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A Comparison of Methods for Generating Bivariate Non-normally Distributed Random Variables

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A Comparison of Methods for Generating
Bivariate Non-normally Distributed Random Variables

By

Jaimee E. Stewart

A thesis submitted to the
Department of Mathematics and Statistics
in partial fulfillment of the requirements for the degree of

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COLLEGE OF ARTS AND SCIENCES

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ABSTRACT

Many distributions of multivariate data in the real world follow a non-normal model with distributions being skewed and/or heavy tailed. In studies in which multivariate non-normal distributions are needed, it is important for simulations of those variables to provide data that is close to the desired parameters while also being fast and easy to perform.

Three algorithms for generating multivariate non-normal distributions are reviewed for accuracy, speed and simplicity. They are the Fleishman Power Method, the Fifth-Order Polynomial Transformation Method, and the Generalized Lambda Distribution Method.

Simulations were run in order to compare the three methods by how well they generate bivariate distributions with the desired means, variances, skewness, kurtoses, and correlation, simplicity of the algorithms, and how quickly the desired distributions were calculated.

Chapter 1

Introduction

Multivariate data consists of two or more random variables that are usually correlated, which are obtained as a result of an experiment. The most convenient and familiar multivariate distribution is the multivariate normal. Its popularity is likely due to its ease of simulation and ability to allow for closed-form theoretical results. Rarely though, do the distributions of data follow the symmetric multivariate normal model. In practice, most distributions are non-normal, with the data being skewed and/or heavy-tailed. Examples of random variables with non-normal distributions would be waiting times, lengths of time between malfunctions in machinery, growth data such as bacterial growth, and proportional data.

Monte Carlo simulations needing correlated normal and non-normal distributions have been used to investigate the small sample properties of competing statistics or the comparison of estimation techniques. Cases of this include the dependent sample t-test (Blair and Higgins, 1985), regression (Iman & Conover, 1979), and meta-analysis (Sawilowsky, Kelley, Blair, & Markman, 1994).

The simulation of multivariate data is an integral component of data analysis methodologies. Simulation studies are used in computer evaluations to verify theoretical, large sample properties of statistical methods, estimators, and test statistics. The generation of multivariate random variables is also an important part of data analysis methodologies, such as bootstrap resampling and Markov Chain Monte Carlo.

A good non-normal random number generator is able to produce data that will satisfy the requirements for certain parameters. It would be preferable that the generated random variables match all moments of the desired distribution, but matching the mean, the standard deviation, skewness, and kurtosis tends to produce adequate results.

Skewness, γ_1 , can be identified as the third standardized moment and is defined as:

$$\gamma_1 = \frac{m_3}{\sigma^3},$$

where m_3 is the third moment about the mean of a random variable and σ is the standard deviation. It can also be described as the standardized cumulant, which is the ratio of the third cumulant κ_3 and the third power of the square root of the second cumulant κ_2 :

$$\gamma_1 = \frac{\kappa_3}{\sqrt{\kappa_2^3}}.$$

Kurtosis, γ_2 , is the fourth standardized moment and is defined as:

$$\gamma_2 = \frac{m_4}{\sigma^4} - 3,$$

where m_4 is the fourth moment about the mean of a random variable and σ is the standard deviation. It can also be defined as the standardized cumulant, which is the fourth cumulant divided by the square of the second cumulant:

$$\gamma_2 = \frac{\kappa_4}{\kappa_2^2}.$$

For two correlated random variables, the correlation is defined as:

$$\rho_{12} = \frac{\mu_{12} - \mu_1\mu_2}{\sigma_1\sigma_2},$$

Where μ_1 and μ_2 are the means of the two random variables considered and μ_{12} is the mean of the products of the two random variables.

A procedure for developing non-normal univariate data was developed by Fleishman (1978). This method uses the polynomial transformation

$$Y = a + bZ + cZ^2 + dZ^3 \quad (1.1)$$

where Z is the standard normal random variable. The constants a , b , c , and d are chosen so that Y has the desired coefficients of skewness and kurtosis.

A two-step approach to generate a non-normal set of correlated data was proposed by Olejnick and Algina (1984, 1987) in their study on ANCOVA and its rank transformation analog. This approach first uses the Fleishman (1978) transformation procedure and then uses the algorithm

$$X_i = \rho Y_i + \sqrt{1 - \rho^2} E_i \quad (1.2)$$

or the model

$$X_i = \beta Y_i + E_i \quad (1.3)$$

to generate the X_i which are correlated with the Y_i at the specified level of correlation, ρ ,

in (1.2) and $\rho = \frac{\beta}{\sqrt{\beta^2 + 1}}$ in (1.3). The variables Y_i and E_i are the independent random

variables generated by first using the Fleishman (1978) transformation procedure.

There is a problem with this two-step procedure in that the values of skewness and kurtosis for the X_i are dependent on ρ . So, as ρ varies from 0 to 1, the values of skewness and kurtosis will change. Vale and Maurelli (1983) created a procedure that would circumvent this problem. Their method involves an initial step involving principal

components (or other factorization method) decomposition on the population correlation matrix.

Headrick and Sawilowsky (1999) developed another method that extends the Fleishman (1978) method to multivariate data, which creates a procedure simpler than the one developed earlier by Vale and Maurelli (1983). The algorithm is easier to use because it avoids the preliminary step of the factorization procedure and is simpler to code in a programming language.

The method developed by Headrick and Sawilowsky (1999) seems to work well because it avoids the issue from the two-step procedure in which the values of skewness and kurtosis are dependent on ρ . It is also simpler than the Vale and Maurelli (1983) method and has shown to be better at generating desired correlations when distributions are highly skewed and/or heavy tailed and when sample sizes are small to moderate (Headrick and Sawilowsky, 1999).

The multivariate Fleishman power transformation methods developed by Headrick and Sawilowsky (1999) and Vale and Maurelli (1983) have been used for studies involving analysis of covariance (Harwell and Serlin, 1988; Headrick and Sawilowsky, 1999; Olejnick and Algina, 1984, 1987, Seamen et al., 1985), hierarchical linear models (Shieh, 2000), multivariate nonparametric tests (Habib and Harwell, 1989), regression (Harwell and Serlin, 1989; Whittaker et al., 2001), and repeated measures (Harwell and Serlin, 1997). Monte Carlo simulations are also studied with topics and techniques such as continuous non-normal distributions correlated with ranked or ordinal structures (Headrick and Beasley, 2003), ranked data (Headrick, 2004), systems of linear

statistical equations (Headrick and Beasley, 2004), and distributions with specified intraclass correlations (Headrick and Zumbo, 2004).

One limitation to the Fleishman power method developed by Headrick and Sawilowsky (1999) is that the procedure is bounded. In general, for a specified value of γ_1 , there is an associated lower bound of γ_2 , defined by the following inequality (Devroye, 1986, p. 688):

$$\gamma_2 \geq \gamma_1^2 - 2. \quad (1.4)$$

Specifically for the Fleishman power method, it has been found that the lower boundary point of kurtosis for the given value of $\gamma_1 = 0$ is $\gamma_2 = -1.15132$ (Headrick and Sawilowsky, 2000b).

Headrick (2002) developed the fifth-order polynomial transformation method in order to derive a family of distributions that span a larger space in the (γ_1^2, γ_2) plane as well as improve on the accuracy of the Fleishman power method. The fifth-order polynomial transformation procedure generates univariate and multivariate non-normal distributions based on the first six standardized cumulants. The transformation is defined as:

$$X = c_0 + c_1Z + c_2Z^2 + c_3Z^3 + c_4Z^4 + c_5Z^5$$

where Z follows a standard normal distribution and $c_0, c_1, c_2, c_3, c_4,$ and c_5 are constants chosen in such a way that X has the desired skewness, kurtosis, and fifth and sixth standardized cumulant.

A large number of studies (Harwell and Serlin, 1988; Headrick and Sawilowsky, 1999; Olejnick and Algina, 1984, 1987, Seamen et al., 1985; Shieh, 2000; Habib and Harwell, 1989; Harwell and Serlin, 1989; Whittaker et al., 2001; Harwell and serlin,

1997; Headrick and Beasley, 2003; Headrick and Beasley, 2004; Headrick, 2004; Headrick and Zumbo, 2004) that have already employed the Fleishman power method to simulate correlated non-normal distributions indicate that those algorithms are efficient and easy to use. The fifth-order polynomial transformation method was developed to be as simple to use as the Fleishman power method, yet more precise and with larger span in the (γ_1^2, γ_2) plane.

The method for simulating multivariate non-normal distributions from the generalized lambda distribution (GLD) was created by Headrick and Mugdadi (2006) to extend the univariate GLD developed by Ramberg and Schmeiser (1974) to multivariate data generation. Ramberg and Schmeiser (1974) generalized Tukey's power transformation of uniform random variables to obtain a GLD that is summarized by the inverse distribution function

$$X = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2}$$

where p is uniform $(0,1)$. Then, λ_1 and λ_2 are the location and scale parameters of X respectively, and λ_3 and λ_4 are the shape parameters that determine its skewness and kurtosis of X .

The GLD method for generating univariate variables has been used for studies involving data mining (Dudewicz and Karian, 1999), independent component analysis (Karvanen, 2003 and Mutihac and Van Hulle, 2003), micro array research (Beasley et al., 2004), operations research (Ganeshan, 2001), option pricing (Corrado, 2001), psychometrics (Bradley, 1993, Bradley and Fleisher, 1994 and Delaney and Vargha, 2000), and structural equation modeling (Reinartz et al., 2002).

Headrick and Mugdadi (2006) reported that previous attempts to extend the GLD method to multivariate generation have proven to be difficult. This is due to having to take several steps to overcome the problem of generating biased correlation coefficients (Bradley and Fleisher, 1994) and having access to (or reliance on) commercial software packages and ensuring the accuracy of numerical solutions to complicated integrals (Corrado, 2001). Therefore, other methods for generating multivariate non-normal distributions, such as the power method transformations, proved to be more popular. Since real-world distributions are often non-normal, it is vital to have a variety of procedures available for multivariate non-normal data generation. Headrick and Mugdadi (2006) believed that their method of extending the univariate GLD to a multivariate technique for generating non-normal variables provides a “viable competitor to the power method because of its simplicity and ease of execution” (p. 3352).

The purpose of this study is to evaluate the three existing methods for generating bivariate non-normally distributed random variables. Each is evaluated on its ease of use, how efficiently it performs, and how accurately the generated random variables replicate the desired distributions with the specified skewness, kurtosis, and correlation. The methods considered are the Fleishman power method transformation, the fifth-order polynomial transformation method, and the generalized lambda distribution method.

In this study, bivariate random variables with zero mean, unit variance, and specified skewness, kurtoses, and correlation were produced using each method. Each method was then compared to determine which produced the most accurate distributions with regards to the correlations between the variables, and each variable’s mean,

variance, skewness, and kurtosis. The length of time needed to generate the variables was compared for each method as well as ease of use.

Hence, the results of this study should provide a guide for those wanting the most effective method for generating bivariate non-normally distributed variables.

The methods under studied are detailed in Chapter 2. In Chapter 3, the simulation study is given for comparing the three different methods for generating bivariate non-normally distributed random variables. The results of the simulation are discussed in Chapter 4. A conclusion for the study is given in Chapter 5.

Chapter 2

The Proposed Method

Three methods for generating bivariate, non-normally distributed random variables are considered. They are the Fleishman power method (Headrick and Sawilowsky, 1999), the fifth-order polynomial transform method (Headrick, 2002), and the generalized lambda distribution method (Headrick and Mugadi, 2006).

2.1 The Fleishman Power Method

The Fleishman power method developed by Headrick and Sawilowsky (1999) uses a third-order polynomial transformation to produce non-normal data. In the univariate case, the data is generated by using the following equation:

$$X = a + bZ + cZ^2 + dZ^3 \quad (2.1)$$

Where Z has a standard normal distribution, and a , b , c , and d are constants chosen in such a way that X has the desired coefficients of skewness and kurtosis. Fleishman (1978) showed that $a = -c$ and the constants b , c , and d can be determined by simultaneously solving the Fleishman Equations

$$b^2 + 6bd + 2c^2 + 15d^2 - 1 = 0 \quad (2.2)$$

$$2c(b^2 + 24bd + 105d^2 + 2) - \gamma_1 = 0$$

$$24\{bd + c^2(1 + b^2 + 28bd) + (12 + 48bd + 141c^2 + 225d^2)\} - \gamma_2 = 0$$

for the specified values of skewness, γ_1 , and kurtosis, γ_2 . The equations are solved by using a modified Powell hybrid algorithm and a finite-difference approximation to the Jacobian. The values of a , b , c , and d are then substituted into (2.1) to transform the standard normal variable Z to X .

If X_1 and X_2 are bivariate non-normal variables generated using the Fleishman method, the correlation coefficient between X_1 and X_2 determined by Headrick and Sawilowsky (1999) is

$$\rho_{X_1X_2} = \rho_{Z_1Z_2} (b_1b_2 + 3b_1d_2 + 9d_1d_2 + 2a_1a_2\rho_{Z_1Z_2} + 6d_1d_2\rho_{Z_1Z_2}^2) \quad (2.3)$$

where $\rho_{Z_1Z_2}$ is the intermediate correlation.

The procedure for generating bivariate random variables with specified skewness, kurtoses, and correlation begins with obtaining the Fleishman constants for each variable. Then the value of the intermediate correlation, or $\rho_{Z_1Z_2}$, is determined by substituting the calculated Fleishman constants into equation (2.3), setting the equation equal to the specified post-correlation, and solving for $\rho_{Z_1Z_2}$. This value is then used to generate standard normal random variables correlated at the intermediate level by substituting in the following equations developed by Headrick and Sawilowsky (1999):

$$\begin{aligned} Z_1 &= \sqrt{\rho_{Z_1Z_2}} Z_1^* + \sqrt{1 - \rho_{Z_1Z_2}} E_1 \\ Z_2 &= \sqrt{\rho_{Z_1Z_2}} Z_2^* + \sqrt{1 - \rho_{Z_1Z_2}} E_2 \end{aligned}$$

where Z_i^* and E_i are independent standard normal variates. Finally, to generate non-normal distributions with the desired skewness, kurtoses, and correlation, substitute Z_1 and Z_2 into the subsequent equations:

$$\begin{aligned} X_1 &= a_1 + b_1Z_1 + c_1Z_1^2 + d_1Z_1^3 \\ X_2 &= a_2 + b_2Z_2 + c_2Z_2^2 + d_2Z_2^3 \end{aligned}$$

2.2 The Fifth-Order Polynomial Transform Method

The fifth-order polynomial transformation method proposed by Headrick (2002) attempts to improve on the approximations of non-normal distributions which are generated using the Fleishman power method by using a fifth-order polynomial transformation. In the univariate case, it simulates non-normal distributions based on a moment-matching procedure involving the first six standardized cumulants, with the transformation expressed as follows:

$$X = c_0 + c_1Z + c_2Z^2 + c_3Z^3 + c_4Z^4 + c_5Z^5$$

where Z is a standard normal variate and $c_0, c_1, c_2, c_3, c_4,$ and c_5 are constants chosen in such a way that X has the desired coefficients of skewness (γ_1), kurtosis (γ_2), and fifth and sixth standardized cumulants (γ_3 and γ_4 respectively). Using a modified Powell hybrid algorithm and a finite-difference approximation to the Jacobian to simultaneously solve the following systems of equations developed by Headrick (2002) yields the solution values of $c_1, c_2, c_3, c_4,$ and c_5 .

$$(c_1^2 + 2c_2^2 + 24c_2c_4 + 6c_1(c_3 + 5c_5) + 3(5c_3^2 + 32c_4^2 + 70c_3c_5 + 315c_5^2)) - 1 = 0 \quad (2.4)$$

$$\begin{aligned} & (2(4c_2^3 + 108c_2^2c_4 + 3c_1^2(c_2 + 6c_4) + 18c_1(2c_2c_3 + 16c_3c_4 + 15c_2c_5 + 150c_4c_5) \\ & + 9c_2(15c_3^2 + 128c_4^2 + 280c_3c_5 + 1575c_5^2) \\ & + 54c_4(25c_3^2 + 88c_4^2 + 560c_3c_5 + 3675c_5^2)) - \gamma_1 = 0 \end{aligned} \quad (2.5)$$

$$\begin{aligned} & (24(2c_2^4 + 96c_2^3c_4 + c_1^3(c_3 + 10c_5) + 30c_2^2(6c_3^2 + 64c_4^2 + 140c_3c_5 + 945c_5^2)) \\ & + c_1^2(2c_2^2 + 18c_3^2 + 36c_2c_4 + 192c_4^2 + 375c_3c_5 + 2250c_5^2) \\ & + 36c_2c_4(125c_3^2 + 528c_4^2 + 3360c_3c_5 + 25725c_5^2) \\ & + 3c_1(45c_3^3 + 1584c_3c_4^2 + 1590c_3^2c_5 + 21360c_4^2c_5 + 21525c_3c_5^2 + 110250c_5^3 \\ & + 12c_2^2(c_3 + 10c_5) + 8c_2c_4(32c_3 + 375c_5) + 9(45c_3^4 + 8704c_4^4 + 2415c_3^3c_5 \\ & + 932400c_4^2c_5^2 + 3018750c_5^4 + 20c_3^2(178c_4^2 + 2765c_5^2) \\ & + 35c_3(3104c_4^2c_5 + 18075c_5^3)))) - \gamma_2 = 0 \end{aligned} \quad (2.6)$$

$$\begin{aligned}
& (24(16c_2^5 + 5c_1^4c_4 + 1200c_2^4c_4 + 10c_1^3(3c_2c_3 + 42c_3c_4 + 40c_2c_5 + 570c_4c_5) \\
& + 300c_2^3(10c_3^2 + 128c_4^2 + 280c_3c_5 + 2205c_5^2) + 1080c_2^2c_4(125c_3^2 + 3920c_3c_5 \\
& + 28(22c_4^2 + 1225c_5^2)) + 10c_1^2(2c_2^3 + 72c_2^2c_4 + 3c_2(24c_3^2 + 320c_4^2 \\
& + 625c_3c_5 + 4500c_5^2) + 9c_4(109c_3^2 + 528c_4^2 + 3130c_3c_5 + 24975c_5^2)) \\
& + 30c_1(8c_2^3(2c_3 + 25c_5) + 40c_2^2c_4(16c_3 + 225c_5) \\
& + 3c_2(75c_3^3 + 3168c_3c_4^2 + 3180c_3^2c_5 + 49840c_4^2c_5 + 50225c_3c_5^2 + 294000c_5^3) \\
& + 6c_4(555c_3^3 + 8704c_3c_4^2 + 26225c_3^2c_5 + 152160c_4^2c_5 + 459375c_3c_5^2 \\
& + 2963625c_5^3)) + 90c_2(270c_3^4 + 16905c_3^3c_5 + 280c_3^2(89c_4^2 + 1580c_5^2) \\
& + 35c_3(24832c_4^2c_5 + 162675c_5^3 + 4(17408c_4^4 + 2097900c_4^2c_5^2 + 7546875c_5^4)) \\
& + 27c_4(14775c_3^4 + 1028300c_3^3c_5 + 50c_3^2(10144c_4^2 + 594055c_5^2) \\
& + 700c_3(27904c_4^2c_5 + 598575c_5^3) \\
& + 3(316928c_4^4 + 68908000c_4^2c_5^2 + 806378125c_5^4)))) - \gamma_3 = 0
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
& (120(32c_2^6 + 3456c_2^5c_4 + 6c_1^5c_5 + 3c_1^4(9c_3^2 + 16c_2c_4 + 168c_5^2 + 330c_3c_5 \\
& + 2850c_5^2) + 720c_2^4(15c_3^2 + 224c_4^2 + 490c_3c_5 + 4410c_5^2) + 6048c_2^3c_4(125c_3^2 \\
& + 704c_4^2 + 4480c_3c_5 + 44100c_5^2 + 12c_1^3(4c_2^2(3c_3 + 50c_5) \\
& + 60c_2c_4(7c_3 + 114c_5) + 3(24c_3^3 + 1192c_3c_4^2 + 1170c_3^2c_5 + 20440c_4^2c_5 \\
& + 20150c_3c_5^2 + 124875c_5^3)) + 216c_2^2(945c_3^4 + 67620c_3^3c_5 + 560c_3^2(178c_4^2 \\
& + 3555c_5^2) + 315c_3(12416c_4^2c_5 + 90375c_5^3) + 6(52224c_4^4 + 6993000c_4^2c_5^2 \\
& + 27671875c_5^4)) + 6c_1^2(8c_2^4 + 480c_2^3c_4 + 180c_2^2(4c_3^2 + 64c_4^2 + 125c_3c_5 \\
& + 1050c_5^2) + 72c_2c_4(327c_3^2 + 1848c_4^2 + 10955c_3c_5 + 99900c_5^2) \\
& + 9(225c_3^4 + 22824c_3^2c_4^2 + 69632c_4^4 + 15090c_3^3c_5 + 830240c_3c_4^2c_5 \\
& + 412925c_3^2c_5^2 + 8239800c_4^2c_5^2 + 5475750c_3c_5^3 + 29636250c_5^4)) \\
& + 1296c_2c_4(5910c_3^4 + 462735c_3^3c_5 + c_3^2(228240c_4^2 + 14851375c_5^2) \\
& + 175c_3(55808c_4^2c_5 + 1316865c_5^3) + 3(158464c_4^4 + 37899400c_4^2c_5^2 \\
& + 483826875c_5^4)) + 27(9945c_3^6 + 92930048c_4^6 + 1166130c_3^5c_5 \\
& + 35724729600c_4^4c_5^2 + 977816385000c_4^2c_5^4 + 1907724656250c_5^6 \\
& + 180c_3^4(16082c_4^2 + 345905c_5^2) + 140c_3^3(1765608c_4^2c_5 + 13775375c_5^3) \\
& + 15c_3^2(4076032c_4^4 + 574146160c_4^2c_5^2 + 2424667875c_5^4)
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
& + 210c_3(13526272c_4^4c_5 + 687499200c_4^2c_5^3 + 1876468125c_5^5)) \\
& + 18c_1(80c_2^4(c_3 + 15c_5) + 160c_2^3c_4(32c_3 + 525c_5) + 12c_2^2(225c_3^3 \\
& + 11088c_3c_4^2 + 11130c_3^2c_5 + 199360c_4^2c_5 + 200900c_3c_5^2 + 1323000c_5^3) \\
& + 24c_2c_4(3885c_3^3 + 69632c_3c_4^2 + 209800c_3^2c_5 + 1369440c_4^2c_5 \\
& + 4134375c_3c_5^2 + 29636250c_5^3) + 9(540c_3^5 + 48585c_3^4c_5) \\
& + 20c_3^3(4856c_4^2 + 95655c_5^2) + 80c_3^2(71597c_4^2c_5 + 513625c_5^3) \\
& + 4c_3(237696c_4^4 + 30726500c_4^2c_5^2 + 119844375c_5^4) \\
& + 5c_5(4076032c_4^4 + 191074800c_4^2c_5^2 + 483826875c_5^4)))) - \gamma_4 = 0
\end{aligned} \tag{2.8 cont.}$$

The value for c_0 is found by using the equation $c_0 = -c_2 - 3c_4$. It should be noted that the formulas in the paper by Headrick (2002) contained typos. The above formulas were taken from a Mathematica program written by the author, Headrick. They are used to solve for the constants and verified to be correct.

