can buy a rug that is the right size to put on it. Or what if you wanted to paint your treehouse walls, what would you count to see how much space there is so that you could buy enough paint?"

PASS OUT THE PACKETS AND RULERS AT THIS TIME.

Step 3. **ACTIVITY--SQUARE UNITS AND AREA**

There are 2 groups of packets, Group X uses metric measurements and Group Y uses standard measurements.
Both groups have 3 cut-out shapes (1-rectangle, 1-square, 1-irregular shape and Group X has different size shapes than Group Y), 3 sheets of graph paper.
Ask students to trace the rectangle and square onto the graph paper. They should be sure to line up the sides with the lines on the paper. DO AN EXAMPLE ON THE OVERHEAD TRANSPARENCY.

Then ask them to count the number of squares inside the surface. Place their answers on the board. Make sure students with the same group of packets agree on the answers. State and write their answers as:

Group X: The area of the rectangle is ___little squares.
The area of the square is ___little squares.

Group Y: The area of the rectangle is ___little squares.
The area of the square is ___little squares.

EXPLAIN: THE MEANING OF "SQUARE UNIT"
The reason the answers are different between the groups is due to the fact that each group has different size squares on their graph paper (SHOW THEM). ASK STUDENTS TO MEASURE, WITH THE RULER, ALL FOUR SIDES OF ONE LITTLE SQUARE ON THEIR PAPER.
DISCUSS WITH STUDENTS:

1. the meaning of square inch, square centimeter, and square foot emphasizing the word "square". Compare visually (use the prepared poster) an "inch" and a "square inch"; "centimeter" and "square centimeter"; "foot" and "square foot" (display a square foot tile from the visual box since it is not on the poster). Discuss the differences and the uses. (use the previous examples and non-examples). Inch, centimeter, and foot are not the only units that can be squared. ASK STUDENTS FOR OTHER EXAMPLES. (METERS, YARDS, KILOMETERS, OR ANYTHING THAT MEASURES A DISTANCE).

2. the number of squares that are needed depends on the size of the square--display various size ceramic tiles and vinyl tiles use for flooring and compare how many are needed to cover the same surface.

3. that a square unit is the best and most convenient way to measure the surface area.

NOW, GO BACK TO THE STATEMENTS ON THE BOARD AND REPRESS THEM AS:

The area of the rectangle is ___ square inches/centimeters.

The area of the square is ___ square
inches/centimeters.

GO TO THE DEFINITION OF AREA ON THE BOARD: AREA IS THE AMOUNT OF SPACE COVERING THE INSIDE OF ANY FLAT SURFACE. DISCUSS REWRITING IT:

T. QUESTION: Now that you have seen that you can cover a flat surface with square units in order to count the area, how can we rewrite the definition using this new information?

T. QUESTION: Rather than saying "the amount of space" can you be more specific? Who can think of a better way to say what area is?

Elicit a student response that area is the number of squares that covers the inside of any flat surface.

T. Continue to give positive feedback for correct responses. Then discuss the usage of the word units. Units is a generic term that can represent any measurement. REPLACE THE DEFINITION WITH: AREA IS THE NUMBER OF SQUARE UNITS THAT COVERS THE INSIDE OF ANY FLAT SURFACE

Step 4. ACTIVITY—DEVELOPING A=L x W

Have students measure, with the ruler, (Group X-centimeters and Group Y-inches) the length (the number of squares across) and width (the number of squares down) of the rectangle and square in the
packet. Compare the dimensions to the area (number of counted squares) and guide students to see the relationship between the two.

QUESTION: Could there be a shorter, quicker method of measuring the area instead of counting squares?

QUESTION: How are the length, which is the number of squares across, and width, which is the number of squares down, related to the actual area? (Looking back at the traced figures may help in seeing the relationship?)

Students should be reaching the conclusion that multiplying the length by the width will yield the number of square units i.e. area. Once this has been established proceed with the following line of questions:

T. "Do you suppose this method will work for all rectangles and squares?" HOLD UP THE 7"x2" RECTANGLE THAT HAS GRAPH PAPER OVERLAY ON THE BACK SHOW THEM THE NON-GRAPH PAPER SIDE. ASK A STUDENT TO MULTIPLY THE LENGTH BY THE WIDTH. THEN HAND THE CARD TO ANOTHER STUDENT TO COUNT THE SQUARES ON THE BACK. HAVE THEM MATCH THEIR ANSWERS. DO
THIS PROCEDURE WITH 2 OTHER EXAMPLES.

