Curve Reconstruction from the Apparent Contour Graph

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MOTIVATION

The research group of Bellettini, Beorchia, Paolini, and Pasquarelli worked on the **Scenery Reconstruction Problem (3D-AC).** This is a pivotal problem in computer vision and optics. My advisor, Dr. Dreibelbis was interested in a similar problem called the **Bitangency Reconstruction Problem (3D-BP).** To help understand 3D-BP, our research project looks at 2D-BP. As a warmup, we tackled 2D-AC, a simplified version of the work of Bellettini et al.

APPARENT CONTOUR AND SCENERY RECONSTRUCTION

Given a surface, the apparent contour of the surface is the edge of the shadow that we get when we project the surface into a plane. The Scenery Reconstruction Problem asks the following question: given a collection of curves, can we construct a surface that has these curves for its apparent contour? If the curves satisfy an admissibility condition, then the answer is yes [1].

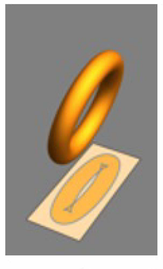

Figure 1: Reconstruction of the surface. Credit to Dr. Daniel Dreibelbis.

CONTOUR GENERATOR AND APPARENT CONTOUR FOR CURVES

Given a simple, closed curve, its **contour generator** is the collection of points that map to the edge when projected onto the x-axis. These are the points on the curve that have vertical tangencies. The **apparent contour** is the image of those points [2].

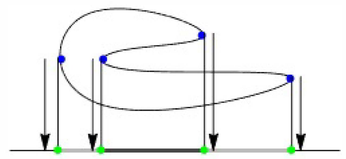

Figure 2: An example of contour generators (blue points) and apparent contours (green points).

APPARENT CONTOUR GRAPH

We label each apparent contour point with the number of times that a vertical line intersects the curve between the contour generator and the x-axis. We label each interval with the number of times our vertical line intersects the curve. We call this labeling an **apparent contour graph**.

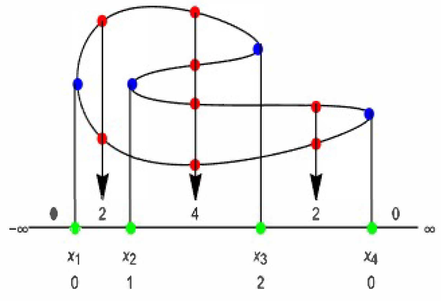

Figure 3: An example of an apparent contour graph.

LABELING THEOREM

Theorem 1: Let C be a simple, closed, smooth curve in general position. The apparent contour graph satisfies the following conditions:

- The label for each interval is even.
- The label on the point in the apparent contour is always less than or equal to the interval labels next to it.
- Interval labels must either increase or decrease by two between each adjacent interval.
- The number of apparent contour points must be even.

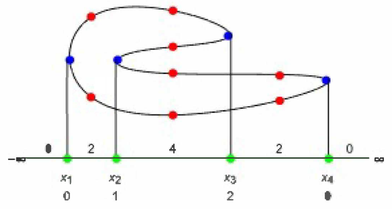

Figure 4: An example of our theorem being executed.

2D-AC

Given an admissible apparent contour graph, is there a curve with this graph as its apparent contour? The answer is yes.

Theorem 2: Given an admissible graph, there exists a simple, closed curve C whose labeled apparent contour is this graph. The curve C is unique up to a vertical diffeomorphism of the plane.

CURVE RECONSTRUCTION

We developed an algorithm that begins with an admissible apparent contour graph and reconstructs a curve with this apparent contour

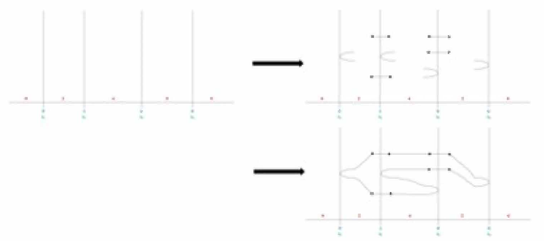

Figure 5: An example of our algorithm being executed.

MOVING FORWARD

We can ask similar questions about admissible labeling and reconstruction of bitangencies between two curves. This is the bitangency reconstruction problem, that we are currently working on.

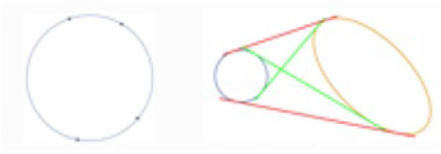

Figure 6: Our 2D-BP project. Credit to Dr. Daniel Dreibelbis.

REFERENCES

[1] G. Bellettini, V. Beorchia, M. Paolini, F. Pasquarelli. *Shape Reconstruction from Apparent Contours: Theory and Algorithms*. Computational Imaging and Vision, 44. *Springer, Heidelberg,* 2015.

[2] J. W. Bruce, P. J. Giblin. *Curves and Singularities. A Geometrical Introduction to Singularity Theory*. Second edition. *Cambridge University Press, Cambridge, 1992.*