The method for generating bivariate non-normally distributed random variables using the fifth-order polynomial transformation is similar to the Fleishman power method. The six constants are calculated for both distributions. The intermediate correlation developed by Headrick (2002) is then calculated by using the formula

$$\begin{aligned}
\rho_{X_1, X_2} = & 3c_{4(1)}c_{0(2)} + 3c_{4(1)}c_{2(2)} + 9c_{4(1)}c_{4(2)} + c_{0(1)}(c_{0(2)} + c_{2(2)} + 3c_{4(2)}) \\
& + c_{1(1)}c_{1(2)}\rho_{Z_1, Z_2} + c_{3(1)}c_{1(2)}\rho_{Z_1, Z_2} + 15c_{5(1)}c_{1(2)}\rho_{Z_1, Z_2} \\
& + 3c_{1(1)}c_{3(2)}\rho_{Z_1, Z_2} + 9c_{3(1)}c_{3(2)}\rho_{Z_1, Z_2} + 45c_{5(1)}c_{3(2)}\rho_{Z_1, Z_2} \\
& + 15c_{1(1)}c_{5(2)}\rho_{Z_1, Z_2} + 45c_{3(1)}c_{5(2)}\rho_{Z_1, Z_2} + 225c_{5(1)}c_{5(2)}\rho_{Z_1, Z_2} \\
& + 12c_{4(1)}c_{2(2)}\rho_{Z_1, Z_2}^2 + 72c_{4(1)}c_{4(2)}\rho_{Z_1, Z_2}^2 + 6c_{3(1)}c_{3(2)}\rho_{Z_1, Z_2}^3 \\
& + 60c_{5(1)}c_{3(2)}\rho_{Z_1, Z_2}^3 + 60c_{3(1)}c_{5(2)}\rho_{Z_1, Z_2}^3 + 600c_{5(1)}c_{5(2)}\rho_{Z_1, Z_2}^3 \\
& + 24c_{4(1)}c_{4(2)}\rho_{Z_1, Z_2}^4 + 120c_{5(1)}c_{5(2)}\rho_{Z_1, Z_2}^5 \\
& + c_{2(1)}(c_{0(2)} + c_{2(2)} + 3c_{4(2)} + 2c_{2(2)}\rho_{Z_1, Z_2}^2 + 12c_{4(2)}\rho_{Z_1, Z_2}^2)
\end{aligned} \tag{2.9}$$

where $\rho_{X_1X_2}$ is the desired correlation and $c_{j(i)}$ represents the j^{th} coefficient, $j = 1, 2, \dots, 6$, of the i^{th} variable, $i = 1, 2$, to solve for the intermediate correlation $\rho_{Z_1Z_2}$. Substitute this

value into the following equations:

$$\begin{aligned} Z_1 &= \sqrt{\rho_{Z_1Z_2}} Z_1^* + \sqrt{1 - \rho_{Z_1Z_2}} E_1 \\ Z_2 &= \sqrt{\rho_{Z_1Z_2}} Z_2^* + \sqrt{1 - \rho_{Z_1Z_2}} E_2 \end{aligned}$$

to generate standard normal deviates correlated at the intermediate level. The variables Z_i^* and E_i are independently normally distributed random variables with zero means and unit variances. Lastly, substitute Z_1 and Z_2 and their associated constants into

$$\begin{aligned} X_1 &= c_{0(1)} + c_{1(1)}Z_1 + c_{2(1)}Z_1^2 + c_{3(1)}Z_1^3 + c_{4(1)}Z_1^4 + c_{5(1)}Z_1^5 \\ X_2 &= c_{0(2)} + c_{1(2)}Z_2 + c_{2(2)}Z_2^2 + c_{3(2)}Z_2^3 + c_{4(2)}Z_2^4 + c_{5(2)}Z_2^5 \end{aligned}$$

to generate the desired bivariate non-normal distributions with the specified post-intercorrelations.

2.3 The Generalized Lambda Distribution Method

The generalized lambda distribution (GLD) method (Headrick and Mugadi, 2006) uses the inverse distribution function

$$X = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2}$$

to generate non-normally distributed random variables, where p is uniform $(0,1)$, λ_1 and λ_2 are its location and scale parameters respectively, and λ_3 and λ_4 are its shape parameters that determine its skewness and kurtosis. It works to generate simulated data

from a distribution with finite support, which is determined by the values of skewness and kurtosis.

For any given γ_1 and γ_2 , λ_3 and λ_4 are determined by solving the equations developed by Ramberg and Schmeiser (1974) using the successive quadratic programming algorithm and a finite difference gradient:

$$\begin{aligned} & \{[1/(3\lambda_3 + 1) - 3Beta(2\lambda_3 + 1, \lambda_4 + 1) + 3Beta(\lambda_3 + 1, 2\lambda_4 + 1) - 1/(3\lambda_4 + 1) \\ & - 3[1/(2\lambda_3 + 1) - 2Beta(\lambda_3 + 1, \lambda_4 + 1) + 1/(2\lambda_4 + 1)][1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)] \\ & + 2[1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)]^3\} - \gamma_1 = 0 \end{aligned} \quad (2.10)$$

$$\begin{aligned} & \{[1/(4\lambda_3 + 1) - 4Beta(3\lambda_3 + 1, \lambda_4 + 1) + 6Beta(2\lambda_3 + 1, 2\lambda_4 + 1) - 4Beta(\lambda_3 + 1, 3\lambda_4 + 1) \\ & + 1/(4\lambda_4 + 1) - 4[1/(3\lambda_3 + 1) - 3Beta(2\lambda_3 + 1, \lambda_4 + 1) + 3Beta(\lambda_3 + 1, 2\lambda_4 + 1) \\ & - 1/(3\lambda_4 + 1)][1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)] + 6[1/(2\lambda_3 + 1) - 2Beta(\lambda_3 + 1, \lambda_4 + 1) + \\ & 1/(2\lambda_4 + 1)][1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)]^2 - 3[1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)]^4\} - \gamma_2 = 0 \end{aligned} \quad (2.11)$$

With the values for λ_3 and λ_4 , it is possible to find λ_1 and λ_2 with the following Ramberg and Schmeiser (1974) equations:

$$\lambda_2 = \sqrt{[1/(2\lambda_3 + 1) - 2Beta(\lambda_3 + 1, \lambda_4 + 1) + 1/(2\lambda_4 + 1)] - [1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)]^2} \quad (2.12)$$

$$\lambda_1 = -[1/(\lambda_3 + 1) - 1/(\lambda_4 + 1)] / \lambda_2. \quad (2.13)$$

In order to simulate bivariate non-normally distributed random variables, the lambdas for distributions are calculated. The following method from Headrick and Muggadi (2006) is then used to find the intermediate correlation.

Let Z_1 and Z_2 be standard normal random variables:

$$f_1 = f_{z_1} = (2\pi)^{(-1/2)} \exp\{-z_1^2 / 2\} \text{ and} \quad (2.14)$$

$$f_2 = f_{z_2} = (2\pi)^{(-1/2)} \exp\{-z_2^2 / 2\}, \quad (2.15)$$

with bivariate standard normal distribution:

$$f_{1,2} = f_{z_1 z_2}(z_1, z_2, \rho_{z_1 z_2}) = (2\pi\sqrt{1-\rho_{z_1 z_2}^2})^{-1} \exp\{-(2\sqrt{1-\rho_{z_1 z_2}^2})^{-1} \times (z_1^2 - 2\rho_{z_1 z_2} z_1 z_2 + z_2^2)\}.$$

So, the distribution functions related to (2.14) and (2.15) are denoted as

$$\Phi(z_1) = \int_{-\infty}^{z_1} 2\pi^{(-1/2)} \exp\{-u_1^2 / 2\} du_1,$$

$$\Phi(z_2) = \int_{-\infty}^{z_2} 2\pi^{(-1/2)} \exp\{-u_2^2 / 2\} du_2$$

where $\Phi(z_1) \sim U[0,1]$, $\Phi(z_2) \sim U[0,1]$ with correlation $\rho_{\Phi(z_1), \Phi(z_2)} = (6/\pi) \sin^{-1}(\rho_{z_1 z_2} / 2)$.

Let $x_1(z_1, \lambda_{1k})$ and $x_2(z_2, \lambda_{2k})$ where $k = 1, 2, 3, 4$ be standardized GLDs that take the form of

$$x = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2}$$

for the bivariate case as

$$x_1(z_1, \lambda_{1k}) = \lambda_{11} + \left((\Phi(z_1))^{\lambda_{13}} - (1 - (\Phi(z_1))^{\lambda_{14}}) \right) / \lambda_{12},$$

$$x_2(z_2, \lambda_{2k}) = \lambda_{21} + \left((\Phi(z_2))^{\lambda_{23}} - (1 - (\Phi(z_2))^{\lambda_{24}}) \right) / \lambda_{22}.$$

The correlation between $x_1(z_1, \lambda_{1k})$ and $x_2(z_2, \lambda_{2k})$ can be expressed as

$$\rho_{x_1 x_2} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x_1(z_1, \lambda_{1k}) * x_2(z_2, \lambda_{2k})) f_{12} dz_1 dz_2$$

or as an algorithm of Riemann sums:

$$\begin{aligned} \rho_{x_1 x_2} \cong & \sum_{u_{1\min}}^{u_{1\max}} \sum_{u_{2\min}}^{u_{2\max}} [(\lambda_{11} + (\sum_{u_{1\min}}^{z_1} (f_1 \Delta u_1)^{\lambda_{13}} - (1 - \sum_{u_{1\min}}^{z_1} (f_1 \Delta u_1)^{\lambda_{14}}) / \lambda_{12})) \\ & * (\lambda_{21} + (\sum_{u_{2\min}}^{z_2} (f_2 \Delta u_2)^{\lambda_{23}} - (1 - \sum_{u_{2\min}}^{z_2} (f_2 \Delta u_2)^{\lambda_{24}}) / \lambda_{22})) \\ & * f_{12} \Delta u_1 \Delta u_2] \end{aligned} \quad (2.16)$$

where u_1, u_2, z_1, z_2 start with $u_1 = u_{1_{\min}}, u_2 = u_{2_{\min}}, z_1 = u_{1_{\min}}, z_2 = u_{2_{\min}}$ and use steps of $\Delta u_1, \Delta u_2, \Delta u_1, \Delta u_2$, respectively. Note that $\rho_{z_1 z_2}$ in f_{12} in the above equations is the intermediate correlation.

Two standard normal random variates, Z_1 and Z_2 , with a correlation of the intermediate correlation value are generated. Then, the cumulative probability for each z_1 and z_2 is calculated. The values of these probabilities become the uniform deviates U_1 and U_2 . These uniform deviates are then used in the following equations

$$x_1 = \lambda_{11} + \frac{u_1^{\lambda_{13}} - (1 - u_1)^{\lambda_{14}}}{\lambda_{12}}$$

$$x_2 = \lambda_{21} + \frac{u_2^{\lambda_{23}} - (1 - u_2)^{\lambda_{24}}}{\lambda_{22}}$$

to generate the bivariate distributions with the desired skewness, kurtoses, and correlation.

Chapter 3

Simulation Study

The purpose of this study is to compare three different methods for generating bivariate, non-normally distributed random variables. To do this, bivariate random variables with zero mean, unit variance, and specified skewness, kurtoses, and correlation were generated using the Fleishman power method (Headrick and Sawilowsky, 1999), the fifth-order polynomial transform method (Headrick, 2002), and the generalized lambda distribution (GLD) method (Headrick and Mugdadi, 2006). The methods were then evaluated on their accuracy in producing the specified distributions with the desired correlation, ease of use, and time needed to produce the variables.

The programs used to generate the bivariate distributions were written in Fortran 90 for Windows on a Dell Optiplex GX260 computer. A total of 242 combinations of 22 distributions were generated with correlations of 0.1, 0.5, and 0.9 for each combination. The 22 distributions include seven symmetrical distributions: Gaussian, Logistic, Uniform, Laplace, Triangular, $t(7df)$, and $t(10df)$, seven χ^2 distributions with varying degrees of freedom of $\nu = 1, 2, 3, 4, 8, 16,$ and 32 , four Beta distributions with parameters $(\alpha=4, \beta=4), (\alpha=4, \beta=2), (\alpha=4, \beta=3/2),$ and $(\alpha=4, \beta=5/4)$, the Weibull $(\alpha=6, \beta=10)$ distribution, the Gamma $(\alpha=\beta=10)$ distribution, the Rayleigh $(\alpha=1/2, \mu=\sqrt{\pi/2})$ distribution, and the Pareto $(\theta=10, \alpha=1)$ distribution. Eight of these distributions are symmetrical with kurtoses ranging from -0.545455 to 3. The remaining distributions have skewnesses ranging from -0.848164 for the Beta $(\alpha=4, \beta=5/4)$ distribution to 2.811057 for the Pareto distribution. The values of kurtoses range from -0.545455 for the

Beta ($\alpha=4, \beta=4$) distribution to 14.828571 for the Pareto distribution. Sample sizes of 1,000,000 were produced for each combination in order to evaluate the accuracy of each method. Sample sizes of 30, 100, and 1,000 were simulated 10,000 times with selected combinations to see how the methods performed with smaller sample sizes.

3.1 Fleishman Power Method

The steps for generating bivariate random variables using the Fleishman power method (Headrick and Sawilowsky, 1999) are as follows:

1. Given the parameters $\mu = 0, \sigma^2 = 1$, and the desired values for skewness and kurtosis, γ_1 and γ_2 respectively:
2. Obtain the Fleishman constants, a, b, c, d , for each variable using equations (2.2).
3. Solve for the intermediate correlation, $\rho_{Z_1 Z_2}$, in equation (2.4).
4. Generate four independent standard normal random variables,

Z_1^*, Z_2^*, E_1, E_2 , and let $r = \sqrt{\rho_{Z_1 Z_2}}$. Substitute these values into the equations:

$$Z_1 = rZ_1^* + \sqrt{1-r^2} E_1 \quad (3.1)$$

$$Z_2 = rZ_2^* + \sqrt{1-r^2} E_2 \quad (3.2)$$

to generate standard random normal deviates, Z_1 and Z_2 correlated at the intermediate level.

5. Substitute Z_1 and Z_2 into the Fleishman equations:

$$X_1 = a_1 + b_1 Z_1 + c_1 Z_1^2 + d_1 Z_1^3$$

$$X_2 = a_2 + b_2 Z_2 + c_2 Z_2^2 + d_2 Z_2^3$$

to generate the bivariate non-normal distributions with the desired skewness, kurtoses, and correlation.

3.2 Fifth-Order Polynomial Transformation Method

In order to generate bivariate non-normal deviates using the fifth-order polynomial transformation method (Headrick, 2002), the following steps are taken:

1. Given the parameters $\mu = 0$, $\sigma^2 = 1$, and the desired values for γ_1 , γ_2 , γ_3 , and γ_4 , where γ_3 and γ_4 are the fifth and sixth standardized cumulants respectively:
2. Obtain the constants, c_0 , c_1 , c_2 , c_3 , c_4 , and c_5 using equations (2.4), (2.5), (2.6), (2.7), and (2.8).
3. Solve for the intermediate correlation, $\rho_{Z_1 Z_2}$ with equation (2.9).
4. Let $r = \sqrt{\rho_{Z_1 Z_2}}$ and generate four independent standard normal random variables, Z_1^* , Z_2^* , E_1 , E_2 . Substitute those values into (3.1) and (3.2) to generate standard random normal deviates, Z_1 and Z_2 correlated at the intermediate level.
5. Substitute Z_1 and Z_2 into the equations:

$$X_1 = c_{0(1)} + c_{1(1)}Z_1 + c_{2(1)}Z_1^2 + c_{3(1)}Z_1^3 + c_{4(1)}Z_1^4 + c_{5(1)}Z_1^5$$

$$X_2 = c_{0(2)} + c_{1(2)}Z_2 + c_{2(2)}Z_2^2 + c_{3(2)}Z_2^3 + c_{4(2)}Z_2^4 + c_{5(2)}Z_2^5$$

to generate the bivariate non-normal distributions with the desired skewness, kurtoses, and correlation.

3.3 Generalized Lambda Distribution Method

Generating bivariate non-normal random variables with the generalized lambda distribution method (Headrick and Mugdadi, 2006) is done with the following procedure:

1. Given the parameters $\mu = 0$, $\sigma^2 = 1$, and the desired values for skewness and kurtosis, γ_1 and γ_2 respectively:
2. Solve for the lambdas for each distribution using equations (2.12), (2.13), (2.14), and (2.15).
3. Determine the intermediate correlation by solving for $\rho_{z_1 z_2}$ using partitioning steps of $\Delta u_1 = \Delta u_2 = 0.05$ and interpolating equation (2.18) until the correct value for $\rho_{z_1 z_2}$ is obtained.
4. Generate two standard normal distributions, Z_1 and Z_2 , with the intermediate correlation, $\rho_{Z_1 Z_2}$.
5. Find the cumulative probability for each z_1 and z_2 to create uniform deviates, U_1 and U_2 .
6. Substitute U_1 and U_2 into the equations:

$$x_1 = \lambda_{11} + \frac{u_1^{\lambda_{13}} - (1-u_1)^{\lambda_{14}}}{\lambda_{12}}$$

$$x_2 = \lambda_{21} + \frac{u_2^{\lambda_{23}} - (1-u_2)^{\lambda_{24}}}{\lambda_{22}}$$

to generate the bivariate distributions with the desired skewness, kurtoses, and correlation.

In step three there were cases when it was necessary to use partitioning steps of 0.02 to solve for the correct intermediate correlation. This would significantly decrease the efficiency of the program.

In all three methods, the mean, variance, skewness, and kurtosis were calculated for X_1 and X_2 , as well as the correlation between X_1 and X_2 . These values were then

compared to the desired values for the parameters and correlation. Times needed to generate each method were recorded as well.

Chapter 4

Simulation Results

4.1 Comparison of Accuracy

Tables 1-10 show a sample of the comparisons of accuracy when sample sizes of 1,000,000 were generated for each combination of distributions. This sample size of $N = 1,000,000$ is used to analyze each method so there is less risk of generating biased sample estimates which could occur with smaller sample sizes. Appendix I contains the comprehensive results from the combinations of all 22 distributions.

While the sample sizes of 1,000,000 give a good overall indication of how well each method performs, realistically the methods will be used to generate smaller samples for real life analysis. Tables 11-17 show comparisons of accuracy for the correlation between the variables of two distributions, and for the values of the mean, variance, skewness, and kurtosis for each variable generated with smaller samples. Independent sample sizes of 30, 100, and 1,000 were simulated 10,000 times. Values for the correlation, means, variances, skewness, and kurtoses were calculated for each simulation. The average of the absolute differences between each of these values and the desired value was then calculated.

Some observations can be made in terms of comparisons of accuracy between the methods. In general, as correlation increases precision tends to increase across methods. Different values for skewness and kurtosis also have an effect on accuracy. In addition, increasing sample size give values that are in closer proximity to the desired parameters.

4.1.1 Accuracy as correlation changes

Within samples sizes, precision increases in calculating correlation as the desired correlation increases. There does not appear to be an effect on accuracy for the mean, variance, skewness, or kurtosis as correlation changes.

There are cases (see tables 1 and 11) in which the generalized lambda distribution (GLD) method (Headrick and Mugdadi, 2006) was not able to calculate an accurate value for the intermediate correlation with a delta value of 0.05, where the delta value is the step size for equation (2.16). This seemed to occur most often when the desired correlation was equal to 0.1 or 0.9. A solution to this issue is to change the delta value to 0.025 or lower. This gives accurate values for the intermediate correlation for all combinations except for the case with two correlated Gaussian distributions. In trying to generate random variables from two correlated Gaussian distributions, the GLD method could not calculate an accurate intermediate correlation. Attempts were made using delta values of 0.05, 0.02, and 0.01.

When the desired correlation is 0.5 or greater, there are circumstances in which the Fleishman power method (Headrick and Sawilowsky, 1999) and the fifth-order polynomial transformation method (Headrick, 2002) do not give viable solutions for the intermediate correlation (see table 14). When using equations (2.3) and (2.9) to solve for the intermediate correlation the obtained solutions are greater than one, which is outside the possible range for the value of correlation. The Maple program was used to confirm these results. These complications generally occur when one of the distributions has a very heavy tail with a kurtosis at least equal to four. Further study is needed to determine why this occurs. In most of these cases, the GLD method does give the desired

correlation, or a close approximation, which would make it a possible alternative. There are a few combinations of distributions in which no method gives a value close to the desired correlation when the correlation is equal to 0.09 (see table 7). It would require more studying to see why these methods do not perform well with the larger correlations. Possibly, another method could be developed that would do well in these situations.

4.1.2 Accuracy as skewness and kurtoses change

As a general rule, the values for skewness, γ_1 , and kurtosis, γ_2 , are bounded by the equation (Devroye, 1986, p. 688):

$$\gamma_2 \geq \gamma_1^2 - 2. \tag{4.1}$$

Neither the Fleishman power method nor the fifth-order polynomial transformation method covers the entire plane defined by (4.1), but the fifth-order polynomial transformation method covers a wider range of the plane. For instance, it is not possible to simulate a uniform distribution using the Fleishman power method as the lower boundary point of kurtosis for the given value of $\gamma_1 = 0$ is $\gamma_2 = -1.15132$. Headrick (2002) gives a table for the lower bounds of kurtosis given the values of γ_1 , γ_3 , and γ_4 for both the Fleishman power method and fifth-order polynomial transform method, which is summarized in Appendix A. Headrick and Mugdadi (2006) do not discuss the bounds for skewness and kurtosis for the GLD method, but simulations were possible for all of the combinations skewness and kurtosis generated in this study.

For all three methods, accuracy in producing the desired skewness and kurtosis decreased as the values for the desired skewness and kurtosis increased. The Fleishman power method appears to perform best with generating distributions with heavy tails, but

it also is most likely to have issues with calculating the intermediate correlation when the desired kurtosis is large. At this time there is no explanation for why the Fleishman method does better with generating distributions with heavy tails but also has intermittent problems with calculating the intermediate correlation. Additional study would be needed in order to determine why this occurs.

4.1.3 Accuracy as sample size changes

Accuracy remains fairly consistent between methods regardless of sample size. In generating sample sizes of 30, the absolute differences between the generated parameters and desired parameters varies greatly depending on the value of the desired parameter. As mentioned in the previous section, accuracy diminishes for higher values of skewness and kurtosis. Headrick (2002) notes that “the higher the standardized cumulant simulated, the larger the sample size required to obtain a very close agreement to the population parameters.” This is true for all three methods.

In Appendix I, where sample sizes of 1,000,000 were generated for all combinations for each method, it appears that the fifth-order polynomial transformation method is the most accurate overall.

4.2 Comparison of Ease of Use

Of the three methods, the Fleishman power method (Headrick and Sawilowsky, 1999) is the easiest to use. Generating random variable using the GLD method can be very time consuming, which is discussed in the next section. Another drawback to the GLD method is that the initial guesses for solving the lambdas needs to change for

different combinations of skewness and kurtosis. If λ_3 and λ_4 are both negative, the initial guess must be negative. Positive values for λ_3 and λ_4 require positive initial guesses. The book *Fitting Statistical Distributions: The Generalized Lambda Distribution and Generalized Bootstrap Methods* (Karian & Dudewicz, 2000) gives an extensive list of combinations of skewness and kurtoses with their corresponding lambdas, but it is not entirely comprehensive. Therefore, some estimation needs to be made when choosing the initial guesses. Therefore, if the initial guess is wrong changes need to be made within the program.

The fifth-order polynomial transformation method (Headrick, 2002) uses lengthy equations to solve for the constants, c_0 , c_1 , c_2 , c_3 , c_4 , and c_5 , which increases the probability for error when inputting the formulas into the simulation program. Also, it is likely that the fifth and sixth standardized cumulants of the desired distribution are not known. In this case, it is not possible to use the fifth-order polynomial transform method. The distributions and their associated standardized cumulants of γ_1 , γ_2 , γ_3 , and γ_4 , are listed in Appendix B.

The Fleishman power method is simpler to code than the fifth order polynomial transform method, while needing much less time than the GLD method to produce the variables. It is nearly as accurate as the fifth-order polynomial transform method, but there are more instances when it is unable to give the correct intermediate correlation than the fifth-order polynomial transform method. Further analysis is needed to determine why this happens.