T. ASK STUDENT TO THINK OF A RULE FOR FINDING THE AREA OF RECTANGLES OR SQUARES THAT CAN BE USED INSTEAD OF COUNTING.
Elicit a student response to the effect that area is the length times the width.
T. WRITE ON THE BOARD: AREA= LENGTH X WIDTH.
"In code this can be written: A = L x W"
T. HOLD UP A RECTANGLE WITH THE DIMENSIONS LABELED AND ASK STUDENTS TO FIND THE AREA. BE SURE THEY LABEL THE ANSWER PROPERLY. CONTINUE WITH THE FOLLOWING EXAMPLES ORALLY:
Find the area of a rectangle with L= 10 cm. & W=13 cm.
Find the area of a square with L= 15 in. & W= 15 in.

Step 5. ACTIVITY
Ask students to draw a rectangle that has an area of 20 square units. Then ask them for the dimensions.
(answers will vary, 20x1, 4x5, 2x10)
Now ask students to draw a rectangle that has an area of 18 square units. Ask them for the dimensions
(answers will vary, 1x18, 2x9, 3x6)
One final time ask student to draw a square that has an
area of 25 square units. (answers should be 5x5).

**QUESTION:** Why is there only one way of drawing a square with an area of 25 square units? Will there be only one way to draw any square? Why or why not?

**Step 6. SOLVING PROBLEMS**

On their last piece of graph paper ask students to trace their hand (keep their fingers together) and try to estimate the area of their hand without counting the squares. They should draw a rectangle around their hand and calculate the area of the rectangle.

* * * * *

Ask students to estimate the area of the irregular shape cut-out in their packet, without counting the squares. Students should try to box in the irregular shape and estimate its area to be that of the area of the rectangle or square.

* * * * *

T. "Suppose you wanted to cover the floor of your treehouse with a carpet. The floor measures 7 ft. long and 5 ft. wide. How much carpet do you need for the treehouse?"  
Elicit student response of 35 square feet.  
T. Discuss what would they would do if they didn't know the length and width of the treehouse floor. What would have to be done first in order to find the area?
Elicit student response that measuring the length and width is necessary before calculating the area.

* * * * *

T. Now they want to wallpaper the wall in this room with green polka-dotted wallpaper. The wall measures 10' x 14' how much wallpaper is needed?" Elicit student response of 140 square feet.


**DAY 3:** Review homework. Administer Posttest.
Appendix C
Which ones are examples of area problems?

TRUE OR FALSE. Write True if the example describes a problem that can be solved by finding the area and False if it does not.

1. It's "Be Nice To Your Dog" week. The doghouse roof leaks, so you are going to cover it with new shingles. You want to know how many shingles are needed to cover the roof.

2. The dog needs a new rope when he is tied outside. You have to be sure he can't reach the garden. You need to know the distance from the doghouse to the garden.

3. The weather is cold and you want the dog to stay warm. Your mother said you could have some cheap carpet to cover the floor of the doghouse. You want to know how much carpet is needed to cover the floor.

4. The gumball machine is empty. You want to know how many gumballs are needed to fill up the gumball machine.
Appendix D
The soccer field is 120 yards long and 75 yards wide. What is the area of the soccer field?

**Remember**

The area of a rectangle is its length times its width. A soccer field has the shape of a rectangle.

\[ A = l \times w \]
\[ A = 120 \times 75 \]
\[ A = 9,000 \]

The soccer field has an area of 9,000 square yards.

Count to find the area.

1. 
   
   
   
   

2. 
   
   
   
   

3. 
   
   
   
   

Multiply to find the area.

4. 
   
   
   

5. 
   
   
   

6. 
   
   
   

Multiply to find the area of the rectangle or square.

7. \( l = 12 \text{ cm}, \ w = 5 \text{ cm} \) 
8. \( l = 15 \text{ ft}, \ w = 3 \text{ ft} \)
9. \( l = 10 \text{ in.}, \ w = 10 \text{ in.} \)
10. \( l = 8 \text{ cm}, \ w = 8 \text{ cm} \)
11. \( l = 49 \text{ yd}, \ w = 15 \text{ yd} \)
12. \( l = 10 \text{ m}, \ w = 4 \text{ m} \)
References


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