4.3 Comparison of Efficiency

Generation of bivariate non-normally distributed random variables using the Fleishman power method (Headrick and Sawilowsky, 1999) and the fifth order polynomial transform method (Headrick, 2002) generally took less than one second. The GLD method (Headrick and Mugdadi, 2006) relies on an interpolation of a Reimann sum to calculate the intermediate correlation. The time needed for the interpolation depends on the size of the partitioning steps, or delta value. Using a partitioning step of 0.05 for calculating the intermediate correlation in the GLD method (Headrick and Mugdadi, 2006) would on average take just under three minutes. If it was necessary to use a delta value of 0.025 the time would increase to approximately 20 minutes, a delta value of 0.02 would take 40 minutes, and a delta value of 0.01 would be close to five hours.

Table 1
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	
<i>Desired Parameters</i>		ρ	0.2007**	0.0998	0.0998	0.6009**	0.4997	0.4992	1.000**	0.8997	0.8997
Gaussian	0.0000	μ	-0.0006	0.0000	0.0000	0.0007	0.0018	-0.0001	-0.0007	0.0020	-0.0001
	1.0000	σ^2	0.9971	1.0001	1.0001	1.0028	1.0012	0.9987	1.0006	1.0003	0.9975
	0.0000	γ_1	0.0001	-0.0017	-0.0017	-0.0013	0.0013	0.0004	0.0038	0.0030	0.0027
Gaussian	0.0000	γ_2	0.0011	0.0034	0.0034	0.0003	0.0028	0.0068	-0.0021	0.0093	0.0074
	0.0000	μ	0.0004	-0.0003	-0.0003	0.0012	0.0042	-0.0003	-0.0007	0.0031	-0.0002
	1.0000	σ^2	0.9994	1.0005	1.0005	0.9999	1.0061	0.9993	1.0006	1.0027	0.9978
	0.0000	γ_1	0.0013	-0.0032	-0.0032	-0.0024	0.0162	-0.0009	0.0038	0.0119	0.0016
	0.0000	γ_2	-0.0023	0.0003	0.0003	0.0011	0.0014	-0.0005	-0.0021	0.0060	0.0008
<i>Desired Parameters</i>		ρ	0.1001*	0.0999	0.0995	0.4999	0.4980	0.5001	0.8996	0.8997	0.8998
Triangular	0.0000	μ	0.0009	0.0000	-0.0007	0.0009	0.0017	0.0003	0.0009	0.0024	-0.0017
	1.0000	σ^2	1.0025	0.9997	1.0017	1.0025	1.0007	0.9992	1.0025	0.9987	1.0004
	0.0000	γ_1	-0.0030	-0.0003	0.0007	-0.0030	-0.0008	0.0032	-0.0030	0.0024	-0.0086
Beta ($\alpha=4, \beta=4$)	-0.6000	γ_2	-0.6017	-0.6011	-0.5870	-0.6017	-0.6018	-0.5847	-0.6017	-0.5914	-0.5850
	0.0000	μ	0.0000	-0.0006	-0.0016	0.0004	0.0034	0.0000	0.0008	0.0039	-0.0019
	1.0000	σ^2	1.0005	0.9991	1.0006	1.0010	1.0002	0.9988	1.0022	0.9989	1.0007
	0.0000	γ_1	-0.0008	0.0010	-0.0010	-0.0020	0.0186	0.0013	-0.0029	-0.0003	-0.0088
	-0.5455	γ_2	-0.5477	-0.5449	-0.5502	-0.5467	-0.5467	-0.5484	-0.5461	-0.5371	-0.5494

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

** attempts were made to calculate an accurate correlation using delta values of 0.025, 0.02, and 0.01

Table 2
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	
<i>Desired Parameters</i>	ρ	0.1000*	0.1005	0.0989	0.5019*	0.4993	0.4986	0.9036*	0.8997	0.9004	
Weibull	0.0000	μ	-0.0003	-0.0005	0.0003	0.0014	0.0000	-0.0005	0.0000	0.0001	0.0002
($\alpha=6, \beta=10$)	1.0000	σ^2	1.0010	1.0010	1.0010	1.0017	0.9987	1.0002	1.0008	0.9972	1.0016
	-0.3733	γ_1	-0.3739	-0.3764	-0.3702	-0.3758	-0.3739	-0.3745	-0.3736	-0.3714	-0.3724
	0.0355	γ_2	0.0261	0.0398	0.0325	0.0376	0.0393	0.0398	0.0420	0.0370	0.0343
Weibull	0.0000	μ	-0.0002	-0.0015	0.0008	0.0009	-0.0002	0.0009	0.0000	0.0000	-0.0002
($\alpha=6, \beta=10$)	1.0000	σ^2	0.9991	0.9969	0.9998	1.0000	0.9994	0.9982	1.0007	0.9976	1.0022
	-0.3733	γ_1	-0.3753	-0.3715	-0.3737	-0.3764	-0.3738	-0.3718	-0.3727	-0.3711	-0.3690
	0.0355	γ_2	0.0363	0.0265	0.0294	0.0409	0.0339	0.0378	0.0361	0.0327	0.0279
<i>Desired Parameters</i>	ρ	0.0982	0.0999	0.0998	0.4919	0.4973	0.5004	0.9059*	0.8995	0.8999	
Beta	0.0000	μ	0.0000	-0.0004	0.0000	-0.0003	0.0020	0.0006	0.0009	-0.0039	-0.0002
($\alpha=4, \beta=3/2$)	1.0000	σ^2	1.0003	0.9996	0.9992	1.0026	0.9999	0.9985	0.9983	0.9970	0.9995
	-0.6939	γ_1	-0.6960	-0.6950	-0.6921	-0.6944	-0.6974	-0.6886	-0.6946	-0.6921	-0.6963
	-0.0686	γ_2	-0.0640	-0.0633	-0.0697	-0.0723	-0.0730	-0.0732	-0.0690	-0.0834	-0.0665
Beta	0.0000	μ	-0.0003	-0.0001	-0.0005	0.0007	0.0002	-0.0008	0.0008	-0.0043	-0.0005
($\alpha=4, \beta=5/4$)	1.0000	σ^2	0.9987	1.0005	1.0003	0.9979	0.9989	1.0018	0.9974	0.9946	1.0009
	-0.8482	γ_1	-0.8443	-0.8491	-0.8468	-0.8473	-0.8385	-0.8466	-0.8477	-0.8442	-0.8514
	0.2210	γ_2	0.2102	0.2222	0.2158	0.2195	0.2071	0.2184	0.2197	0.1956	0.2201

GLD = Generalized Lambda Distribution Method, FPM = Fliesman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 3
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9		
Method:		<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>
<i>Desired Parameters</i>	ρ	0.1014	0.0998	0.0996	0.5086	0.5018	0.4994	0.8735		
Gaussian	0.0000	μ	0.0008	0.0012	0.0007	0.0008	0.0017	0.0009	-0.0008	
	1.0000	σ^2	1.0031	0.9997	0.9993	1.0000	0.9967	0.9990	0.9991	
	0.0000	γ_1	-0.0037	-0.0009	-0.0005	-0.0024	-0.0031	-0.0018	0.0007	unable to calculate
	0.0000	γ_2	0.0000	0.0018	-0.0097	0.0020	0.0186	-0.0031	0.0163	intermediate correlation
$\chi^2_{(1)}$	0.0000	μ	0.0000	0.0004	0.0009	0.0010	-0.0022	0.0005	-0.0008	unable to calculate
	1.0000	σ^2	0.9976	1.0027	1.0026	1.0009	0.9911	1.0011	1.0025	intermediate correlation
	2.8284	γ_1	2.5777	2.8379	2.8274	2.5905	2.7811	2.8283	2.6308	
	12.0000	γ_2	11.6070	12.1106	11.9865	11.8973	11.3773	11.9646	12.3417	
<i>Desired Parameters</i>	ρ	0.1012		0.0993	0.5056		0.4988	0.8982		
Uniform	0.0000	μ	-0.0002	0.0005	0.0013		0.0001	-0.0012		
	1.0000	σ^2	1.0001	unable to calculate	1.0018	1.0008	unable to calculate	1.0000	0.9994	
	0.0000	γ_1	0.0007	constants	-0.0015	-0.0027	constants	0.0018	0.0030	unable to calculate
	-1.2000	γ_2	-1.2006	constants	-1.2023	-1.2000	constants	-1.2006	-1.1996	unable to calculate
$\chi^2_{(3)}$	0.0000	μ	-0.0009	for	-0.0016	0.0004	for	-0.0001	-0.0005	intermediate correlation
	1.0000	σ^2	0.9983	uniform distribution	0.9980	0.9997	uniform distribution	1.0005	1.0038	intermediate correlation
	1.6330	γ_1	1.6299		1.6309	1.6216		1.6495	1.6491	
	4.0000	γ_2	3.9802		3.9570	3.9010		4.1872	4.0981	

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 4
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	<u>GLD</u>	<u>FPM</u>	<u>Fifth-Order</u>	
<i>Desired Parameters</i>	ρ	0.1003	0.1005	0.0980	0.5045	0.4983	0.5000	0.9106	0.8996	0.9002	
Logistic	0.0000	μ	-0.0004	-0.0007	-0.0017	-0.0011	0.0007	0.0016	-0.0003	0.0029	-0.0004
	1.0000	σ^2	0.9977	0.9997	0.9997	1.0011	1.0077	1.0020	1.0008	1.0003	0.9996
	0.0000	γ_1	-0.0057	-0.0018	-0.0037	-0.0051	-0.0066	0.0016	0.0019	-0.0112	0.0081
$\chi^2_{(4)}$	1.2000	γ_2	1.5814	1.1928	1.1545	1.2018	1.2902	1.2721	1.5919	1.2140	1.1878
	0.0000	μ	-0.0003	0.0019	-0.0002	0.0001	-0.0011	0.0009	0.0000	0.0023	-0.0003
	1.0000	σ^2	0.9968	1.0027	1.0007	1.0011	0.9987	1.0033	0.9998	1.0039	1.0005
	1.4142	γ_1	1.4083	1.4131	1.4127	1.4224	1.4164	1.4250	1.4167	1.4000	1.4165
	3.0000	γ_2	2.9784	2.9730	2.9889	3.0620	2.9707	3.0774	3.0238	2.9175	2.9808
<i>Desired Parameters</i>	ρ	0.0990		0.0994	0.4985		0.5009	0.8997		0.9003	
Uniform	0.0000	μ	0.0014		-0.0003	-0.0012	0.0008	0.0021		0.0004	
	1.0000	σ^2	1.0009	unable to calculate	1.0005	0.9989	unable to calculate	1.0007	0.9992	unable to calculate	1.0003
	0.0000	γ_1	-0.0017	constants	-0.0014	0.0002	constants	0.0031	-0.0036	constants	-0.0005
Triangular	-1.2000	γ_2	-1.2005	constants	-1.1965	-1.1987	constants	-1.2009	-1.1987	constants	-1.1988
	0.0000	μ	0.0016	for	-0.0002	-0.0021	for	0.0004	0.0015	for	0.0002
	1.0000	σ^2	0.9994	uniform distribution	0.9985	1.0002	uniform distribution	1.0006	0.9997	uniform distribution	1.0003
	0.0000	γ_1	-0.0019		-0.0005	-0.0001		0.0074	-0.0030		-0.0054
	-0.6000	γ_2	-0.5988		-0.5853	-0.5999		-0.5844	-0.5992		-0.5879

GLD = Generalized Lambda Distribution Method, FPM = Flieshrman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 5
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	
<i>Desired Parameters</i>		ρ	0.0991	0.0998	0.1018	0.4993	0.5011	0.5001	0.8988	0.8989	0.9001
Laplace	0.0000	μ	0.0021	-0.0018	-0.0006	-0.0011	-0.0002	0.0001	-0.0014	0.0028	0.0002
	1.0000	σ^2	1.0029	0.9991	1.0020	1.0023	0.9998	1.0026	1.0000	0.9985	1.0023
	0.0000	γ_1	0.0021	-0.0008	-0.0018	-0.0028	-0.0016	0.0070	-0.0140	0.0107	0.0052
t(10df)	3.0000	γ_2	3.0730	3.0935	2.9846	3.0231	2.8943	3.0506	3.0071	4.0793	3.0607
	0.0000	μ	0.0006	0.0007	0.0002	-0.0015	0.0022	0.0009	-0.0006	0.0022	0.0002
	1.0000	σ^2	0.9984	0.9973	1.0001	1.0000	1.0015	1.0008	1.0000	0.9936	1.0012
	0.0000	γ_1	-0.0046	0.0054	0.0045	-0.0009	0.0183	0.0098	-0.0088	0.0320	-0.0030
	1.0000	γ_2	0.9892	0.9689	1.0177	0.9860	1.1038	0.9775	0.9796	1.0398	0.9976
<i>Desired Parameters</i>		ρ	0.1004*	0.1012	0.0998	0.4984	0.4979	0.4993	0.8968	0.8997	0.8999
Triangular	0.0000	μ	0.0009	0.0003	-0.0007	0.0009	-0.0025	-0.0005	0.0009	-0.0059	-0.0016
	1.0000	σ^2	1.0025	0.9976	0.9985	1.0025	0.9988	1.0013	1.0025	0.9982	1.0001
	0.0000	γ_1	-0.0030	-0.0005	0.0021	-0.0030	-0.0077	0.0004	-0.0030	-0.0145	-0.0043
Weibull ($\alpha=6, \beta=10$)	-0.6000	γ_2	-0.6017	-0.5982	-0.5839	-0.6017	-0.5942	-0.5847	-0.6017	-0.5925	-0.5850
	0.0000	μ	-0.0001	0.0016	-0.0006	0.0002	0.0030	-0.0007	0.0005	-0.0066	-0.0021
	1.0000	σ^2	1.0006	0.9979	0.9995	1.0013	0.9857	1.0024	1.0028	0.9987	1.0016
	-0.3733	γ_1	-0.3743	-0.3729	-0.3735	-0.3758	-0.3612	-0.3752	-0.3767	-0.3868	-0.3797
	0.0355	γ_2	0.0333	0.0415	0.0336	0.0380	0.0477	0.0408	0.0397	0.0221	0.0358

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 6
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	
<i>Desired Parameters</i>	ρ	0.1015	0.0993	0.1002	0.5101	0.4985	0.5002	0.8848			
t(7df)	0.0000	μ	0.0006	-0.0005	0.0014	0.0006	0.0008	0.0001	0.0006		
	1.0000	σ^2	1.0041	1.0006	0.9967	1.0041	0.9982	1.0009	1.0041		
	0.0000	γ_1	-0.0039	-0.0038	-0.0185	-0.0039	-0.0034	-0.0114	-0.0039	unable to calculate	unable to calculate
$\chi^2_{(1)}$	2.0000	γ_2	1.9973	1.9647	1.9739	1.9973	1.8576	2.1943	1.9973	intermediate correlation	intermediate correlation
	0.0000	μ	0.0000	-0.0004	0.0008	0.0006	0.0031	0.0003	0.0015		
	1.0000	σ^2	0.9977	0.9994	1.0036	1.0012	1.0075	1.0046	1.0036		
	2.8284	γ_1	2.5777	2.8329	2.8398	2.6083	2.8400	2.8400	2.6059		
	12.0000	γ_2	11.6076	12.1251	12.1807	12.1493	11.8646	12.1067	12.1886		
<i>Desired Parameters</i>	ρ	0.1001	0.1010	0.1000	0.5029	0.5023	0.5002	0.9049	0.8995	0.9000	
t(10df)	0.0000	μ	0.0007	-0.0007	-0.0016	0.0007	0.0008	0.0007	0.0007	-0.0018	0.0012
	1.0000	σ^2	1.0037	0.9978	1.0001	1.0037	1.0019	1.0019	1.0037	1.0095	0.9981
	0.0000	γ_1	-0.0044	0.0026	0.0052	-0.0044	-0.0022	0.0014	-0.0044	-0.0209	0.0058
$\chi^2_{(16)}$	1.0000	γ_2	1.0020	0.9996	1.0061	1.0020	1.0291	1.0018	1.0020	0.9918	1.0204
	0.0000	μ	0.0000	-0.0003	-0.0005	0.0004	0.0060	0.0000	0.0009	-0.0010	0.0008
	1.0000	σ^2	1.0001	0.9997	1.0009	1.0008	1.0048	1.0000	1.0026	1.0023	1.0008
	0.7071	γ_1	0.7040	0.7072	0.7072	0.7039	0.7321	0.7097	0.7045	0.6900	0.7117
	0.7500	γ_2	0.7385	0.7465	0.7487	0.7473	0.7663	0.7615	0.7504	0.7312	0.7581

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 7
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9		
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order
<i>Desired Parameters</i> $\chi^2_{(1)}$	ρ	0.1010	0.1002	0.1014	0.4686		0.5005	0.7328		
	0.0000	μ	0.0015	-0.0017	-0.0012	0.0015		0.0014	0.0015	
	1.0000	σ^2	1.0035	0.9950	0.9942	1.0035		1.0019	1.0035	
	2.8284	γ_1	2.6052	2.8175	2.8193	2.6052	unable to calculate	2.8212	2.6052	unable to calculate
	12.0000	γ_2	12.1789	11.7807	11.9213	12.1789	intermediate correlation	11.9683	12.1789	intermediate correlation
Beta ($\alpha=4, \beta=5/4$)	0.0000	μ	-0.0001	-0.0012	0.0004	0.0002		0.0013	0.0004	
	1.0000	σ^2	1.0008	0.9998	0.9980	1.0017		1.0002	1.0031	
	-0.8482	γ_1	-0.8481	-0.8456	-0.8480	-0.8500		-0.8494	-0.8512	
	0.2210	γ_2	0.2196	0.2126	0.2248	0.2245		0.2244	0.2260	
<i>Desired Parameters</i> $\chi^2_{(2)}$	ρ	0.1026	0.1005	0.1005	0.5145	0.5028	0.5009	0.9247	0.9004	0.9000
	0.0000	μ	0.0015	0.0006	0.0003	0.0015	-0.0003	0.0008	0.0015	0.0019
	1.0000	σ^2	1.0029	0.9991	0.9989	1.0029	0.9979	1.0033	1.0029	1.0073
	2.0000	γ_1	1.9959	1.9862	1.9992	1.9959	2.0039	2.0090	1.9959	2.0374
	6.0000	γ_2	6.0176	5.8673	6.0437	6.0176	6.0052	6.0975	6.0176	6.1701
$\chi^2_{(3)}$	0.0000	μ	0.0000	0.0010	0.0005	0.0005	0.0022	-0.0009	0.0012	0.0017
	1.0000	σ^2	0.9992	1.0013	0.9979	1.0006	1.0064	1.0014	1.0028	1.0136
	1.6330	γ_1	1.6262	1.6342	1.6185	1.6323	1.6202	1.6282	1.6337	1.6768
	4.0000	γ_2	3.9477	4.0144	3.8914	4.0159	3.9974	3.9798	4.0195	4.1904

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 8
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	
<i>Desired Parameters</i>	ρ	0.1009	0.0997	0.1012	0.5071	0.4990	0.4997	0.9133	0.9004	0.9001	
$\chi^2_{(4)}$	0.0000	μ	-0.0005	-0.0014	-0.0005	-0.0001	0.0011	-0.0012	-0.0004	0.0031	0.0010
	1.0000	σ^2	0.9996	0.9953	0.9990	1.0022	1.0008	0.9990	0.9993	1.0016	1.0004
	1.4142	γ_1	1.4161	1.4162	1.4168	1.4183	1.4120	1.4187	1.4099	1.3981	1.4169
	3.0000	γ_2	3.0145	3.0269	3.0121	3.0289	3.0426	3.0397	2.9724	2.8539	3.0565
$\chi^2_{(32)}$	0.0000	μ	-0.0012	0.0000	-0.0003	-0.0004	-0.0007	-0.0019	-0.0002	0.0030	0.0007
	1.0000	σ^2	1.0020	0.9991	1.0010	1.0017	1.0043	0.9975	1.0014	0.9985	1.0000
	0.5000	γ_1	0.5042	0.4957	0.5053	0.4995	0.4871	0.4920	0.4984	0.4958	0.5003
	0.3750	γ_2	0.3850	0.3627	0.3909	0.3783	0.3255	0.3730	0.3674	0.3271	0.3757
<i>Desired Parameters</i>	ρ	0.1001	0.0988	0.0986	0.4995	0.4996	0.5005	0.8883			
$\chi^2_{(8)}$	0.0000	μ	-0.0002	0.0004	-0.0017	-0.0002	-0.0022	0.0000	-0.0008		
	1.0000	σ^2	0.9999	0.9987	0.9999	1.0004	0.9926	0.9991	0.9974	unable to calculate	unable to calculate
	1.0000	γ_1	1.0004	0.9974	0.9970	0.9975	0.9859	0.9998	1.0009	intermediate correlation	intermediate correlation
	1.5000	γ_2	1.5007	1.4769	1.4714	1.4801	1.4734	1.5209	1.5037	intermediate correlation	intermediate correlation
Beta ($\alpha=4, \beta=5/4$)	0.0000	μ	0.0004	0.0000	0.0006	0.0000	-0.0038	0.0007	-0.0003	correlation	correlation
	1.0000	σ^2	0.9974	1.0016	0.9978	1.0006	1.0024	1.0016	0.9986		
	-0.8482	γ_1	-0.8470	-0.8481	-0.8493	-0.8477	-0.8428	-0.8477	-0.8460		
	0.2210	γ_2	0.2202	0.2166	0.2253	0.2184	0.2153	0.2218	0.2135		

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 9

n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	
<i>Desired Parameters</i>		ρ	0.1024	0.1002	0.1010	0.5027	0.4962	0.4992	0.9055	0.9007	0.9002
Beta ($\alpha=4, \beta=4$)	0.0000	μ	-0.0006	0.0004	0.0014	0.0006	0.0020	-0.0005	-0.0005	0.0034	0.0012
	1.0000	σ^2	1.0000	1.0001	0.9990	0.9991	1.0007	0.9996	1.0012	1.0040	1.0003
	0.0000	γ_1	0.0022	-0.0008	-0.0019	-0.0034	-0.0044	-0.0047	0.0017	0.0194	0.0001
	-0.5455	γ_2	-0.5478	-0.5459	-0.5477	-0.5446	-0.5463	-0.5446	-0.5480	-0.5458	-0.5480
Gamma ($\alpha=\beta=10$)	0.0000	μ	0.0009	0.0002	-0.0012	0.0001	-0.0017	-0.0013	-0.0007	0.0034	0.0013
	1.0000	σ^2	1.0003	0.9984	1.0010	0.9999	1.0009	0.9994	0.9997	1.0067	1.0020
	0.8222	γ_1	0.8221	0.8227	0.8195	0.8281	0.8124	0.8214	0.8235	0.8279	0.8204
	0.6000	γ_2	0.6037	0.6012	0.6161	0.6221	0.6077	0.6212	0.6103	0.5599	0.5872
<i>Desired Parameters</i>		ρ	0.0997	0.1007	0.0991	0.4989	0.4996	0.4992	0.8980	0.8997	0.8997
Beta ($\alpha=4, \beta=3/2$)	0.0000	μ	-0.0003	-0.0003	-0.0010	-0.0002	0.0042	0.0004	-0.0013	-0.0008	0.0004
	1.0000	σ^2	1.0033	1.0013	1.0005	0.9989	0.9963	1.0009	1.0030	0.9965	1.0003
	-0.6939	γ_1	-0.6932	-0.6946	-0.6920	-0.6946	-0.6878	-0.6697	-0.6943	-0.7015	-0.6921
	-0.0686	γ_2	-0.0729	-0.0639	-0.0749	-0.0687	-0.0977	0.5110	-0.0693	-0.0623	-0.0659
Rayleigh ($\alpha=1/2, \mu=\sqrt{(\pi/2)}$)	0.0000	μ	-0.0004	-0.0001	0.0002	0.0000	0.0076	-0.0006	-0.0008	-0.0017	0.0002
	1.0000	σ^2	1.0001	0.9990	1.0012	0.9999	1.0094	0.9995	1.0025	0.9963	1.0006
	0.6311	γ_1	0.6318	0.6293	0.6310	0.6314	0.6679	0.6332	0.6309	0.6161	0.6343
	0.2451	γ_2	0.2415	0.2417	0.2489	0.2483	0.2928	0.2416	0.2522	0.2505	0.2461

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 10
n=1, 000, 000

		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9		
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order
<i>Desired Parameters</i>	ρ	0.0985	0.0993	0.1011	0.5053	0.4990	0.4995	0.9088	0.8997	0.8999
Gamma	0.0000	μ	0.0001	0.0023	-0.0010	0.0001	-0.0003	0.0013	-0.0015	-0.0010
($\alpha=\beta=10$)	1.0000	σ^2	0.9985	1.0023	1.0012	1.0014	0.9987	1.0020	0.9992	0.9993
	0.8222	γ_1	0.6267	0.8225	0.8278	0.6310	0.8245	0.8263	0.6287	0.8262
	0.6000	γ_2	0.2348	0.6051	0.6004	0.2406	0.6189	0.6681	0.2360	0.6238
Gamma	0.0000	μ	0.0012	0.0001	0.0002	-0.0007	-0.0004	0.0008	-0.0019	-0.0005
($\alpha=\beta=10$)	1.0000	σ^2	0.9979	1.0004	0.9994	1.0002	0.9988	0.9994	0.9994	0.9999
	0.8222	γ_1	0.6327	0.8242	0.8247	0.6305	0.8215	0.8288	0.6290	0.8238
	0.6000	γ_2	0.2488	0.6097	0.5934	0.2392	0.6015	0.6013	0.2329	0.6071
<i>Desired Parameters</i>	ρ	0.1007	0.1015	0.0998	0.5016	0.5018		0.8189		
Weibull	0.0000	μ	-0.0022	0.0016	0.0019	0.0006	0.0022	-0.0004		
($\alpha=6, \beta=10$)	1.0000	σ^2	1.0011	0.9960	0.9996	1.0014	0.9952	0.9987		
	-0.3733	γ_1	-0.3720	-0.3741	-0.3721	-0.3758	-0.3605	unable to calculate	-0.3738	unable to calculate
	0.0355	γ_2	0.0346	0.0301	0.0313	0.0363	0.0294	intermediate correlation	0.0361	intermediate correlation
Pareto	0.0000	μ	-0.0003	0.0010	0.0020	0.0011	-0.0001	intermediate correlation	-0.0010	intermediate correlation
($\theta=10, \alpha=1$)	1.0000	σ^2	0.9951	1.0012	1.0055	1.0019	0.9846	0.9960		
	2.8111	γ_1	2.7732	2.8248	2.8005	2.8358	2.6974	2.8201		
	14.8286	γ_2	13.9209	15.1452	14.2546	15.2307	12.9086	15.4302		

GLD = Generalized Lambda Distribution Method, FPM = Flieshman Power Method, and Fifth-Order = Fifth-Order Polynomial Transformation Method

ρ = correlation, μ = mean, σ^2 = variance, γ_1 = skewness, and γ_2 = kurtosis

* a delta value of 0.025 was needed to calculate an accurate correlation

Table 11

Average Absolute Differences between Generated Parameters and Desired Parameters

			Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method			
<i>n</i>		<i>correlation</i>	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9	
<i>n</i> = 30	Gaussian	Correlation		0.149	0.113	0.029	0.149	0.113	0.029	0.166	0.130	0.100
		Mean	0.000	0.146	0.146	0.145	0.146	0.146	0.145	0.146	0.146	0.146
		Variance	1.000	0.207	0.206	0.204	0.207	0.206	0.204	0.205	0.205	0.205
		skewness	0.000	0.318	0.320	0.320	0.318	0.320	0.320	0.323	0.323	0.323
		kurtosis	0.000	0.576	0.580	0.578	0.576	0.580	0.578	0.566	0.566	0.566
	Gaussian	mean	0.000	0.145	0.145	0.144	0.145	0.145	0.144	0.147	0.147	0.146
		variance	1.000	0.205	0.205	0.205	0.205	0.205	0.205	0.205	0.207	0.205
		skewness	0.000	0.317	0.316	0.315	0.317	0.316	0.315	0.316	0.318	0.323
		kurtosis	0.000	0.575	0.580	0.581	0.575	0.580	0.581	0.562	0.564	0.566
<i>n</i> =100												
Gaussian	correlation		0.079	0.060	0.015	0.079	0.060	0.015	0.115	0.103	0.100	
	mean	0.000	0.080	0.079	0.079	0.080	0.079	0.079	0.080	0.080	0.080	
	variance	1.000	0.113	0.113	0.113	0.113	0.113	0.113	0.114	0.114	0.114	
	skewness	0.000	0.188	0.189	0.189	0.188	0.189	0.189	0.186	0.186	0.186	
	kurtosis	0.000	0.357	0.356	0.357	0.357	0.356	0.357	0.340	0.340	0.340	
Gaussian	mean	0.000	0.080	0.080	0.079	0.080	0.080	0.079	0.080	0.080	0.080	
	variance	1.000	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.114	
	skewness	0.000	0.187	0.187	0.190	0.187	0.187	0.190	0.184	0.187	0.186	
	kurtosis	0.000	0.354	0.359	0.357	0.354	0.359	0.357	0.339	0.341	0.340	
<i>n</i> =1,000												
Gaussian	correlation		0.025	0.019	0.005	0.025	0.019	0.005	0.100	0.100	0.100	
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	
	variance	1.000	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	
	skewness	0.000	0.062	0.062	0.062	0.062	0.062	0.062	0.059	0.059	0.059	
	kurtosis	0.000	0.122	0.123	0.124	0.122	0.123	0.124	0.112	0.112	0.112	
Gaussian	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	
	variance	1.000	0.035	0.035	0.036	0.035	0.035	0.036	0.036	0.035	0.036	
	skewness	0.000	0.061	0.062	0.062	0.061	0.062	0.062	0.060	0.060	0.059	
	kurtosis	0.000	0.121	0.123	0.120	0.121	0.123	0.120	0.113	0.113	0.112	

Table 12

Average Absolute Differences between Generated Parameters and Desired Parameters

		Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method		
<i>n</i> = 30	correlation	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
Laplace	correlation	0.149	0.110	0.024	0.231	0.110	0.025	0.149	0.111	0.025
	mean	0.000	0.145	0.146	0.144	0.146	0.146	0.144	0.146	0.146
	variance	1.000	0.312	0.313	0.311	0.306	0.306	0.304	0.300	0.300
	skewness	0.000	0.638	0.644	0.646	0.602	0.604	0.606	0.597	0.597
Beta (4,4)	kurtosis	3.000	2.254	2.258	2.254	2.414	2.418	2.412	2.455	2.455
	mean	0.000	0.145	0.145	0.144	0.145	0.145	0.144	0.147	0.147
	variance	1.000	0.178	0.177	0.176	0.177	0.177	0.176	0.176	0.178
	skewness	0.000	0.244	0.243	0.243	0.242	0.241	0.241	0.244	0.247
	kurtosis	-0.545	0.344	0.350	0.346	0.341	0.344	0.340	0.344	0.342
<i>n</i> =100										
Laplace	correlation	0.079	0.058	0.014	0.079	0.058	0.015	0.080	0.059	0.015
	mean	0.000	0.079	0.079	0.079	0.079	0.079	0.079	0.080	0.080
	variance	1.000	0.175	0.176	0.177	0.173	0.174	0.175	0.173	0.173
	skewness	0.000	0.489	0.492	0.492	0.478	0.480	0.481	0.471	0.471
Beta (4,4)	kurtosis	3.000	1.673	1.681	1.684	1.917	1.930	1.943	1.979	1.979
	mean	0.000	0.080	0.080	0.079	0.080	0.080	0.079	0.080	0.080
	variance	1.000	0.097	0.097	0.097	0.097	0.097	0.097	0.096	0.097
	skewness	0.000	0.133	0.133	0.136	0.131	0.131	0.134	0.132	0.134
	kurtosis	-0.545	0.187	0.189	0.187	0.181	0.182	0.181	0.186	0.189
<i>n</i> =1,000										
Laplace	correlation	0.025	0.018	0.005	0.025	0.018	0.006	0.025	0.018	0.005
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.057	0.057	0.057	0.057	0.057	0.057	0.056	0.056
	skewness	0.000	0.194	0.195	0.193	0.211	0.217	0.218	0.215	0.215
Beta (4,4)	kurtosis	3.000	0.729	0.736	0.740	1.022	1.093	1.144	1.147	1.147
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
	skewness	0.000	0.042	0.043	0.043	0.041	0.042	0.042	0.042	0.042
	kurtosis	-0.545	0.059	0.060	0.059	0.056	0.057	0.056	0.059	0.058

Table 13

Average Absolute Differences between Generated Parameters and Desired Parameters

		Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method		
<i>n</i> = 30		0.10	0.50	0.90	0.10	0.50	0.90	0.10	0.50	0.90
χ^2 (1df)	correlation									
	mean	0.000								
	variance	1.000								
	skewness	2.828								
	kurtosis	12.000								
Pareto	mean	0.000								
	variance	1.000								
	skewness	2.811								
	kurtosis	14.283								
<i>n</i> =100										
χ^2 (1df)	correlation									
	mean	0.000								
	variance	1.000								
	skewness	2.828								
	kurtosis	12.000								
Pareto	mean	0.000								
	variance	1.000								
	skewness	2.811								
	kurtosis	14.283								
<i>n</i> =1,000										
χ^2 (1df)	correlation									
	mean	0.000								
	variance	1.000								
	skewness	2.828								
	kurtosis	12.000								
Pareto	mean	0.000								
	variance	1.000								
	skewness	2.811								
	kurtosis	14.283								

Table 14

Average Absolute Differences between Generated Parameters and Desired Parameters

			Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method		
<i>n</i> = 30			0.10	0.50	0.90	0.10	0.50	0.90	0.10	0.50	0.90
	correlation		0.146	0.097		0.146	0.097		0.146	0.099	0.026
χ^2 (8df)	mean	0.000	0.146	0.146	unable	0.146	0.146	unable	0.146	0.146	0.146
	variance	1.000	0.265	0.266	to	0.264	0.265	to	0.265	0.265	0.265
	skewness	1.000	0.435	0.439	calculate	0.435	0.439	calculate	0.440	0.440	0.440
	kurtosis	1.500	1.595	1.596	intermediate	1.600	1.602	intermediate	1.521	1.521	1.521
Beta (4,5/4)	mean	0.000	0.145	0.145	correlation	0.145	0.145	correlation	0.147	0.147	0.145
	variance	1.000	0.215	0.216		0.215	0.216		0.218	0.219	0.218
	skewness	-0.848	0.297	0.298		0.301	0.301		0.293	0.297	0.298
	kurtosis	0.221	0.804	0.810		0.824	0.831		0.790	0.789	0.798
<i>n</i> =100											
	correlation		0.077	0.051		0.077	0.051		0.078	0.052	0.017
χ^2 (8df)	mean	0.000	0.080	0.079	unable	0.080	0.079	unable	0.080	0.080	0.080
	variance	1.000	0.148	0.148	to	0.148	0.148	to	0.149	0.149	0.149
	skewness	1.000	0.278	0.279	calculate	0.279	0.279	calculate	0.267	0.267	0.267
	kurtosis	1.500	1.206	1.213	intermediate	1.214	1.221	intermediate	1.090	1.090	1.090
Beta (4,5/4)	mean	0.000	0.080	0.080	correlation	0.080	0.080	correlation	0.080	0.080	0.080
	variance	1.000	0.119	0.118		0.119	0.118		0.120	0.119	0.120
	skewness	-0.848	0.165	0.164		0.169	0.169		0.160	0.162	0.162
	kurtosis	0.221	0.494	0.488		0.520	0.514		0.470	0.478	0.477
<i>n</i> =1,000											
	correlation		0.024	0.016		0.024	0.016		0.024	0.016	0.012
χ^2 (8df)	mean	0.000	0.025	0.025	unable	0.025	0.025	unable	0.025	0.025	0.025
	variance	1.000	0.047	0.048	to	0.047	0.048	to	0.047	0.047	0.047
	skewness	1.000	0.101	0.102	calculate	0.101	0.102	calculate	0.091	0.091	0.091
	kurtosis	1.500	0.535	0.546	intermediate	0.542	0.554	intermediate	0.429	0.429	0.429
Beta (4,5/4)	mean	0.000	0.025	0.025	correlation	0.025	0.025	correlation	0.025	0.025	0.025
	variance	1.000	0.038	0.038		0.038	0.038		0.037	0.037	0.037
	skewness	-0.848	0.051	0.052		0.053	0.054		0.051	0.051	0.050
	kurtosis	0.221	0.156	0.156		0.168	0.169		0.152	0.152	0.150

Table 15

Average Absolute Differences between Generated Parameters and Desired Parameters

			Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method		
<i>n</i> = 30	correlation		0.10	0.50	0.90	0.10	0.50	0.90	0.10	0.50	0.90
Rayleigh	correlation		0.150	0.117	0.031	0.150	0.117	0.031	0.150	0.117	0.030
	mean	0.000	0.146	0.146	0.144	0.146	0.146	0.144	0.146	0.146	0.146
	variance	1.000	0.218	0.218	0.217	0.218	0.218	0.217	0.217	0.217	0.217
	skewness	0.631	0.322	0.325	0.326	0.322	0.325	0.326	0.322	0.322	0.322
	kurtosis	0.245	0.825	0.828	0.832	0.822	0.824	0.828	0.763	0.763	0.763
Rayleigh	mean	0.000	0.146	0.145	0.145	0.146	0.145	0.145	0.147	0.147	0.147
	variance	1.000	0.218	0.218	0.217	0.218	0.218	0.217	0.217	0.219	0.217
	skewness	0.631	0.323	0.324	0.325	0.323	0.324	0.325	0.319	0.320	0.322
	kurtosis	0.245	0.831	0.835	0.837	0.827	0.831	0.833	0.763	0.764	0.763
<i>n</i> = 100											
Rayleigh	correlation		0.080	0.063	0.016	0.080	0.063	0.016	0.080	0.062	0.017
	mean	0.000	0.080	0.079	0.079	0.080	0.079	0.079	0.080	0.080	0.080
	variance	1.000	0.120	0.120	0.120	0.120	0.120	0.120	0.121	0.121	0.121
	skewness	0.631	0.191	0.192	0.193	0.190	0.191	0.192	0.180	0.180	0.180
	kurtosis	0.245	0.572	0.573	0.577	0.564	0.565	0.568	0.476	0.476	0.476
Rayleigh	mean	0.000	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.081
	variance	1.000	0.119	0.120	0.121	0.119	0.120	0.121	0.119	0.119	0.121
	skewness	0.631	0.188	0.190	0.192	0.188	0.189	0.191	0.180	0.180	0.181
	kurtosis	0.245	0.568	0.574	0.573	0.560	0.567	0.565	0.476	0.473	0.477
<i>n</i> = 1,000											
Rayleigh	correlation		0.025	0.020	0.005	0.025	0.020	0.005	0.025	0.020	0.009
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.038	0.038	0.038	0.038	0.038	0.038	0.038	0.038	0.038
	skewness	0.631	0.063	0.064	0.064	0.062	0.063	0.063	0.057	0.057	0.057
	kurtosis	0.245	0.217	0.220	0.222	0.210	0.212	0.213	0.158	0.158	0.158
Rayleigh	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.037	0.037	0.038	0.037	0.037	0.038	0.038	0.038	0.037
	skewness	0.631	0.063	0.063	0.064	0.062	0.062	0.063	0.058	0.057	0.057
	kurtosis	0.245	0.215	0.216	0.214	0.208	0.209	0.206	0.159	0.158	0.158

Table 16

Average Absolute Differences between Generated Parameters and Desired Parameters

			Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method		
$n = 30$	correlation		0.10	0.50	0.90	0.10	0.50	0.90	0.10	0.50	0.90
Beta (4,2)	correlation		0.149	0.117	0.030	0.150	0.116	0.030	0.150	0.118	0.098
	mean	0.000	0.146	0.147	0.145	0.146	0.147	0.145	0.145	0.145	0.145
	variance	1.000	0.188	0.187	0.186	0.188	0.187	0.185	0.187	0.187	0.187
	skewness	-0.468	0.258	0.258	0.257	0.260	0.259	0.257	0.256	0.256	0.256
Beta (4, 3/2)	kurtosis	-0.375	0.487	0.486	0.489	0.495	0.494	0.497	0.466	0.466	0.466
	mean	0.000	0.145	0.145	0.144	0.145	0.145	0.144	0.147	0.147	0.145
	variance	1.000	0.201	0.202	0.201	0.201	0.202	0.201	0.203	0.204	0.203
	skewness	-0.694	0.277	0.277	0.276	0.281	0.280	0.279	0.273	0.277	0.278
	kurtosis	-0.069	0.655	0.660	0.657	0.671	0.676	0.673	0.635	0.635	0.642
$n=100$											
Beta (4,2)	correlation		0.079	0.062	0.016	0.079	0.062	0.016	0.080	0.063	0.098
	mean	0.000	0.080	0.079	0.079	0.080	0.079	0.079	0.080	0.080	0.080
	variance	1.000	0.101	0.101	0.101	0.101	0.101	0.101	0.103	0.103	0.103
	skewness	-0.468	0.139	0.139	0.140	0.139	0.139	0.140	0.138	0.138	0.138
Beta (4, 3/2)	kurtosis	-0.375	0.283	0.282	0.281	0.289	0.289	0.287	0.266	0.266	0.266
	mean	0.000	0.080	0.080	0.079	0.080	0.080	0.079	0.080	0.080	0.080
	variance	1.000	0.111	0.110	0.111	0.111	0.110	0.110	0.112	0.111	0.112
	skewness	-0.694	0.153	0.152	0.154	0.156	0.156	0.157	0.148	0.150	0.150
	kurtosis	-0.069	0.399	0.395	0.395	0.417	0.412	0.413	0.371	0.376	0.376
$n=1,000$											
Beta (4,2)	correlation		0.025	0.020	0.005	0.025	0.020	0.005	0.025	0.020	0.098
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032
	skewness	-0.468	0.045	0.044	0.044	0.045	0.044	0.044	0.043	0.043	0.043
Beta (4, 3/2)	kurtosis	-0.375	0.092	0.091	0.091	0.094	0.093	0.092	0.082	0.082	0.082
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035
	skewness	-0.694	0.048	0.048	0.048	0.049	0.049	0.049	0.047	0.047	0.046
	kurtosis	-0.069	0.126	0.126	0.124	0.133	0.134	0.131	0.119	0.119	0.117

Table 17

Average Absolute Differences between Generated Parameters and Desired Parameters

			Fifth Order Transformation Method			Fleishman Power Method			Generalized Lambda Distribution Method		
<i>n</i> = 30		correlation	0.10	0.50	0.90	0.10	0.50	0.90	0.10	0.50	0.90
t (7df)	correlation		0.149	0.113	0.029	0.149	0.114	0.030	0.148	0.115	0.030
	mean	0.000	0.145	0.146	0.144	0.146	0.146	0.144	0.146	0.146	0.146
	variance	1.000	0.266	0.268	0.267	0.278	0.279	0.278	0.276	0.276	0.276
	skewness	0.000	0.475	0.480	0.481	0.524	0.528	0.530	0.535	0.535	0.535
t (7df)	kurtosis	2.000	1.939	1.948	1.945	1.757	1.763	1.760	1.755	1.755	1.755
	mean	0.000	0.145	0.145	0.144	0.145	0.145	0.144	0.146	0.146	0.146
	variance	1.000	0.265	0.264	0.265	0.276	0.276	0.277	0.275	0.277	0.277
	skewness	0.000	0.478	0.476	0.475	0.526	0.525	0.523	0.518	0.522	0.531
	kurtosis	2.000	1.944	1.931	1.937	1.760	1.749	1.755	1.743	1.754	1.756
<i>n</i> =100											
t (7df)	correlation		0.079	0.060	0.016	0.079	0.061	0.016	0.079	0.061	0.016
	mean	0.000	0.079	0.079	0.079	0.079	0.079	0.079	0.080	0.080	0.080
	variance	1.000	0.152	0.153	0.154	0.156	0.157	0.158	0.158	0.158	0.158
	skewness	0.000	0.375	0.377	0.378	0.393	0.395	0.396	0.394	0.394	0.394
t (7df)	kurtosis	2.000	1.650	1.664	1.685	1.360	1.371	1.381	1.351	1.351	1.351
	mean	0.000	0.080	0.080	0.079	0.080	0.080	0.079	0.080	0.080	0.080
	variance	1.000	0.153	0.152	0.152	0.157	0.156	0.157	0.156	0.155	0.157
	skewness	0.000	0.374	0.372	0.374	0.393	0.392	0.394	0.392	0.390	0.394
	kurtosis	2.000	1.656	1.661	1.654	1.365	1.369	1.370	1.366	1.354	1.342
<i>n</i> =1,000											
t (7df)	correlation		0.025	0.019	0.005	0.025	0.019	0.005	0.025	0.019	0.005
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.050	0.051	0.052	0.051	0.051	0.051	0.050	0.050	0.050
	skewness	0.000	0.181	0.192	0.200	0.162	0.166	0.167	0.161	0.161	0.161
t (7df)	kurtosis	2.000	1.074	1.247	1.406	0.672	0.710	0.746	0.673	0.673	0.673
	mean	0.000	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	variance	1.000	0.050	0.050	0.050	0.050	0.050	0.051	0.051	0.050	0.050
	skewness	0.000	0.176	0.179	0.179	0.159	0.161	0.160	0.163	0.162	0.162
	kurtosis	2.000	1.043	1.054	1.071	0.661	0.666	0.662	0.683	0.678	0.676

Chapter 5

Conclusions

This study analyzed three different methods for generating bivariate non-normally distributed random variables, the Fleishman power method (Headrick and Sawilowsky, 1999), the fifth-order polynomial transform method (Headrick, 2002), and the generalized lambda distribution method (Headrick and Mugadi, 2006). Each method was compared in terms of accuracy, simplicity, and efficiency.

While the fifth-order polynomial transform method provides the most accuracy, the Fleishman power method offers nearly the same amount of accuracy with a greater ease of use. It also is a much faster method for generating random variates than the generalized lambda distribution method. The Fleishman power method is not able to generate the range of distributions that the fifth-order polynomial transform method or generalized lambda distribution method are capable of producing, though. In addition, the Fleishman power method and fifth-order polynomial transform method have cases in which it is not possible to generate the intermediate correlation. In those instances, the generalized lambda distribution method provides an acceptable alternative.

All three methods perform less well when distributions have larger values for skewness and/or kurtosis. Therefore, it would be helpful for another method to be developed that could handle simulations of bivariate non-normal data for these extreme circumstances.

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Appendix A

The lower bounds of kurtosis (γ_2) for the Fleishman power method and fifth-order polynomial method, given the values of skewness (γ_1), and the fifth and sixth standardized cumulants (γ_3 and γ_4).

γ_1	γ_2 (fifth-order polynomial method)	γ_2 (Fleishman power method)	γ_3	γ_4
0.00	-1.385081	-1.151320	0.00	28.50
0.24	-1.209981	-1.053310	-1.00	11.00
0.48	-0.92574	-0.772778	-2.00	6.25
0.72	-0.480129	-0.321231	-2.50	2.50
0.96	0.133374	0.303505	-2.25	-0.25
1.20	0.907509	1.106980	-1.20	-3.08
1.44	1.775770	2.094340	0.40	6.00
1.68	2.762360	3.272010	2.38	6.00
1.92	4.172850	4.647430	11.00	195.00
2.16	5.199340	6.231700	10.00	37.00
2.40	6.606610	8.042820	15.00	200.00

Appendix B

Theoretical densities and their associated values of skewness (γ_1), kurtosis (γ_2), fifth standardized cumulant (γ_3), and sixth standardized cumulant (γ_4).

	<i>Gaussian</i>	<i>Logistic</i>	<i>Uniform</i>	<i>Laplace</i>	<i>Triangular</i>	<i>t (7df)</i>	<i>t (10df)</i>
γ_1	0	0	0	0	0	0	0
γ_2	0	$\frac{6}{5}$	$-\frac{6}{5}$	3	$-\frac{3}{5}$	2	1
γ_3	0	0	0	0	0	0	0
γ_4	0	$\frac{48}{7}$	$\frac{48}{7}$	30	$\frac{12}{7}$	80	10

	$\chi^2_{(1)}$	$\chi^2_{(2)}$	$\chi^2_{(3)}$	$\chi^2_{(4)}$	$\chi^2_{(8)}$	$\chi^2_{(16)}$	$\chi^2_{(32)}$
γ_1	$\sqrt{8}$	2	$2\sqrt{2/3}$	$\sqrt{2}$	1	$1/\sqrt{2}$	$\frac{1}{2}$
γ_2	12	6	4	3	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{3}{8}$
γ_3	$48\sqrt{2}$	24	$16\sqrt{2/3}$	$6\sqrt{2}$	3	$3/2\sqrt{2}$	$\frac{3}{8}$
γ_4	480	120	$\frac{160}{3}$	30	$\frac{15}{2}$	$\frac{15}{8}$	$\frac{15}{32}$

	Beta ($\alpha=4, \beta=4$)	Beta ($\alpha=4, \beta=2$)	Beta ($\alpha=4, \beta=3/2$)	Beta ($\alpha=4, \beta=5/4$)
γ_1	0	-0.467707	-0.693889	-0.848164
γ_2	-0.545455	-0.375	-0.068627	0.221003
γ_3	0	1.403122	1.828171	1.906805
γ_4	1.678322	-0.426136	-3.379486	-5.827789

	<i>Weibull</i> ($\alpha=4, \beta=10$)	<i>Gamma</i> ($\alpha=\beta=10$)	<i>Rayleigh</i> ($\alpha=1/2, \mu=\sqrt{\pi/2}$)	<i>Pareto</i> ($\theta=10, \alpha=1$)
γ_1	-0.373262	0.822192	0.631111	2.811057
γ_2	0.035455	0.6	0.245089	14.828571
γ_3	0.447065	-1.134200	-0.313137	130.208155
γ_4	-1.022066	-1.56	-0.868288	1808.899592

Appendix C

c Program for Generalized Lambda Distribution

c This program is correct.

use numerical_libraries

C Declare variables

implicit real*8 (a-h, o-z)

integer I, IRANK, ISEED, J, K, LDR, LDRSIG, NOUT, NR

c L=NR, N=K

parameter (L=1000000, N=2)

REAL*8 C11,C12,C13,C14,C21,C22,C23,C24,PI,sq2pi,int,DELTA,
& UMIN,UMAX,VMIN,VMAX,Z1,Z2,ST,rho, time, sprho, x(2),
& aint, bint, estint, rhoa, rhob, fofa, fest, rhoest,
& COV(2,2), R(L,N), RSIG(2,2), z1gen(L), z2gen(L),
& x1(L), x2(L), sumx1, sumx2,u1(L), u2(L), g1(L), g2(L),
& avex1, avex2, spx1x2, apx1x2,
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2

open (unit=9,file='c:\GLD.out')

c specified skews and kurtoses

data skew1/0.0d+00/

data skurt1/6.0d+00/

data skew2/-0.467707d+00/

data skurt2/2.625d+00/

c specified correlation

data sprho/0.1d+00/

c delta used in Reimann sums to calculate intermediate correlation

data delta/0.05d+00/

umin = -5.0d+00

vmin = -5.0d+00

umax = 5.0d+00

vmax = 5.0d+00

aint = sprho

bint = sprho+0.1d+00

PI=3.14159265358979323846d+00

sq2pi = dSQRT(2.0d+00*PI)

```

NR          = L
K          = N
LDRSIG     = 2
LDR        = L

```

```

time = CPSEC()

```

```

c      x(1) = 0.05d+00
c      x(2) = 0.05d+00
x(1) = -0.1d+00
x(2) = -0.1d+00

```

```

call gdlambda(x, skew1,skurt1, onelam, twolam, thrlam, fourlam)
c11=onelam
c12=twolam
c13=thrlam
c14=fourlam

```

```

print*, 'c11= ', c11
print*, 'c12= ', c12
print*, 'c13= ', c13
print*, 'c14= ', c14

```

```

x(1) = 0.05d+00
x(2) = 0.05d+00
c      x(1) = -0.1d+00
c      x(2) = -0.1d+00

```

```

call gdlambda(x, skew2,skurt2, onelam2, twolam2, thrlam2, fourlam2)
c21=onelam2
c22=twolam2
c23=thrlam2
c24=fourlam2

```

```

print*, 'c21= ', c21
print*, 'c22= ', c22
print*, 'c23= ', c23
print*, 'c24= ', c24

```

```

c      Calculate the intermediate correlation

```

```

do while (dabs(bint-aint).gt.0.000001d+00)

```

```

rhoa = 0.0d+00
rhoest = 0.0d+00

```

```
estint = (aint+bint)/2.0d+00
```

```
DO 10 Z1=UMIN,UMAX,DELTA
```

```
DO 20 Z2=VMIN,VMAX,DELTA
```

```
rhoa=rhoa+ST(C11,C12,C13,C14,C21,C22,C23,C24,PI,sq2pi,aint,  
&      UMIN,VMIN,DELTA,Z1,Z2)*DELTA**2.0d+00
```

```
rhoest=rhoest+ST(C11,C12,C13,C14,C21,C22,C23,C24,PI,sq2pi,estint,  
&      UMIN,VMIN,DELTA,Z1,Z2)*DELTA**2.0d+00
```

```
20 CONTINUE
```

```
10 CONTINUE
```

```
fofa=sprho-rhoa
```

```
fest=sprho-rhoest
```

```
if (fofa*fest.gt.0.0d+00) then
```

```
    aint=estint
```

```
    else
```

```
    bint=estint
```

```
end if
```

```
end do
```

```
c  print*, 'aint= ', aint
```

```
c  print*, 'bint= ', bint
```

```
print*, 'estint= ', estint
```

```
c  *****
```

```
Generate random variables using GLD
```

```
*****
```

```
c  Generate Z1 and Z2 with correlations of intermediate rho
```

```
c  R matrix represents Z1 and Z2
```

```
COV(1,1) = 1.0d+00
```

```
COV(1,2) = estint
```

```
COV(2,1) = estint
```

```
COV(2,2) = 1.0d+00
```

```
C      Obtain the Cholesky factorization.
```

```
CALL dCHFAC (K, COV, K, 0.00001d+00, IRANK, RSIG, LDRSIG)
```

```

C           Initialize seed of random number generator.
c   ISEED = 123457
c       CALL RNSET (ISEED)
CALL dRNMVN (NR, K, RSIG, LDRSIG, R, LDR)

      do 100 i100=1,L
          z1gen(i100)=r(i100,1)
          z2gen(i100)=r(i100,2)
c   print*, 'z1gen= ', r(i100, 1), ' z2gen= ', r(i100, 2)
100  continue

c           Transform standard normal deviates to uniform deviates
      do 50 m=1,L
          u1(m)=dnordf(z1gen(m))
          u2(m)=dnordf(z2gen(m))
50  continue

c           Generate X1 and X2 by GLD with the desired post-correlation and the specified
skew and kurtosis

      do 3 j=1,L
          g1(j)= c11+((u1(j)**c13)-(1.0d+00-u1(j))**c14)/c12
3  continue

      do 4 j1=1,L
          g2(j1)= c21+((u2(j1)**c23)-(1.0d+00-u2(j1))**c24)/c22
4  continue

      call outsum(g1, g2, L, sumx1, sumx2,
& avex1, avex2, spx1x2, apx1x2,
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2)

      write (*,*) 'E[x1]= ', avex1, ' E[x2]= ', avex2
      write (*,*) 'std dev x1 = ', Sx1, ' std dev x2 =', Sx2
      write (*,*) 'corrx = ', corrx, ' skewx1=', skewx1, ' skewx2=',
& skewx2, ' kurtx1=', skurtx1, ' kurtx2=', skurtx2

      time = CPSEC()

```

```

write(9,*) corrX
write(9,*) avex1
write(9,*) Sx1**2.0d+00
write(9,*) skewx1
write(9,*) skurtx1
write(9,*) avex2
write(9,*) Sx2**2.0d+00
write(9,*) skewx2
write(9,*) skurtx2
write(9,*) time

```

END

C*****

```

double precision FUNCTION ST(C11,C12,C13,C14,C21,C22,C23,C24,
& PI,sq2pi,int,UMIN,VMIN,DELTA,Z1,Z2)

```

```

REAL*8 R, C11,C12,C13,C14,C21,C22,C23,C24,PI,int,
& sq2pi, UMIN,VMIN,DELTA,Z1,Z2

```

```

X1=C11+(PHI(sq2PI,UMIN,Z1,DELTA)**C13-
& (1.0d+00-PHI(sq2PI,UMIN,Z1,DELTA))**C14)/C12

```

```

X2=C21+(PHI(sq2pi,VMIN,Z2,DELTA)**C23-
& (1.0d+00-PHI(sq2PI,VMIN,Z2,DELTA))**C24)/C22

```

```

R=((X1*X2)*(((2.0d+00*PI)*(dSQRT(1.0d+00-int**2)))**(-1.0d+00))*
& dEXP((-1.0d+00/(2.0d+00*(1.0d+00-int**2.0d+00))))*
*((Z1**2.0d+00)-2.0d+00*int*(Z1*Z2)+(Z2**2.0d+00)))

```

ST=R

```

RETURN
END

```

C*****

```

double precision FUNCTION PHI(sq2PI,UMIN,Z1,DELTA)

```

```

REAL*8 SUM1,U, delta, z1, umin, sq2pi

```

```
SUM1=0.0d+00
```

```
DO 200 U=UMIN,Z1,DELTA
```

```
SUM1=SUM1+1.0d+00/(sq2pi)*  
*dEXP(-(U**2.0d+00)/2.0d+00)*DELTA
```

```
200 CONTINUE
```

```
PHI=SUM1
```

```
RETURN  
END
```

C Used to find $E[x_1]$, $E[x_2]$, $E[x_1x_2]$, standard deviations for x_1 and x_2 ,
c correlation between x_1 and x_2 , skew and kurtosis

```
subroutine outsum(x1, x2, L, sumx1, sumx2,  
& avex1, avex2, spx1x2, apx1x2,  
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,  
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2)
```

```
implicit REAL*8 (a-h, o-z)  
integer L  
REAL*8 x1(L), x2(L), sumx1, sumx2,  
& avex1, avex2, spx1x2, apx1x2,  
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,  
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2
```

```
sumx1=0.0d+00  
sumx2=0.0d+00  
spx1x2 = 0.0d+00  
ssqx1=0.0d+00  
ssqx2=0.0d+00  
scux1=0.0d+00  
scux2=0.0d+00  
sumqux1=0.0d+00  
sumqux2=0.0d+00
```

```
do 20 i=1,L  
sumx1=sumx1+x1(i)  
sumx2=sumx2+x2(i)  
spx1x2 = spx1x2 + x1(i)*x2(i)
```


20 continue

```
avex1 = sumx1/float(L)
avex2 = sumx2/float(L)
apx1x2 = spx1x2/float(L)
```

```
do 30 i30=1,L
ssqx1 = ssqx1 + (x1(i30)-avex1)**2.0d+00
ssqx2 = ssqx2 + (x2(i30)-avex2)**2.0d+00
scux1 = scux1 + (x1(i30)-avex1)**3.0d+00
scux2 = scux2 + (x2(i30)-avex2)**3.0d+00
sumqux1 = sumqux1 + (x1(i30)-avex1)**4.0d+00
sumqux2 = sumqux2 + (x2(i30)-avex2)**4.0d+00
30 continue
```

```
Sx1 = dsqrt(ssqx1/(float(L)))
Sx2 = dsqrt(ssqx2/(float(L)))
corr = (apx1x2-avex1*avex2)/(Sx1*Sx2)
skewx1=scux1/(float(L)*Sx1**3.0d+00)
skewx2=scux2/(float(L)*Sx2**3.0d+00)
skurtx1=(sumqux1/(float(L)*Sx1**4.0d+00))-3.0d+00
skurtx2=(sumqux2/(float(L)*Sx2**4.0d+00))-3.0d+00
```

```
RETURN
END
```

c *****subroutine to calculate lambdas*****

```
subroutine gdlambda(x, skew, skurt, onelam, twolam, thrlam, fourlam)
implicit real*8 (a-h, o-z)
```

```
INTEGER LDC, LDDG, LWK, M, ME, N
```

```
PARAMETER (M=1, ME=0, N=2, LDC=N+1, LDDG=M,
& LWK=2*N*(N+16)+9*M+68)
```

C

```
INTEGER IBTYPE, IDO, IPRINT, IWK(19+M), MAXFUN, MAXITN,
& MODE
```

```
REAL*8 C(LDC,N+1), CONWK(M), D(N+1), DF(N),
& DG(LDDG,N), G(M), U(M+N+N+2), WK(LWK), X(N), XLB(N), XUB(N)
```

```
LOGICAL ACTIVE(2*M+13)
INTRINSIC dSQRT
```

C
 DATA IBTYPE/0/, MAXITN/5000/, MODE/2/
 data MAXFUN/10000/, IPRINT/1/

DATA XLB(1)/-0.25d+00/, XUB(1)/1.0d+00/
 DATA XLB(2)/-0.25d+00/, XUB(2)/1.0d+00/
 DATA SCBOU/1000.0d+00/

C Set final accuracy (ACC)
 ACC = dSQRT(dMACH(3))

C ACTIVE(1) = .true.
 IDO = 0

10 IF (IDO.EQ.0 .OR. IDO.EQ.1) THEN

C Evaluate the function at X.

FVALUE = (((((1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*dbeta(1.0d+00+
 2.0d+00
 *x(1),1.0d+00+x(2))+3.0d+00*dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*
 *x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2)))-3.0d+00*(1.0d+00/(1.0d+00+
 x(1))-1.0d+00/(1.0d+00+x(2)))(1.0d+00/(1.0d+00+2.0d+00*x(1))-
 *2.0d+00*dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/(1.0d+00+
 *2.0d+00*x(2)))+2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+
 *x(2)))**3.0d+00)/((1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*dbeta
 *(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))-
 *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00)**
 *(3.0d+00/2.0d+00))-skew)**2.0d+00
 *+
 *(((1.0d+00/(1.0d+00+4.0d+00*x(1))-4.0d+00*dbeta(1.0d+00+
 *3.0d+00*x(1),1.0d+00+x(2))+6.0d+00*dbeta(1.0d+00+2.0d+00*x(1),
 *1.0d+00+2.0d+00*x(2))-4.0d+00*dbeta(1.0d+00+x(1),1.0d+00+3.0d+00*
 *x(2))+1.0d+00/(1.0d+00+4.0d+00*x(2)))-4.0d+00*(1.0d+00/(1.0d+00+
 x(1))-1.0d+00/(1.0d+00+x(2)))(1.0d+00/(1.0d+00+3.0d+00*x(1))-
 *3.0d+00*dbeta(1.0d+00+2.0d+00*x(1),1.0d+00+x(2))+3.0d+00*
 *dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+
 *3.0d+00*x(2)))+6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+
 *x(2)))**2.0d+00*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*dbeta

```

*(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2))-
*3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*4.0d+00)/
*((1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),
*1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))-(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2)))*2.0d+00)*2.0d+00)-skurt)*2.0d+00

```

C Evaluate the constraints at X.

$$G(1) = x(1)*x(2)$$

END IF

C IF (IDO.EQ.0 .OR. IDO.EQ.2) THEN

C Evaluate the function gradient at X.

```

DF(1) = (2.0d+00*((1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta
*(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1))+3.0d+00*Dbeta(1.0d+00+x(1),
*1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2))-(3.0d+00*
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/
*(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+
*1.0d+00/(1.0d+00+2.0d+00*x(2)))+2.0d+00*(1.0d+00/(1.0d+00+x(1))-
*1.0d+00/(1.0d+00+x(2)))*3.0d+00)/(1.0d+00/(1.0d+00+2.0d+00*
*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/
*(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))*2.0d+00)*(3.0d+00/2.0d+00)-skew))*((-3.0d+00/
*(1.0d+00+3.0d+00*x(1))*2.0d+00-(3.0d+00*(2.0d+00*Dpsi(1.0d+00+
*2.0d+00*x(1))-2.0d+00*Dpsi(2.0d+00+x(2)+2.0d+00*x(1))))*Dbeta
*(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1)))+(3.0d+00*(Dpsi(1.0d+00+
*x(1))-Dpsi(2.0d+00+x(1)+2.0d+00*x(2))))*Dbeta(1.0d+00+x(1),
*1.0d+00+2.0d+00*x(2)))+(3.0d+00*(1.0d+00/(1.0d+00+2.0d+00*
*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/
*(1.0d+00+2.0d+00*x(2))))/(1.0d+00+x(1))*2.0d+00-(3.0d+00*
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(-2.0d+00/
*(1.0d+00+2.0d+00*x(1))*2.0d+00-(2.0d+00*(Dpsi(1.0d+00+x(1))-
*Dpsi(2.0d+00+x(2)+x(1))))*Dbeta(1.0d+00+x(2), 1.0d+00+x(1)))-
*6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*
*2.0d+00/(1.0d+00+x(1))*2.0d+00)/(1.0d+00/
*(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+
*1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))*2.0d+00)*(3.0d+00/2.0d+00)-(3.0d+00/2.0d+00)*
*(1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+x(2),
*1.0d+00+2.0d+00*x(1))+3.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*
*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2))-(3.0d+00*(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+2.0d+00*x(1))-
*2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/(1.0d+00+
*2.0d+00*x(2)))+2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+

```

$$\begin{aligned}
& *x(2)))**3.0d+00)*(-2.0d+00/(1.0d+00+2.0d+00*x(1))**2.0d+00- \\
& *(2.0d+00* \\
& *(Dpsi(1.0d+00+x(1))-Dpsi(2.0d+00+x(2)+x(1))))*Dbeta(1.0d+00+ \\
& *x(2),1.0d+00+x(1))+(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))/(1.0d+00+x(1))**2.0d+00)/(1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2)))**2.0d+00)**(5.0d+00/2.0d+00))+(2.0d+00*((1.0d+00/(1.0d+00+ \\
& *4.0d+00*x(1))-4.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+3.0d+00*x(1))+ \\
& *6.0d+00*Dbeta(1.0d+00+2.0d+00*x(2),1.0d+00+2.0d+00*x(1))- \\
& *4.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+3.0d+00*x(2))+1.0d+00/ \\
& *(1.0d+00+4.0d+00*x(2))-(4.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))*(1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta \\
& *(1.0d+00+x(2),1.0d+00+2.0d+00*x(1))+3.0d+00*Dbeta(1.0d+00+x(1), \\
& *1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2)))+6.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00* \\
& *(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), \\
& *1.0d+00+x(1))+1.0d+00/(1.0d+00+2.0d+00*x(2))-3.0d+00*(1.0d+00/ \\
& *(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**4.0d+00)/(1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+ \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))**2.0d+00)**2.0d+00-skurt))*((-4.0d+00/(1.0d+00+ \\
& *4.0d+00*x(1))**2.0d+00-(4.0d+00*(3.0d+00*Dpsi(1.0d+00+3.0d+00* \\
& *x(1))-3.0d+00*Dpsi(2.0d+00+x(2)+3.0d+00*x(1))))*Dbeta(1.0d+00+ \\
& *x(2),1.0d+00+3.0d+00*x(1)))+(6.0d+00*(2.0d+00*Dpsi(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dpsi(2.0d+00+2.0d+00*x(2)+2.0d+00*x(1))))* \\
& *Dbeta(1.0d+00+2.0d+00*x(2),1.0d+00+2.0d+00*x(1))-(4.0d+00* \\
& *(Dpsi(1.0d+00+x(1))-Dpsi(2.0d+00+x(1)+3.0d+00*x(2))))*Dbeta \\
& *(1.0d+00+x(1),1.0d+00+3.0d+00*x(2)))+(4.0d+00*(1.0d+00/ \\
& *(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+ \\
& *2.0d+00*x(1))+3.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))- \\
& *1.0d+00/(1.0d+00+3.0d+00*x(2)))/(1.0d+00+x(1))**2.0d+00- \\
& *(4.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))* \\
& *(-3.0d+00/(1.0d+00+3.0d+00*x(1))**2.0d+00-(3.0d+00*(2.0d+00* \\
& *Dpsi(1.0d+00+2.0d+00*x(1))-2.0d+00*Dpsi(2.0d+00+x(2)+2.0d+00* \\
& *x(1))))*Dbeta(1.0d+00+x(2),1.0d+00+2.0d+00*x(1)))+(3.0d+00* \\
& *(Dpsi(1.0d+00+x(1))-Dpsi(2.0d+00+x(1)+2.0d+00*x(2))))*Dbeta \\
& *(1.0d+00+x(1),1.0d+00+2.0d+00*x(2)))-12.0d+00*(1.0d+00/ \\
& *(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*(1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2)))/(1.0d+00+x(1))**2.0d+00+6.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00* \\
& *(-2.0d+00/(1.0d+00+2.0d+00*x(1))**2.0d+00-(2.0d+00*(Dpsi(1.0d+00+ \\
& *x(1))-Dpsi(2.0d+00+x(2)+x(1))))*Dbeta(1.0d+00+x(2),1.0d+00+ \\
& *x(1)))+12.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2)))**3.0d+00/(1.0d+00+x(1))**2.0d+00)/(1.0d+00/(1.0d+00+
\end{aligned}$$

$$\begin{aligned}
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+ \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))- \\
& *1.0d+00/(1.0d+00+x(2)))**2.0d+00)**2.0d+00-(2.0d+00*(1.0d+00/ \\
& *(1.0d+00+4.0d+00*x(1))-4.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+ \\
& *3.0d+00*x(1))+6.0d+00*Dbeta(1.0d+00+2.0d+00*x(2),1.0d+00+2.0d+00* \\
& *x(1))-4.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+3.0d+00*x(2))+ \\
& *1.0d+00/(1.0d+00+4.0d+00*x(2))-(4.0d+00*(1.0d+00/(1.0d+00+ \\
& *x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+3.0d+00*x(1))- \\
& *3.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1))+3.0d+00* \\
& *Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+ \\
& *3.0d+00*x(2)))+6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2)))**2.0d+00*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta \\
& *(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/(1.0d+00+2.0d+00*x(2)))- \\
& *3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
& *4.0d+00))*(-2.0d+00/(1.0d+00+2.0d+00*x(1))**2.0d+00-(2.0d+00*(Dpsi \\
& *(1.0d+00+x(1))-Dpsi(2.0d+00+x(2)+x(1))))*Dbeta(1.0d+00+x(2), \\
& *1.0d+00+x(1))+(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2))))/(1.0d+00+x(1))**2.0d+00)/(1.0d+00/(1.0d+00+2.0d+00* \\
& *x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))**2.0d+00)**3.0d+00)
\end{aligned}$$

$$\begin{aligned}
DF(2)= & (2.0d+00*((1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta \\
& *(1.0d+00+2.0d+00*x(1), 1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1), \\
& *1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2))-(3.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+ \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2)))+2.0d+00*(1.0d+00/(1.0d+00+x(1))- \\
& *1.0d+00/(1.0d+00+x(2)))**3.0d+00)/(1.0d+00/(1.0d+00+2.0d+00* \\
& *x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))**2.0d+00)**(3.0d+00/2.0d+00-skew)))* \\
& *((-(3.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+2.0d+00*x(1))+ \\
& *x(2))))*Dbeta(1.0d+00+2.0d+00*x(1), 1.0d+00+x(2)))+(3.0d+00* \\
& *(2.0d+00*Dpsi(1.0d+00+2.0d+00*x(2))-2.0d+00*Dpsi(2.0d+00+x(1))+ \\
& *2.0d+00*x(2))))*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))+3.0d+00/ \\
& *(1.0d+00+3.0d+00*x(2))**2.0d+00-(3.0d+00*(1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))))/(1.0d+00+x(2))**2.0d+00-(3.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(-(2.0d+00* \\
& *(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+x(1)+x(2))))*Dbeta(1.0d+00+ \\
& *x(1),1.0d+00+x(2))-2.0d+00/(1.0d+00+2.0d+00*x(2))**2.0d+00)+ \\
& *6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
& *2.0d+00/(1.0d+00+x(2))**2.0d+00)/(1.0d+00/(1.0d+00+2.0d+00*x(1))- \\
& *2.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+
\end{aligned}$$

$$\begin{aligned}
& *2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
& *2.0d+00)**(3.0d+00/2.0d+00)-(3.0d+00/2.0d+00)*(1.0d+00/(1.0d+00+ \\
& *3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+2.0d+00*x(1),1.0d+00+x(2))+ \\
& *3.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2))-1.0d+00/ \\
& *(1.0d+00+3.0d+00*x(2))-(3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta \\
& *(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))+ \\
& *2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
& *3.0d+00)*(-(2.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+x(1)+ \\
& *x(2))))*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))-2.0d+00/(1.0d+00+ \\
& *2.0d+00*x(2))**2.0d+00-(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2))))/(1.0d+00+x(2))**2.0d+00)/(1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2)))**2.0d+00)**(5.0d+00/2.0d+00)+(2.0d+00*((1.0d+00/(1.0d+00+ \\
& *4.0d+00*x(1))-4.0d+00*Dbeta(1.0d+00+3.0d+00*x(1),1.0d+00+x(2))+ \\
& *6.0d+00*Dbeta(1.0d+00+2.0d+00*x(1), 1.0d+00+2.0d+00*x(2))- \\
& *4.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+3.0d+00*x(2))+1.0d+00/ \\
& *(1.0d+00+4.0d+00*x(2))-(4.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta \\
& *(1.0d+00+2.0d+00*x(1),1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1), \\
& *1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2)))+6.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00* \\
& *(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1), \\
& *1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))-3.0d+00*(1.0d+00/ \\
& *(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**4.0d+00)/(1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+ \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))**2.0d+00)**2.0d+00-skurt))*((-4.0d+00*(Dpsi \\
& *(1.0d+00+x(2))-Dpsi(2.0d+00+3.0d+00*x(1)+x(2))))*Dbeta(1.0d+00+ \\
& *3.0d+00*x(1), 1.0d+00+x(2)))+(6.0d+00*(2.0d+00*Dpsi(1.0d+00+ \\
& *2.0d+00*x(2))-2.0d+00*Dpsi(2.0d+00+2.0d+00*x(1)+2.0d+00*x(2))))* \\
& *Dbeta(1.0d+00+2.0d+00*x(1), 1.0d+00+2.0d+00*x(2))-(4.0d+00* \\
& *(3.0d+00*Dpsi(1.0d+00+3.0d+00*x(2))-3.0d+00*Dpsi(2.0d+00+x(1)+ \\
& *3.0d+00*x(2))))*Dbeta(1.0d+00+x(1), 1.0d+00+3.0d+00*x(2))- \\
& *4.0d+00/(1.0d+00+4.0d+00*x(2))**2.0d+00-(4.0d+00*(1.0d+00/ \\
& *(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+2.0d+00*x(1), \\
& *1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))- \\
& *1.0d+00/(1.0d+00+3.0d+00*x(2)))/(1.0d+00+x(2))**2.0d+00- \\
& *(4.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(- \\
& *(3.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+2.0d+00*x(1)+x(2))))* \\
& *Dbeta(1.0d+00+2.0d+00*x(1),1.0d+00+x(2)))+(3.0d+00*(2.0d+00* \\
& *Dpsi(1.0d+00+2.0d+00*x(2))-2.0d+00*Dpsi(2.0d+00+x(1)+2.0d+00* \\
& *x(2))))*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))+3.0d+00/ \\
& *(1.0d+00+3.0d+00*x(2))**2.0d+00)+(12.0d+00*(1.0d+00/(1.0d+00+ \\
& *x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+2.0d+00*x(1))-
\end{aligned}$$

```

*2.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+
*2.0d+00*x(2))/(1.0d+00+x(2))**2.0d+00+6.0d+00*(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00*(-(2.0d+00*(Dpsi(1.0d+00+
*x(2))-Dpsi(2.0d+00+x(1)+x(2))))*Dbeta(1.0d+00+x(1),1.0d+00+x(2))-
*2.0d+00/(1.0d+00+2.0d+00*x(2))**2.0d+00)-12.0d+00*(1.0d+00/
*(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**3.0d+00/(1.0d+00+
*x(2))**2.0d+00)/(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta
*(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2))-
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00)**
*2.0d+00-(2.0d+00*(1.0d+00/(1.0d+00+4.0d+00*x(1))-4.0d+00*Dbeta
*(1.0d+00+3.0d+00*x(1),1.0d+00+x(2))+6.0d+00*Dbeta(1.0d+00+
*2.0d+00*x(1), 1.0d+00+2.0d+00*x(2))-4.0d+00*Dbeta(1.0d+00+x(1),
*1.0d+00+3.0d+00*x(2))+1.0d+00/(1.0d+00+4.0d+00*x(2)))-(4.0d+00*
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/
*(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+2.0d+00*x(1),
*1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2))-
*1.0d+00/(1.0d+00+3.0d+00*x(2)))+6.0d+00*(1.0d+00/(1.0d+00+x(1))-
*1.0d+00/(1.0d+00+x(2)))**2.0d+00*(1.0d+00/(1.0d+00+2.0d+00*x(1))-
*2.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+
*2.0d+00*x(2)))-3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+
*x(2)))**4.0d+00))*(-(2.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+
*x(1)+x(2))))*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))-2.0d+00/(1.0d+00+
*2.0d+00*x(2))**2.0d+00-(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2))))/(1.0d+00+x(2))**2.0d+00)/(1.0d+00/(1.0d+00+
*2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/
*(1.0d+00+2.0d+00*x(2)))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))**2.0d+00)**3.0d+00)

```

C If active evaluate the constraint gradient at X.

```

IF (ACTIVE(1)) THEN
  DG(1,1) = X(2)
  DG(1,2) = X(1)
END IF

```

C

END IF

C

Call N0ONF for the next update.

C

CALL dN0ONF (IDO, M, ME, N, IBTYPE, XLB, XUB, IPRINT, MAXITN, X,

```

& FVALUE, G, DF, DG, LDDG, U, C, LDC, D, ACC, SCBOU,
& MAXFUN, ACTIVE, MODE, WK, IWK, CONWK)

```

C If IDO does not equal 1 or 2, exit.

IF (IDO.EQ.1 .OR. IDO.EQ.2) GO TO 10

C solve for lambda1 and lambda2

```
twolam = dsqrt((1.0d+00/(2.0d+00*x(1)+1.0d+00)-2.0d+00*dbeta
*(x(1)+1.0d+00, x(2)+1.0d+00)+1.0d+00/(2.0d+00*x(2)+1.0d+00))-
*(1.0d+00/(x(1)+1.0d+00)-1.0d+00/(x(2)+1.0d+00))*2.0d+00)
```

```
if (twolam.lt.0.0d+00) then
    onelam=(1.0d+00/(x(1)+1.0d+00)-1.0d+00/(x(2)+1.0d+00))/twolam
end if
```

```
if (twolam.gt.0.0d+00) then
    onelam=-((1.0d+00/(x(1)+1.0d+00)-1.0d+00/(x(2)+1.0d+00))/twolam)
end if
```

C If lambda3 and lambda4 less than zero, switch values

```
if (x(1).lt.0.0d+00) then
    twolam = -1.0d+00*twolam
    thrlam = x(2)
    fourlam = x(1)
end if
```

```
if (x(1).gt.0.0d+00) then
    thrlam = x(1)
    fourlam = x(2)
end if
```

return

END

Appendix D

c Program for Fleishman Power Method

use numerical_libraries

C Declare variables

```
implicit REAL*8 (a-h, o-z)
```

```
PARAMETER (K=1, M=1000000, N=3, NROOT=2)
```

```
data skew1/0.0d+00/
```

```
data skurt1/3.00d+00/
```

```
data skew2/0.0d+00/
```

```
data skurt2/3.0d+00/
```

```
data rho/0.1d+00/
```

```
REAL*8 FNORM, C(N), CGUESS(N), G, R(NROOT), RGUESS(NROOT), Z(M),  
& Eone(M), E2(M), Y1(M), Y2(M), X1(M), X2(M),  
& prodY1Y2(M)
```

```
common skew, skurt, a1, b1, c1, d1, a2, b2, c2, d2, rho
```

```
EXTERNAL FCN, G
```

```
call RNSET(12345)
```

```
open (unit=9, file='c:\FPM.out')
```

C Set values of initial guesses to find Fleishman constants and intermediate correlations

```
DATA CGUESS/3*0.50d+00/
```

```
DATA RGUESS/2*0.5d+00/
```

```
EPS = 0.000001d+00
```

```
ERRABS = 0.000001d+00
```

```
ERRREL = 0.0000001d+00
```

```
ETA = 0.0000001d+00
```

```
ITMAX = 10000
```

```
call UMACH (2, NOUT)
```

C Find the Fleishman constants (a, b, c, d)

```
skew = skew1
```

```
skurt = skurt1
```

```
CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
```

```
a1=-C(2)
```

```

b1=C(1)
c1=C(2)
d1=C(3)

write (*,*) 'a1 = ', a1, ' b1 = ', b1, ' c1 = ', c1,
& ' d1 = ', d1

```

```

skew = skew2
skurt = skurt2
CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
a2=-C(2)
b2=C(1)
c2=C(2)
d2=C(3)

```

```

write (*,*) 'a2 = ', a2, ' b2 = ', b2, ' c2 = ', c2,
& ' d2 = ', d2

```

C Find intermediate correlation (R)

```

CALL DZREAL (G, ERRABS, ERRREL, EPS, ETA, NROOT, ITMAX,
RGUESS,
& R, INFO)

```

```

c write (*,*) 'R = ', R(1)

```

```

sumdsY1=0.0d+00
sumdsY2=0.0d+00
sumdkY1=0.0d+00
sumdkY2=0.0d+00
sumdr=0.0d+00

```

c Generate random variables
using FPM

c Generate X1 and X2 with correlations of R-squared

c Generate Y1 and Y2 by substituting into the Fleishman equations to generate nonnormal distributions

c with the desired post-correlation and the specified skew and kurtosis

```

do 10 j=1,M
Z(j)=drnmof()

```

```

Eone(j)=drnnof()
E2(j)=drnnof()
X1(j)=R(1)*Z(j)+dsqrt(1.0d+00-R(1)**2.0d+00)*Eone(j)
X2(j)=R(1)*Z(j)+dsqrt(1.0d+00-R(1)**2.0d+00)*E2(j)
Y1(j)=a1+b1*X1(j)+c1*X1(j)**2.0d+00+d1*X1(j)**3.0d+00
Y2(j)=a2+b2*X2(j)+c2*X2(j)**2.0d+00+d2*X2(j)**3.0d+00
10  continue
c   write (9,*) 'Y1= ', Y1, ' Y2= ', Y2, ' Z= ', Z, ' E1 = ', Eone,
c   & ' E2= ', E2

call output(Y1, Y2, sumY1, sumY2,
& aveY1, aveY2, sumprodY1Y2, aveprodY1Y2, prodY1Y2,
& sumsqY1, sumsqY2, SY1, SY2, corrY, sumcubeY1, sumcubeY2,
& sumquY1, sumquY2, skewY1, skewY2, skurtY1, skurtY2)

```

```

write(9,*)corrY
write(9,*)aveY1
write(9,*)SY1**2.0d+00
write(9,*)skewY1
write(9,*)skurtY1
write(9,*)aveY2
write(9,*)SY2**2.0d+00
write(9,*)skewY2
write(9,*)skurtY2
END

```

C ***** Subroutine *****

C User-defined subroutine (find a, b, c, d)

```

SUBROUTINE FCN (C, F, N)
implicit REAL*8 (a-h, o-z)
REAL*8 F(N), C(N)
common skew, skurt,a1, b1, c1, d1, a2, b2, c2, d2, rho

```

```

F(1) = C(1)**2.0D+00+6.0D+00*C(1)*C(3)+2.0D+00*C(2)**2.0D+00
&+15.0d+00*C(3)**2.0D+00-1.0D+00

```

```

F(2) =2.0d+00* C(2)*(C(1)**2.0D+00+24.0D+00*C(1)*C(3)+105.0D+00
&*C(3)**2.0D+00+2.0D+00)-skew

```

```

F(3) = 24.0D+00*(C(1)*C(3)+C(2)**2.0D+00*(1.0D+00+C(1)**2.0D+00
&+28.0D+00*C(1)*C(3))+C(3)**2.0D+00*(12.0D+00+48.0D+00*C(1)*C(3)
&+141.0D+00*C(2)**2.0D+00+225.0D+00*C(3)**2.0D+00))-skurt

```



```

sumY2=0.0d+00
sumprodY1Y2 = 0.0d+00
sumsqY1=0.0d+00
sumsqY2=0.0d+00
sumcubeY1=0.0d+00
sumcubeY2=0.0d+00
sumquY1=0.0d+00
sumquY2=0.0d+00

do 20 i=1,M
sumY1=sumY1+Y1(i)
sumY2=sumY2+Y2(i)
prodY1Y2(i) = Y1(i)*Y2(i)
sumprodY1Y2 = sumprodY1Y2 + prodY1Y2(i)
sumsqY1 = sumsqY1 + (Y1(i)-aveY1)**2.0d+00
sumsqY2 = sumsqY2 + (Y2(i)-aveY2)**2.0d+00
sumcubeY1 = sumcubeY1 + (Y1(i)-aveY1)**3.0d+00
sumcubeY2 = sumcubeY2 + (Y2(i)-aveY2)**3.0d+00
sumquY1 = sumquY1 + (Y1(i)-aveY1)**4.0d+00
sumquY2 = sumquY2 + (Y2(i)-aveY2)**4.0d+00
20 continue

aveY1 = sumY1/float(M)
aveY2 = sumY2/float(M)
aveprodY1Y2 = sumprodY1Y2/float(M)
c write (*,*) 'E[Y1]= ', aveY1, ' E[Y2]= ', aveY2,
c & ' E[Y1Y2]= ', aveprodY1Y2

SY1 = dsqrt(sumsqY1/(float(M)-1.0d+00))
SY2 = dsqrt(sumsqY2/(float(M)-1.0d+00))
c write (*,*) 'std dev Y1 = ', SY1, ' std dev Y2 = ', SY2
corrY = (aveprodY1Y2-aveY1*aveY2)/(SY1*SY2)
skewY1=sumcubeY1/(float(M)*SY1**3.0d+00)
skewY2=sumcubeY2/(float(M)*SY2**3.0d+00)
skurtY1=(sumquY1/(float(M)*SY1**4.0d+00))-3.0d+00
skurtY2=(sumquY2/(float(M)*SY2**4.0d+00))-3.0d+00
c write (*,*) 'corrY = ', corrY, ' skewY1=', skewY1, ' skewY2=',
c & skewY2, ' kurtY1=', skurtY1, ' kurtY2=', skurtY2
RETURN
END

```

Appendix E

C Program for Fifth-Order Polynomial Transformation Method

use numerical_libraries

C Declare variables

```
implicit REAL*8 (a-h, o-z)
```

```
PARAMETER (M=10000, N=5, NROOT=3)
```

```
data skew1/0.5d+00/
```

```
data skurt1/0.375d+00/
```

```
data fifth1/0.375d+00/
```

```
data sixth1/0.4687500d+00/
```

```
data skew2/2.811057d+00/
```

```
data skurt2/14.828571d+00/
```

```
data fifth2/130.208155d+00/
```

```
data sixth2/1808.899592d+00/
```

```
data rho/0.5d+00/
```

```
REAL*8 FNORM, C(N), CGUESS(N), G, R(NROOT), RGUESS(NROOT), Z(M),  
& Eone(M), E2(M), Y1(M), Y2(M), X1(M), X2(M),  
& pY1Y2(M)
```

```
common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,  
*c1b,c2b,c3b,c4b,c5b,rho
```

```
EXTERNAL FCN, G
```

```
call RNSET(12345)
```

```
open (unit=9,file='c:\fifth.out')
```

C Set values of initial guesses to find Fleishman constants and intermediate correlations

```
time = CPSEC()
```

```
DATA CGUESS/5*0.01d+00/
```

```
DATA RGUESS/3*0.500d+00/
```

```
EPS = 0.000001d+00
```

```
ERRABS = 0.000001d+00
```

```
ERRREL = 0.0000001d+00
ETA = 0.0000001d+00
ITMAX = 100000
call UMACH (2, NOUT)
```

C Find the values of the constants (c(0), c(1), c(2), c(3), c(4), c(5))

```
skew = skew1
skurt = skurt1
fifth = fifth1
sixth = sixth1
```

```
CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
c0a= -c(2)-3.0d+00*c(4)
c1a=c(1)
c2a=c(2)
c3a=c(3)
c4a=c(4)
c5a=c(5)
```

```
skew = skew2
skurt = skurt2
fifth = fifth2
sixth = sixth2
```

```
CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
c0b= -c(2)-3.0d+00*c(4)
c1b=c(1)
c2b=c(2)
c3b=c(3)
c4b=c(4)
c5b=c(5)
```

C Find intermediate correlation (R)

```
CALL DZREAL (G, ERRABS, ERRREL, EPS, ETA, NROOT, ITMAX,
RGUESS,
& R, INFO)
```

```
write (*, *) 'R = ', R
```

```
sumdsY1=0.0d+00
sumdsY2=0.0d+00
```

```

sumdkY1=0.0d+00
sumdkY2=0.0d+00
sumdr=0.0d+00

```

- c Generate random variables
- c Generate X1 and X2 with correlations of R-squared
- c Generate Y1 and Y2 by substituting into the Fleishman equations to generate
- c nonnormal distributions
- c with the desired post-correlation and the specified skew and kurtosis

```

do 10 j=1,M
Z(j)=drnnof()
Eone(j)=drnnof()
E2(j)=drnnof()
X1(j)=dsqrt(R(1))*Z(j)+dsqrt(1.0d+00-R(1))*Eone(j)
X2(j)=dsqrt(R(1))*Z(j)+dsqrt(1.0d+00-R(1))*E2(j)
Y1(j)=c0a+c1a*X1(j)+c2a*X1(j)**2.0d+00+c3a*X1(j)**3.0d+00+c4a*
*X1(j)**4.0d+00+c5a*X1(j)**5.0d+00
Y2(j)=c0b+c1b*X2(j)+c2b*X2(j)**2.0d+00+c3b*X2(j)**3.0d+00+c4b*
*X2(j)**4.0d+00+c5b*X2(j)**5.0d+00

```

10 continue

```

call outsum(M, Y1, Y2, sumY1, sumY2,
& aveY1, aveY2, spY1Y2, apY1Y2, pY1Y2,
& sumsqY1, sumsqY2, SY1, SY2, corrY, scY1, scY2,
& sumquY1, sumquY2, skewY1, skewY2, skurtY1, skurtY2)

```

```

write(9,*) corrY
write(9,*) aveY1
write(9,*) SY1**2.0d+00
write(9,*) skewY1
write(9,*) skurtY1
write(9,*) aveY2
write(9,*) SY2**2.0d+00
write(9,*) skewY2
write(9,*) skurtY2
time = CPSEC()
write(*,*) 'time= ', time
END

```

C ***** Subroutine *****


```

C           User-defined subroutine (find c(1) - c(5))
SUBROUTINE FCN (C, F, N)
implicit REAL*8 (a-h, o-z)
REAL*8 F(N), C(N)
      common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,
*c1b,c2b,c3b,c4b,c5b,rho

```

```

c      write(*,*) 'skew = ', skew
c      write(*,*) 'skurt = ', skurt
c      write(*,*) 'fifth = ', fifth
c      write(*,*) 'sixth = ', sixth

```

```

F(1) = (c(1)**2.0d+00+2.0d+00*c(2)**2.0d+00+24.0d+00*c(2)*c(4)+
*6.0d+00*c(1)*(c(3)+5.0d+00*c(5))+ 3.0d+00*(5.0d+00*c(3)**2.0d+00
*+32.0d+00*c(4)**2.0d+00+70.0d+00*c(3)*c(5)+315.0d+00*
*c(5)**2.0d+00))-1.0d+00

```

```

F(2) = (2.0d+00*(4.0d+00*c(2)**3.0d+00+108.0d+00*c(2)**2.0d+00*
*c(4)+3.0d+00*c(1)**2.0d+00*(c(2)+6.0d+00*c(4))+18.0d+00*c(1)*
*(2.0d+00*c(2)*c(3)+16.0d+00*c(3)*c(4)+15.0d+00*c(2)*c(5)+
*150.0d+00*c(4)*c(5))+9.0d+00*c(2)*(15.0d+00*c(3)**2.0d+00+
*128.0d+00*c(4)**2.0d+00+280.0d+00*c(3)*c(5)+1575.0d+00*c(5)**
*2.0d+00)+54.0d+00*c(4)*(25.0d+00*c(3)**2.0d+00+88.0d+00*c(4)**
*2.0d+00+560.0d+00*c(3)*c(5)+3675.0d+00*c(5)**2.0d+00)))-skew

```

```

F(3) = (24.0d+00*(2.0d+00*c(2)**4.0d+00+96.0d+00*c(2)**3.0d+00*
*c(4)+c(1)**3.0d+00*(c(3)+10.0d+00*c(5))+30.0d+00*c(2)**2.0d+00*
*(6.0d+00*c(3)**2.0d+00+64.0d+00*c(4)**2.0d+00+140.0d+00*c(3)*c(5)+
*945.0d+00*c(5)**2.0d+00)+c(1)**2.0d+00*(2.0d+00*c(2)**2.0d+00+
*18.0d+00*c(3)**2.0d+00+36.0d+00*c(2)*c(4)+192.0d+00*c(4)**
*2.0d+00+375.0d+00*c(3)*c(5)+2250.0d+00*c(5)**2.0d+00)+36.0d+00*
*c(2)*c(4)*(125.0d+00*c(3)**2.0d+00+528.0d+00*c(4)**2.0d+00+
*3360.0d+00*c(3)*c(5)+25725.0d+00*c(5)**2.0d+00)+3.0d+00*c(1)*
*(45.0d+00*c(3)**3.0d+00+1584.0d+00*c(3)*c(4)**2.0d+00+1590.0d+00*
*c(3)**2.0d+00*c(5)+21360.0d+00*c(4)**2.0d+00*c(5)+21525.0d+00*
*c(3)*c(5)**2.0d+00+110250.0d+00*c(5)**3.0d+00+12.0d+00*c(2)**
*2.0d+00*(c(3)+10.0d+00*c(5))+8.0d+00*c(2)*c(4)*(32.0d+00*c(3)+
*375.0d+00*c(5))+9.0d+00*(45.0d+00*c(3)**4.0d+00+8704.0d+00*c(4)**
*4.0d+00+2415.0d+00*c(3)**3.0d+00*c(5)+932400.0d+00*c(4)**2.0d+00*
*c(5)**2.0d+00+3018750.0d+00*c(5)**4.0d+00+20.0d+00*c(3)**2.0d+00*
*(178.0d+00*c(4)**2.0d+00+2765.0d+00*c(5)**2.0d+00)+35.0d+00*c(3)*
*(3104.0d+00*c(4)**2.0d+00*c(5)+18075.0d+00*c(5)**3.0d+00)))-skurt

```

$$\begin{aligned}
F(4) = & (24.0d+00*(16.0d+00*c(2)**5.0d+00+5.0d+00*c(1)**4.0d+00* \\
& *c(4)+1200.0d+00*c(2)**4.0d+00*c(4)+10.0d+00*c(1)**3.0d+00* \\
& *(3.0d+00* \\
& *c(2)*c(3)+42.0d+00*c(3)*c(4)+40.0d+00*c(2)*c(5)+570.0d+00*c(4)* \\
& *c(5))+300.0d+00*c(2)**3.0d+00*(10.0d+00*c(3)**2.0d+00+128.0d+00* \\
& *c(4)**2.0d+00+280.0d+00*c(3)*c(5)+2205.0d+00*c(5)**2.0d+00)+ \\
& *1080.0d+00*c(2)**2.0d+00*c(4)*(125.0d+00*c(3)**2.0d+00+3920.0d+00* \\
& *c(3)*c(5)+28.0d+00*(22.0d+00*c(4)**2.0d+00+1225.0d+00* \\
& *c(5)**2.0d+00))+10.0d+00*c(1)**2.0d+00*(2.0d+00*c(2)** \\
& *3.0d+00+72.0d+00*c(2)**2.0d+00*c(4)+3.0d+00*c(2)*(24.0d+00*c(3)** \\
& *2.0d+00+320.0d+00*c(4)**2.0d+00+625.0d+00*c(3)*c(5)+4500.0d+00* \\
& *c(5)**2.0d+00)+9.0d+00*c(4)*(109.0d+00*c(3)**2.0d+00+528.0d+00* \\
& *c(4)**2.0d+00+3130.0d+00*c(3)*c(5)+24975.0d+00*c(5)**2.0d+00))+ \\
& *30.0d+00*c(1)*(8.0d+00*c(2)**3.0d+00*(2.0d+00*c(3)+25.0d+00*c(5))+ \\
& *40.0d+00*c(2)**2.0d+00*c(4)*(16.0d+00*c(3)+225.0d+00*c(5))+ \\
& *3.0d+00*c(2)*(75.0d+00*c(3)**3.0d+00+3168.0d+00*c(3)*c(4)** \\
& *2.0d+00+3180.0d+00*c(3)**2.0d+00*c(5)+49840.0d+00*c(4)**2.0d+00* \\
& *c(5)+ \\
& *50225.0d+00*c(3)*c(5)**2.0d+00+294000.0d+00*c(5)**3.0d+00)+ \\
& *6.0d+00*c(4)*(555.0d+00*c(3)**3.0d+00+8704.0d+00*c(3)*c(4)** \\
& *2.0d+00+26225.0d+00*c(3)**2.0d+00*c(5)+152160.0d+00*c(4)**2.0d+00* \\
& *c(5)+459375.0d+00*c(3)*c(5)**2.0d+00+2963625.0d+00*c(5)** \\
& *3.0d+00))+90.0d+00*c(2)*(270.0d+00*c(3)**4.0d+00+16905.0d+00* \\
& *c(3)**3.0d+00*c(5)+280.0d+00*c(3)**2.0d+00*(89.0d+00*c(4)** \\
& *2.0d+00+1580.0d+00*c(5)**2.0d+00)+35.0d+00*c(3)*(24832.0d+00* \\
& *c(4)**2.0d+00*c(5)+162675.0d+00*c(5)**3.0d+00)+4.0d+00* \\
& *(17408.0d+00*c(4)**4.0d+00+2097900.0d+00*c(4)**2.0d+00*c(5)** \\
& *2.0d+00+7546875.0d+00*c(5)**4.0d+00))+27.0d+00*c(4)*(14775.0d+00* \\
& *c(3)**4.0d+00+1028300.0d+00*c(3)**3.0d+00*c(5)+50.0d+00*c(3)** \\
& *2.0d+00*(10144.0d+00*c(4)**2.0d+00+594055.0d+00*c(5)**2.0d+00)+ \\
& *700.0d+00*c(3)*(27904.0d+00*c(4)**2.0d+00*c(5)+598575.0d+00*c(5)** \\
& *3.0d+00)+3.0d+00*(316928.0d+00*c(4)**4.0d+00+68908000.0d+00*c(4)** \\
& *2.0d+00*c(5)**2.0d+00+806378125.0d+00*c(5)**4.0d+00)))- fifth
\end{aligned}$$

$$\begin{aligned}
F(5) = & (120.0d+00*(32.0d+00*c(2)**6.0d+00+3456.0d+00*c(2)** \\
& *5.0d+00*c(4)+6.0d+00*c(1)**5.0d+00*c(5)+3.0d+00*c(1)**4.0d+00* \\
& *(9.0d+00*c(3)**2.0d+00+16.0d+00*c(2)*c(4)+168.0d+00*c(4)**2.0d+00+ \\
& *330.0d+00*c(3)*c(5)+2850.0d+00*c(5)**2.0d+00)+720.0d+00*c(2)** \\
& *4.0d+00*(15.0d+00*c(3)**2.0d+00+224.0d+00*c(4)**2.0d+00+490.0d+00* \\
& *c(3)*c(5)+4410.0d+00*c(5)**2.0d+00)+6048.0d+00*c(2)**3.0d+00*c(4)* \\
& *(125.0d+00*c(3)**2.0d+00+704.0d+00*c(4)**2.0d+00+4480.0d+00*c(3)* \\
& *c(5)+44100.0d+00*c(5)**2.0d+00)+12.0d+00*c(1)**3.0d+00*(4.0d+00* \\
& *c(2)**2.0d+00*(3.0d+00*c(3)+50.0d+00*c(5))+60.0d+00*c(2)*c(4)* \\
& *(7.0d+00*c(3)+114.0d+00*c(5))+3.0d+00*(24.0d+00*c(3)**3.0d+00+ \\
& *1192.0d+00*c(3)*c(4)**2.0d+00+1170.0d+00*c(3)**2.0d+00*c(5)+
\end{aligned}$$

*20440.0d+00*c(4)**2.0d+00*c(5)+20150.0d+00*c(3)*c(5)**2.0d+00+
 *124875.0d+00*c(5)**3.0d+00))+216.0d+00*c(2)**2.0d+00*(945.0d+00*
 *c(3)**4.0d+00+67620.0d+00*c(3)**3.0d+00*c(5)+560.0d+00*c(3)**
 2.0d+00(178.0d+00*c(4)**2.0d+00+3555.0d+00*c(5)**2.0d+00)+
 *315.0d+00*c(3)*(12416.0d+00*c(4)**2.0d+00*c(5)+90375.0d+00*c(5)**
 3.0d+00)+6.0d+00(52224.0d+00*c(4)**4.0d+00+6993000.0d+00*c(4)**
 *2.0d+00*c(5)**2.0d+00+27671875.0d+00*c(5)**4.0d+00))+6.0d+00*
 *c(1)**2.0d+00*(8.0d+00*c(2)**4.0d+00+480.0d+00*c(2)**3.0d+00*c(4)+
 *180.0d+00*c(2)**2.0d+00*(4.0d+00*c(3)**2.0d+00+64.0d+00*c(4)**
 *2.0d+00+125.0d+00*c(3)*c(5)+1050.0d+00*c(5)**2.0d+00)+72.0d+00*
 *c(2)*c(4)*(327.0d+00*c(3)**2.0d+00+1848.0d+00*c(4)**2.0d+00+
 *10955.0d+00*c(3)*c(5)+99900.0d+00*c(5)**2.0d+00)+9.0d+00*
 *(225.0d+00*c(3)**4.0d+00+22824.0d+00*c(3)**2.0d+00*c(4)**2.0d+00+
 *69632.0d+00*c(4)**4.0d+00+15090.0d+00*c(3)**3.0d+00*c(5)+
 *830240.0d+00*c(3)*c(4)**2.0d+00*c(5)+412925.0d+00*c(3)**2.0d+00*
 *c(5)**2.0d+00+
 *8239800.0d+00*c(4)**2.0d+00*c(5)**2.0d+00+5475750.0d+00*c(3)*
 *c(5)**3.0d+00+29636250.0d+00*c(5)**4.0d+00))+1296.0d+00*c(2)*c(4)*
 *(5910.0d+00*c(3)**4.0d+00+462735.0d+00*c(3)**3.0d+00*c(5)+c(3)**
 2.0d+00(228240.0d+00*c(4)**2.0d+00+14851375.0d+00*c(5)**2.0d+00)+
 *175.0d+00*c(3)*(55808.0d+00*c(4)**2.0d+00*c(5)+1316865.0d+00*
 *c(5)**3.0d+00)+3.0d+00*(158464.0d+00*c(4)**4.0d+00+37899400.0d+00*
 *c(4)**2.0d+00*c(5)**2.0d+00+483826875.0d+00*c(5)**4.0d+00))+
 27.0d+00(9945.0d+00*c(3)**6.0d+00+92930048.0d+00*c(4)**6.0d+00+
 *1166130.0d+00*c(3)**5.0d+00*c(5)+35724729600.0d+00*c(4)**4.0d+00*
 *c(5)**2.0d+00+977816385000.0d+00*c(4)**2.0d+00*c(5)**4.0d+00+
 *1907724656250.0d+00*c(5)**6.0d+00+180.0d+00*c(3)**4.0d+00*
 *(16082.0d+00*c(4)**2.0d+00+345905.0d+00*c(5)**2.0d+00)+140.0d+00*
 *c(3)**3.0d+00*(1765608.0d+00*c(4)**2.0d+00*c(5)+13775375.0d+00*
 *c(5)**3.0d+00)+15.0d+00*c(3)**2.0d+00*(4076032.0d+00*c(4)**
 *4.0d+00+574146160.0d+00*c(4)**2.0d+00*c(5)**2.0d+00+
 *2424667875.0d+00*c(5)**4.0d+00)+210.0d+00*c(3)*(13526272.0d+00*
 *c(4)**4.0d+00*c(5)+687499200.0d+00*c(4)**2.0d+00*c(5)**3.0d+00+
 *1876468125.0d+00*c(5)**5.0d+00))+18.0d+00*c(1)*(80.0d+00*c(2)**
 4.0d+00(c(3)+15.0d+00*c(5))+160.0d+00*c(2)**3.0d+00*c(4)*
 *(32.0d+00*c(3)+525.0d+00*c(5))+12.0d+00*c(2)**2.0d+00*
 *(225.0d+00*c(3)**3.0d+00+11088.0d+00*c(3)*c(4)**2.0d+00+
 *11130.0d+00*c(3)**2.0d+00*c(5)+199360.0d+00*c(4)**2.0d+00*c(5)+
 *200900.0d+00*c(3)*c(5)**2.0d+00+1323000.0d+00*c(5)**3.0d+00)+
 *24.0d+00*c(2)*c(4)*(3885.0d+00*c(3)**3.0d+00+69632.0d+00*c(3)*
 *c(4)**2.0d+00+209800.0d+00*c(3)**2.0d+00*c(5)+1369440.0d+00*c(4)**
 *2.0d+00*c(5)+4134375.0d+00*c(3)*c(5)**2.0d+00+29636250.0d+00*
 *c(5)**3.0d+00)+9.0d+00*(540.0d+00*c(3)**5.0d+00+48585.0d+00*
 *c(3)**4.0d+00*c(5)+20.0d+00*c(3)**3.0d+00*(4856.0d+00*c(4)**
 *2.0d+00+95655.0d+00*c(5)**2.0d+00)+80.0d+00*c(3)**2.0d+00*
 *(71597.0d+00*c(4)**2.0d+00*c(5)+513625.0d+00*c(5)**3.0d+00)+

```

*4.0d+00*c(3)*(237696.0d+00*c(4)**4.0d+00+30726500.0d+00*c(4)**
*2.0d+00*c(5)**2.0d+00+119844375.0d+00*c(5)**4.0d+00)+5.0d+00*c(5)*
*(4076032.0d+00*c(4)**4.0d+00+191074800.0d+00*c(4)**2.0d+00*
*c(5)**2.0d+00+483826875.0d+00*c(5)**4.0d+00))))-sixth

```

```

RETURN
END

```

c Function to determine R

```

double precision FUNCTION G (R)
implicit REAL*8 (a-h, o-z)
REAL*8 R
common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,
*c1b,c2b,c3b,c4b,c5b,rho

```

```

G = (3.0d+00*c0b*c4a+3.0d+00*c2b*c4a+9.0d+00*c4a*c4b+
*c0a*(c0b+c2b+3.0d+00*c4b)+c1a* c1b*R+
*3.0d+00*c1b*c3a*R+3.0d+00*c1a*c3b*R+9.0d+00*c3a*c3b*R+
*15.0d+00*c1b*c5a*R+45.0d+00*c3b*c5a*R+
* 15.0d+00*c1a*c5b*R+
*45.0d+00*c3a*c5b*R+225.0d+00*c5a*c5b*R+
* 12.0d+00*c2b*c4a*R**2.0d+00+
*72.0d+00*c4a*c4b*R**2.0d+00+6.0d+00*c3a*c3b*R**3.0d+00+
*60.0d+00*c3b*c5a*R**3.0d+00+60.0d+00*c3a*c5b*R**3.0d+00+
*600.0d+00*c5a*c5b*R**3.0d+00+24.0d+00*c4a*c4b*R**4.0d+00+
*120.0d+00*c5a*c5b*R**5.0d+00+
*c2a*(c0b+c2b+3.0d+00*c4b+2.0d+00*c2b*R**2.0d+00+
*12.0d+00*c4b*R**2.0d+00))-rho

```

```

RETURN
END

```

C ***** Subroutine *****

C Used to find E[Y1], E[Y2], E[Y1Y2], standard deviations for Y1 and Y2,
correlation between Y1 and Y2, skew and kurtosis
subroutine outsum(M, Y1, Y2, sumY1, sumY2,
& aveY1, aveY2, spY1Y2, apY1Y2, pY1Y2,
& sumsqY1, sumsqY2, SY1, SY2, corrY, scY1, scY2,
& sumquY1, sumquY2, skewY1, skewY2, skurtY1, skurtY2)

```

implicit REAL*8 (a-h, o-z)
REAL*8  Y1(M), Y2(M), sumY1, sumY2,
& aveY1, aveY2, spY1Y2, apY1Y2, pY1Y2(M),
& sumsqY1, sumsqY2, SY1, SY2, corrY, scY1, scY2,
& sumquY1, sumquY2, skewY1, skewY2, skurtY1, skurtY2
common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,
*c1b,c2b,c3b,c4b,c5b,rho
sumY1=0.0d+00
sumY2=0.0d+00
spY1Y2 = 0.0d+00
sumsqY1=0.0d+00
sumsqY2=0.0d+00
scY1=0.0d+00
scY2=0.0d+00
sumquY1=0.0d+00
sumquY2=0.0d+00

do 20 i=1,M
sumY1=sumY1+Y1(i)
sumY2=sumY2+Y2(i)
pY1Y2(i) = Y1(i)*Y2(i)
spY1Y2 = spY1Y2 + pY1Y2(i)
sumsqY1 = sumsqY1 + (Y1(i)-aveY1)**2.0d+00
sumsqY2 = sumsqY2 + (Y2(i)-aveY2)**2.0d+00
scY1 = scY1 + (Y1(i)-aveY1)**3.0d+00
scY2 = scY2 + (Y2(i)-aveY2)**3.0d+00
sumquY1 = sumquY1 + (Y1(i)-aveY1)**4.0d+00
sumquY2 = sumquY2 + (Y2(i)-aveY2)**4.0d+00
20 continue

aveY1 = sumY1/float(M)
aveY2 = sumY2/float(M)
apY1Y2 = spY1Y2/float(M)

SY1 = dsqrt(sumsqY1/float(M))
SY2 = dsqrt(sumsqY2/float(M))
corrY = (apY1Y2-aveY1*aveY2)/(SY1*SY2)
skewY1=scY1/(float(M)*SY1**3.0d+00)
skewY2=scY2/(float(M)*SY2**3.0d+00)
skurtY1=(sumquY1/(float(M)*SY1**4.0d+00))-3.0d+00
skurtY2=(sumquY2/(float(M)*SY2**4.0d+00))-3.0d+00

```

```
RETURN
```

```
END
```

Appendix F

```
c Program to calculate average difference
c   between generated and desired parameters using
c   the Generalized Lambda Distribution Method

      use numerical_libraries

C           Declare variables

      implicit real*8 (a-h, o-z)
      integer I, IRANK, ISEED, J, K, LDR, LDRSIG, NOUT, NR

c   L=NR, N=K
      parameter (k1=10000, L=30, N=2)

      REAL*8 C11,C12,C13,C14,C21,C22,C23,C24,PI,sq2pi,int,DELTA,
      & UMIN,UMAX,VMIN,VMAX,Z1,Z2,ST,rho, time, sprho, x(2),
      & aint, bint, estint, rhoa, rhob, fofa, fest, rhoest,
      & COV(2,2), R(L,N), RSIG(2,2), z1gen(L), z2gen(L),
      & x1(L), x2(L), sumx1, sumx2,u1(L), u2(L), g1(L), g2(L),
      & avex1, avex2, spx1x2, apx1x2,
      & ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,
      & sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2

c   specified skews and kurtoses
      data skew1/0.0d+00/
      data skurt1/5.0d+00/
      data skew2/0.0d+00/
      data skurt2/5.0d+00/

c   specified correlation
      data sprho/0.9d+00/

c   delta used in Reimann sums to calculate intermediate correlation
      data delta/0.05d+00/

      umin = -5.0d+00
      vmin = -5.0d+00
      umax = 5.0d+00
      vmax = 5.0d+00
      aint = sprho
      bint = sprho+0.1d+00
      PI=3.14159265358979323846d+00
      sq2pi = dSQRT(2.0d+00*PI)
      NR           = L
```

```

K      = N
LDRSIG = 2
LDR    = L
open (unit=9,file='c:\ave GLD.out')

      call rnsset (12345)

      time = CPSEC()

c      x(1) = 0.05d+00
c      x(2) = 0.05d+00
      x(1) = -0.1d+00
      x(2) = -0.1d+00

      call gdlambda(x, skew1,skurt1, onelam, twolam, thrlam, fourlam)
      c11=onelam
      c12=twolam
      c13=thrlam
      c14=fourlam

      print*, 'c11= ', c11
      print*, 'c12= ', c12
      print*, 'c13= ', c13
      print*, 'c14= ', c14

c      x(1) = 0.05d+00
c      x(2) = 0.05d+00
      x(1) = -0.1d+00
      x(2) = -0.1d+00

      call gdlambda(x, skew2,skurt2, onelam2, twolam2, thrlam2, fourlam2)
      c21=onelam2
      c22=twolam2
      c23=thrlam2
      c24=fourlam2

      print*, 'c21= ', c21
      print*, 'c22= ', c22
      print*, 'c23= ', c23
      print*, 'c24= ', c24

c      Calculate the intermediate correlation

      do while (dabs(bint-aint).gt.0.000001d+00)

```

```
rhoa = 0.0d+00
rhoest = 0.0d+00
```

```
estint = (aint+bint)/2.0d+00
```

```
DO 10 Z1=UMIN,UMAX,DELTA
DO 20 Z2=VMIN,VMAX,DELTA
```

```
rhoa=rhoa+ST(C11,C12,C13,C14,C21,C22,C23,C24,PI,sq2pi,aint,
&          UMIN,VMIN,DELTA,Z1,Z2)*DELTA**2.0d+00
```

```
rhoest=rhoest+ST(C11,C12,C13,C14,C21,C22,C23,C24,PI,sq2pi,estint,
&          UMIN,VMIN,DELTA,Z1,Z2)*DELTA**2.0d+00
```

```
20 CONTINUE
```

```
10 CONTINUE
```

```
fofa=sprho-rhoa
fest=sprho-rhoest
```

```
if (fofa*fest.gt.0.0d+00) then
    aint=estint
    else
    bint=estint
end if
```

```
end do
```

```
c print*, 'aint= ', aint
c print*, 'bint= ', bint
c print*, 'estint= ', estint
```

```
c ***** Generate random variables using GLD
c *****
```

```
c Generate Z1 and Z2 with correlations of intermediate rho
c R matrix represents Z1 and Z2
```

```
sumrho=0.0d+00
sum1=0.0d+00
sum2=0.0d+00
sumvar1=0.0d+00
```



```
sumvar2=0.0d+00
sumsk1=0.0d+00
sumsk2=0.0d+00
sumku1=0.0d+00
sumku2=0.0d+00
```

```
sumdmx1=0.0d+00
sumdmx2=0.0d+00
sumdvx1=0.0d+00
sumdvx2=0.0d+00
sumdsx1=0.0d+00
sumdsx2=0.0d+00
sumdkx1=0.0d+00
sumdkx2=0.0d+00
sumdr=0.0d+00
```

```
do 1 i=1,k1
```

```
COV(1,1) = 1.0d+00
COV(1,2) = estint
  COV(2,1) = estint
  COV(2,2) = 1.0d+00
```

C Obtain the Cholesky factorization.

```
CALL dCHFAC (K, COV, K, 0.00001d+00, IRANK, RSIG, LDRSIG)
```

C Initialize seed of random number generator.

```
c ISEED = 123457
```

```
c CALL RNSET (ISEED)
```

```
CALL dRNMVN (NR, K, RSIG, LDRSIG, R, LDR)
```

```
do 100 i100=1,L
```

```
  z1gen(i100)=r(i100,1)
```

```
  z2gen(i100)=r(i100,2)
```

```
c print*, 'z1gen= ', r(i100, 1), ' z2gen= ', r(i100, 2)
```

```
100 continue
```

c Transform standard normal deviates to uniform deviates

```
do 50 m=1,L
```

```
  u1(m)=dnordf(z1gen(m))
```

```

50          u2(m)=dnordf(z2gen(m))
    continue

```

c Generate X1 and X2 by GLD with the desired post-correlation and the specified skew and kurtosis

```

3      do 3 j=1,L
        g1(j)= c11+((u1(j)**c13)-(1.0d+00-u1(j))**c14)/c12
    continue

```

```

4      do 4 j1=1,L
        g2(j1)= c21+((u2(j1)**c23)-(1.0d+00-u2(j1))**c24)/c22
    continue

```

```

    call outsum(g1, g2, L, sumx1, sumx2,
& avex1, avex2, spx1x2, apx1x2,
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2)

```

```

    sumrho=corrx+sumrho
    sum1=avex1+sum1
    sum2=avex2+sum2
    sumvar1=sx1**2.0d+00+sumvar1
    sumvar2=sx2**2.0d+00+sumvar2
    sumsk1=skewx1+sumsk1
    sumsk2=skewx2+sumsk2
    sumku1=skurtx1+sumku1
    sumku2=skurtx2+sumku2

```

```

    dmx1=dabs(avex1)
    dmx2=dabs(avex2)
    dvarx1=dabs(sx1**2.0d+00-1.0d+00)
    dvarx2=dabs(sx2**2.0d+00-1.0d+00)
    dskx1=dabs(skewx1-skew1)
    dskx2=dabs(skewx2-skew2)
    dkux1=dabs(skurtx1-skurt1)
    dkux2=dabs(skurtx2-skurt2)
    drho=dabs(corrx-sprho)

```

```

    sumdmx1=sumdmx1+dmx1
    sumdmx2=sumdmx2+dmx2
    sumdvx1=sumdvx1+dvarx1
    sumdvx2=sumdvx2+dvarx2
    sumdsx1=sumdsx1+dskx1

```

```
sumdsx2=sumdsx2+dskx2
sumdkx1=sumdkx1+dkux1
sumdkx2=sumdkx2+dkux2
sumdr=sumdr+drho
```

1 continue

```
averho=sumrho/float(k1)
avemean1=sum1/float(k1)
avemean2=sum2/float(k1)
avevar1=sumvar1/float(k1)
avevar2=sumvar2/float(k1)
avesk1=sumsk1/float(k1)
avesk2=sumsk2/float(k1)
aveku1=sumku1/float(k1)-3.0d+00
aveku2=sumku2/float(k1)-3.0d+00
```

```
avedmx1=sumdmx1/float(k1)
avedmx2=sumdmx2/float(k1)
avedvx1=sumdvx1/float(k1)
avedvx2=sumdvx2/float(k1)
avedsx1=sumdsx1/float(k1)
avedsx2=sumdsx2/float(k1)
avedkx1=sumdkx1/float(k1)
avedkx2=sumdkx2/float(k1)
avedr=sumdr/float(k1)
```

```
write(9,*) averho
write(9,*) ''
write(9,*) avemean1
write(9,*) avevar1
write(9,*) avesk1
write(9,*) aveku1
write(9,*) ''
write(9,*) avemean2
write(9,*) avevar2
write(9,*) avesk2
write(9,*) aveku2
write(9,*) ''
write(9,*) ''
write(9,*) avedr
write(9,*) ''
write(9,*) avedmx1
write(9,*) avedvx1
write(9,*) avedsx1
write(9,*) avedkx1
```

```

write(9,*) ''
write(9,*) avedmx2
write(9,*) avedvx2
write(9,*) avedsx2
write(9,*) avedkx2

time = CPSEC()

```

END

```

C*****
*

```

```

double precision FUNCTION ST(C11,C12,C13,C14,C21,C22,C23,C24,
& PI,sq2pi,int,UMIN,VMIN,DELTA,Z1,Z2)

```

```

REAL*8 R, C11,C12,C13,C14,C21,C22,C23,C24,PI,int,
& sq2pi, UMIN,VMIN,DELTA,Z1,Z2

```

```

X1=C11+(PHI(sq2PI,UMIN,Z1,DELTA)**C13-
& (1.0d+00-PHI(sq2PI,UMIN,Z1,DELTA))**C14)/C12

```

```

X2=C21+(PHI(sq2pi,VMIN,Z2,DELTA)**C23-
& (1.0d+00-PHI(sq2PI,VMIN,Z2,DELTA))**C24)/C22

```

```

R=((X1*X2)*(((2.0d+00*PI)*(dSQRT(1.0d+00-int**2)))**(-1.0d+00))*
& dEXP((-1.0d+00/(2.0d+00*(1.0d+00-int**2.0d+00)))*
*((Z1**2.0d+00)-2.0d+00*int*(Z1*Z2)+(Z2**2.0d+00))))

```

ST=R

RETURN
END

```

C*****
*

```

```

double precision FUNCTION PHI(sq2PI,UMIN,Z1,DELTA)

```

```

REAL*8 SUM1,U, delta, z1, umin, sq2pi

```

```
SUM1=0.0d+00
```

```
DO 200 U=UMIN,Z1,DELTA
```

```
SUM1=SUM1+1.0d+00/(sq2pi)*  
*dEXP(-(U**2.0d+00)/2.0d+00)*DELTA
```

```
200 CONTINUE
```

```
PHI=SUM1
```

```
RETURN
```

```
END
```

C Used to find $E[x_1]$, $E[x_2]$, $E[x_1x_2]$, standard deviations for x_1 and x_2 ,
c correlation between x_1 and x_2 , skew and kurtosis

```
subroutine outsum(x1, x2, L, sumx1, sumx2,  
& avex1, avex2, spx1x2, apx1x2,  
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,  
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2)
```

```
implicit REAL*8 (a-h, o-z)  
integer L  
REAL*8 x1(L), x2(L), sumx1, sumx2,  
& avex1, avex2, spx1x2, apx1x2,  
& ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,  
& sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2
```

```
sumx1=0.0d+00  
sumx2=0.0d+00  
spx1x2 = 0.0d+00  
ssqx1=0.0d+00  
ssqx2=0.0d+00  
scux1=0.0d+00  
scux2=0.0d+00  
sumqux1=0.0d+00  
sumqux2=0.0d+00
```

```
do 20 i=1,L  
sumx1=sumx1+x1(i)  
sumx2=sumx2+x2(i)
```

```

20      spx1x2 = spx1x2 + x1(i)*x2(i)
        continue

        avex1 = sumx1/float(L)
        avex2 = sumx2/float(L)
        apx1x2 = spx1x2/float(L)

        do 30 i30=1,L
          ssqx1 = ssqx1 + (x1(i30)-avex1)**2.0d+00
          ssqx2 = ssqx2 + (x2(i30)-avex2)**2.0d+00
          scux1 = scux1 + (x1(i30)-avex1)**3.0d+00
          scux2 = scux2 + (x2(i30)-avex2)**3.0d+00
          sumqux1 = sumqux1 + (x1(i30)-avex1)**4.0d+00
          sumqux2 = sumqux2 + (x2(i30)-avex2)**4.0d+00
30      continue

```

```

        Sx1 = dsqrt(ssqx1/(float(L)))
        Sx2 = dsqrt(ssqx2/(float(L)))
        corrx = (apx1x2-avex1*avex2)/(Sx1*Sx2)
        skewx1=scux1/(float(L)*Sx1**3.0d+00)
        skewx2=scux2/(float(L)*Sx2**3.0d+00)
        skurtx1=(sumqux1/(float(L)*Sx1**4.0d+00))
        skurtx2=(sumqux2/(float(L)*Sx2**4.0d+00))

```

```

        RETURN
END

```

```

c      *****subroutine to calculate lambdas*****

```

```

        subroutine gldlambda(x, skew, skurt, onelam, twolam, thrlam, fourlam)
        implicit real*8 (a-h, o-z)

```

```

        INTEGER LDC, LDDG, LWK, M, ME, N

```

```

        PARAMETER (M=1, ME=0, N=2, LDC=N+1, LDDG=M,
& LWK=2*N*(N+16)+9*M+68)

```

```

C      INTEGER IBTYPE, IDO, IPRINT, IWK(19+M), MAXFUN, MAXITN,
&      MODE

```

```

        REAL*8 C(LDC,N+1), CONWK(M), D(N+1), DF(N),
& DG(LDDG,N), G(M), U(M+N+N+2), WK(LWK), X(N), XLB(N), XUB(N)

```

```

        LOGICAL ACTIVE(2*M+13)

```



```

*x(2))**2.0d+00*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta
*(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))-
*3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*4.0d+00)/
*((1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),
*1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))-(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2)))*2.0d+00)**2.0d+00)-skurt)**2.0d+00

```

C Evaluate the constraints at X.

$$G(1) = x(1)*x(2)$$

END IF

C IF (IDO.EQ.0 .OR. IDO.EQ.2) THEN

C Evaluate the function gradient at X.

```

DF(1) = (2.0d+00*((1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta
*(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1))+3.0d+00*Dbeta(1.0d+00+x(1),
*1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2)))-(3.0d+00*
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/
*(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+
*1.0d+00/(1.0d+00+2.0d+00*x(2)))+2.0d+00*(1.0d+00/(1.0d+00+x(1))-
*1.0d+00/(1.0d+00+x(2)))*3.0d+00)/(1.0d+00/(1.0d+00+2.0d+00*
*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/
*(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))*2.0d+00)**(3.0d+00/2.0d+00)-skew)))*((-3.0d+00/
*(1.0d+00+3.0d+00*x(1))*2.0d+00-(3.0d+00*(2.0d+00*Dpsi(1.0d+00+
*2.0d+00*x(1))-2.0d+00*Dpsi(2.0d+00+x(2)+2.0d+00*x(1))))*Dbeta
*(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1)))+(3.0d+00*(Dpsi(1.0d+00+
*x(1))-Dpsi(2.0d+00+x(1)+2.0d+00*x(2))))*Dbeta(1.0d+00+x(1),
*1.0d+00+2.0d+00*x(2)))+(3.0d+00*(1.0d+00/(1.0d+00+2.0d+00*
*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/
*(1.0d+00+2.0d+00*x(2))))/(1.0d+00+x(1))**2.0d+00-(3.0d+00*
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*(-2.0d+00/
*(1.0d+00+2.0d+00*x(1)))*2.0d+00-(2.0d+00*(Dpsi(1.0d+00+x(1))-
*Dpsi(2.0d+00+x(2)+x(1))))*Dbeta(1.0d+00+x(2), 1.0d+00+x(1)))-
*6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*
*2.0d+00/(1.0d+00+x(1))**2.0d+00)/(1.0d+00/
*(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+x(1))+
*1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))*2.0d+00)**(3.0d+00/2.0d+00)-(3.0d+00/2.0d+00)*
*(1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+x(2),
*1.0d+00+2.0d+00*x(1))+3.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*
*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2)))-(3.0d+00*(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+2.0d+00*x(1))-
*2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/(1.0d+00+

```


$$\begin{aligned}
& *2.0d+00*x(2))) + 2.0d+00*(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+ \\
& *x(2))) ** 3.0d+00) * (-2.0d+00/(1.0d+00+2.0d+00*x(1))) ** 2.0d+00- \\
& *(2.0d+00* \\
& *(Dpsi(1.0d+00+x(1)) - Dpsi(2.0d+00+x(2)+x(1)))) * Dbeta(1.0d+00+ \\
& *x(2), 1.0d+00+x(1)) + (2.0d+00*(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/ \\
& *(1.0d+00+x(2)))) / (1.0d+00+x(1)) ** 2.0d+00) / (1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1)) - 2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1)) + 1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2)) - (1.0d+00/(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+ \\
& *x(2))) ** 2.0d+00) ** (5.0d+00/2.0d+00) + (2.0d+00*((1.0d+00/(1.0d+00+ \\
& *4.0d+00*x(1)) - 4.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+3.0d+00*x(1)) + \\
& *6.0d+00*Dbeta(1.0d+00+2.0d+00*x(2), 1.0d+00+2.0d+00*x(1)) - \\
& *4.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+3.0d+00*x(2)) + 1.0d+00/ \\
& *(1.0d+00+4.0d+00*x(2)) - (4.0d+00*(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/ \\
& *(1.0d+00+x(2)))) * (1.0d+00/(1.0d+00+3.0d+00*x(1)) - 3.0d+00*Dbeta \\
& *(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1)) + 3.0d+00*Dbeta(1.0d+00+x(1), \\
& *1.0d+00+2.0d+00*x(2)) - 1.0d+00/(1.0d+00+3.0d+00*x(2))) + 6.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+x(2))) ** 2.0d+00* \\
& *(1.0d+00/(1.0d+00+2.0d+00*x(1)) - 2.0d+00*Dbeta(1.0d+00+x(2), \\
& *1.0d+00+x(1)) + 1.0d+00/(1.0d+00+2.0d+00*x(2)) - 3.0d+00*(1.0d+00/ \\
& *(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+x(2))) ** 4.0d+00) / (1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(1)) - 2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1)) + \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2)) - (1.0d+00/(1.0d+00+x(1)) - 1.0d+00/ \\
& *(1.0d+00+x(2))) ** 2.0d+00) ** 2.0d+00 - skurt) * ((-4.0d+00/(1.0d+00+ \\
& *4.0d+00*x(1)) ** 2.0d+00 - (4.0d+00*(3.0d+00*Dpsi(1.0d+00+3.0d+00* \\
& *x(1)) - 3.0d+00*Dpsi(2.0d+00+x(2)+3.0d+00*x(1)))) * Dbeta(1.0d+00+ \\
& *x(2), 1.0d+00+3.0d+00*x(1)) + (6.0d+00*(2.0d+00*Dpsi(1.0d+00+ \\
& *2.0d+00*x(1)) - 2.0d+00*Dpsi(2.0d+00+2.0d+00*x(2)+2.0d+00*x(1)))) * \\
& *Dbeta(1.0d+00+2.0d+00*x(2), 1.0d+00+2.0d+00*x(1)) - (4.0d+00* \\
& *(Dpsi(1.0d+00+x(1)) - Dpsi(2.0d+00+x(1)+3.0d+00*x(2)))) * Dbeta \\
& *(1.0d+00+x(1), 1.0d+00+3.0d+00*x(2)) + (4.0d+00*(1.0d+00/ \\
& *(1.0d+00+3.0d+00*x(1)) - 3.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+ \\
& *2.0d+00*x(1)) + 3.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2)) - \\
& *1.0d+00/(1.0d+00+3.0d+00*x(2)))) / (1.0d+00+x(1)) ** 2.0d+00- \\
& *(4.0d+00*(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+x(2)))) * \\
& *(-3.0d+00/(1.0d+00+3.0d+00*x(1)) ** 2.0d+00 - (3.0d+00*(2.0d+00* \\
& *Dpsi(1.0d+00+2.0d+00*x(1)) - 2.0d+00*Dpsi(2.0d+00+x(2)+2.0d+00* \\
& *x(1)))) * Dbeta(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1)) + (3.0d+00* \\
& *(Dpsi(1.0d+00+x(1)) - Dpsi(2.0d+00+x(1)+2.0d+00*x(2)))) * Dbeta \\
& *(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2)) - (12.0d+00*(1.0d+00/ \\
& *(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+x(2)))) * (1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1)) - 2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1)) + 1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2)) / (1.0d+00+x(1)) ** 2.0d+00 + 6.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+x(2))) ** 2.0d+00* \\
& *(-2.0d+00/(1.0d+00+2.0d+00*x(1)) ** 2.0d+00 - (2.0d+00*(Dpsi(1.0d+00+ \\
& *x(1)) - Dpsi(2.0d+00+x(2)+x(1)))) * Dbeta(1.0d+00+x(2), 1.0d+00+ \\
& *x(1)) + 12.0d+00*(1.0d+00/(1.0d+00+x(1)) - 1.0d+00/(1.0d+00+
\end{aligned}$$

$$\begin{aligned}
& *x(2))**3.0d+00/(1.0d+00+x(1))**2.0d+00)/(1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+ \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))- \\
& *1.0d+00/(1.0d+00+x(2)))**2.0d+00)**2.0d+00-(2.0d+00*(1.0d+00/ \\
& *(1.0d+00+4.0d+00*x(1))-4.0d+00*Dbeta(1.0d+00+x(2),1.0d+00+ \\
& *3.0d+00*x(1))+6.0d+00*Dbeta(1.0d+00+2.0d+00*x(2),1.0d+00+2.0d+00* \\
& *x(1))-4.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+3.0d+00*x(2))+ \\
& *1.0d+00/(1.0d+00+4.0d+00*x(2))-(4.0d+00*(1.0d+00/(1.0d+00+ \\
& *x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+3.0d+00*x(1))- \\
& *3.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+2.0d+00*x(1))+3.0d+00* \\
& *Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+ \\
& *3.0d+00*x(2)))+6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2)))**2.0d+00*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta \\
& *(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/(1.0d+00+2.0d+00*x(2)))- \\
& *3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
& *4.0d+00))*(-2.0d+00/(1.0d+00+2.0d+00*x(1))**2.0d+00-(2.0d+00*(Dpsi \\
& *(1.0d+00+x(1))-Dpsi(2.0d+00+x(2)+x(1))))*Dbeta(1.0d+00+x(2), \\
& *1.0d+00+x(1))+(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
& *x(2)))/(1.0d+00+x(1))**2.0d+00)/(1.0d+00/(1.0d+00+2.0d+00* \\
& *x(1))-2.0d+00*Dbeta(1.0d+00+x(2), 1.0d+00+x(1))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))**2.0d+00)**3.0d+00)
\end{aligned}$$

$$\begin{aligned}
DF(2)= & (2.0d+00*((1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta \\
& *(1.0d+00+2.0d+00*x(1), 1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1), \\
& *1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2))-(3.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+ \\
& *1.0d+00/(1.0d+00+2.0d+00*x(2)))+2.0d+00*(1.0d+00/(1.0d+00+x(1))- \\
& *1.0d+00/(1.0d+00+x(2)))**3.0d+00)/(1.0d+00/(1.0d+00+2.0d+00* \\
& *x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
& *(1.0d+00+x(2)))**2.0d+00)**(3.0d+00/2.0d+00)-skew)** \\
& *((-3.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+2.0d+00*x(1)+ \\
& *x(2))))*Dbeta(1.0d+00+2.0d+00*x(1), 1.0d+00+x(2))+(3.0d+00* \\
& *(2.0d+00*Dpsi(1.0d+00+2.0d+00*x(2))-2.0d+00*Dpsi(2.0d+00+x(1)+ \\
& *2.0d+00*x(2))))*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))+3.0d+00/ \\
& *(1.0d+00+3.0d+00*x(2))**2.0d+00-(3.0d+00*(1.0d+00/(1.0d+00+ \\
& *2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/ \\
& *(1.0d+00+2.0d+00*x(2)))/(1.0d+00+x(2)))**2.0d+00-(3.0d+00* \\
& *(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(-2.0d+00*
\end{aligned}$$

$$\begin{aligned}
&*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+x(1)+x(2)))*Dbeta(1.0d+00+ \\
&*x(1),1.0d+00+x(2))-2.0d+00/(1.0d+00+2.0d+00*x(2))**2.0d+00)+ \\
&*6.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
&*2.0d+00/(1.0d+00+x(2))**2.0d+00)/(1.0d+00/(1.0d+00+2.0d+00*x(1))- \\
&*2.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+ \\
&*2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
&*2.0d+00)**(3.0d+00/2.0d+00)-(3.0d+00/2.0d+00)*(1.0d+00/(1.0d+00+ \\
&*3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+2.0d+00*x(1),1.0d+00+x(2))+ \\
&*3.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2))-1.0d+00/ \\
&*(1.0d+00+3.0d+00*x(2))-(3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
&*(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta \\
&*(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))+ \\
&*2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))** \\
&*3.0d+00)*(-(2.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+x(1)+ \\
&*x(2)))*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))-2.0d+00/(1.0d+00+ \\
&*2.0d+00*x(2))**2.0d+00-(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
&*(1.0d+00+x(2))))/(1.0d+00+x(2))**2.0d+00)/(1.0d+00/(1.0d+00+ \\
&*2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/ \\
&*(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+ \\
&*x(2)))**2.0d+00)**(5.0d+00/2.0d+00)+(2.0d+00*((1.0d+00/(1.0d+00+ \\
&*4.0d+00*x(1))-4.0d+00*Dbeta(1.0d+00+3.0d+00*x(1),1.0d+00+x(2))+ \\
&*6.0d+00*Dbeta(1.0d+00+2.0d+00*x(1), 1.0d+00+2.0d+00*x(2))- \\
&*4.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+3.0d+00*x(2))+1.0d+00/ \\
&*(1.0d+00+4.0d+00*x(2))-(4.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
&*(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta \\
&*(1.0d+00+2.0d+00*x(1),1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1), \\
&*1.0d+00+2.0d+00*x(2))-1.0d+00/(1.0d+00+3.0d+00*x(2)))+6.0d+00* \\
&*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**2.0d+00* \\
&*(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1), \\
&*1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2)))-3.0d+00*(1.0d+00/ \\
&*(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))**4.0d+00)/(1.0d+00/ \\
&*(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+ \\
&*1.0d+00/(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/ \\
&*(1.0d+00+x(2)))**2.0d+00)**2.0d+00-skurti)*((-4.0d+00*(Dpsi \\
&*(1.0d+00+x(2))-Dpsi(2.0d+00+3.0d+00*x(1)+x(2))))*Dbeta(1.0d+00+ \\
&*3.0d+00*x(1), 1.0d+00+x(2)))+(6.0d+00*(2.0d+00*Dpsi(1.0d+00+ \\
&*2.0d+00*x(2))-2.0d+00*Dpsi(2.0d+00+2.0d+00*x(1)+2.0d+00*x(2))))* \\
&*Dbeta(1.0d+00+2.0d+00*x(1), 1.0d+00+2.0d+00*x(2))-(4.0d+00* \\
&*(3.0d+00*Dpsi(1.0d+00+3.0d+00*x(2))-3.0d+00*Dpsi(2.0d+00+x(1)+ \\
&*3.0d+00*x(2))))*Dbeta(1.0d+00+x(1), 1.0d+00+3.0d+00*x(2))- \\
&*4.0d+00/(1.0d+00+4.0d+00*x(2))**2.0d+00-(4.0d+00*(1.0d+00/ \\
&*(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+2.0d+00*x(1), \\
&*1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))- \\
&*1.0d+00/(1.0d+00+3.0d+00*x(2)))/(1.0d+00+x(2))**2.0d+00- \\
&*(4.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(- \\
&*(3.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+2.0d+00*x(1)+x(2))))*
\end{aligned}$$

```

*Dbeta(1.0d+00+2.0d+00*x(1),1.0d+00+x(2))+(3.0d+00*(2.0d+00*
*Dpsi(1.0d+00+2.0d+00*x(2))-2.0d+00*Dpsi(2.0d+00+x(1)+2.0d+00*
*x(2))))*Dbeta(1.0d+00+x(1),1.0d+00+2.0d+00*x(2))+3.0d+00/
*(1.0d+00+3.0d+00*x(2))*2.0d+00)+(12.0d+00*(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/(1.0d+00+2.0d+00*x(1))-
*2.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+
*2.0d+00*x(2)))/(1.0d+00+x(2))*2.0d+00+6.0d+00*(1.0d+00/(1.0d+00+
*x(1))-1.0d+00/(1.0d+00+x(2)))*2.0d+00*(-(2.0d+00*(Dpsi(1.0d+00+
*x(2))-Dpsi(2.0d+00+x(1)+x(2))))*Dbeta(1.0d+00+x(1),1.0d+00+x(2))-
*2.0d+00/(1.0d+00+2.0d+00*x(2))*2.0d+00)-12.0d+00*(1.0d+00/
*(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*3.0d+00/(1.0d+00+
*x(2))*2.0d+00)/(1.0d+00/(1.0d+00+2.0d+00*x(1))-2.0d+00*Dbeta
*(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/(1.0d+00+2.0d+00*x(2))-
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2)))*2.0d+00)*
*2.0d+00-(2.0d+00*(1.0d+00/(1.0d+00+4.0d+00*x(1))-4.0d+00*Dbeta
*(1.0d+00+3.0d+00*x(1),1.0d+00+x(2))+6.0d+00*Dbeta(1.0d+00+
*2.0d+00*x(1), 1.0d+00+2.0d+00*x(2))-4.0d+00*Dbeta(1.0d+00+x(1),
*1.0d+00+3.0d+00*x(2))+1.0d+00/(1.0d+00+4.0d+00*x(2)))-(4.0d+00*
*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+x(2))))*(1.0d+00/
*(1.0d+00+3.0d+00*x(1))-3.0d+00*Dbeta(1.0d+00+2.0d+00*x(1),
*1.0d+00+x(2))+3.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+2.0d+00*x(2))-
*1.0d+00/(1.0d+00+3.0d+00*x(2)))+6.0d+00*(1.0d+00/(1.0d+00+x(1))-
*1.0d+00/(1.0d+00+x(2)))*2.0d+00*(1.0d+00/(1.0d+00+2.0d+00*x(1))-
*2.0d+00*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))+1.0d+00/(1.0d+00+
*2.0d+00*x(2))-3.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/(1.0d+00+
*x(2)))*4.0d+00))*(-(2.0d+00*(Dpsi(1.0d+00+x(2))-Dpsi(2.0d+00+
*x(1)+x(2))))*Dbeta(1.0d+00+x(1), 1.0d+00+x(2))-2.0d+00/(1.0d+00+
*2.0d+00*x(2))*2.0d+00-(2.0d+00*(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))/(1.0d+00+x(2))*2.0d+00)/(1.0d+00/(1.0d+00+
*2.0d+00*x(1))-2.0d+00*Dbeta(1.0d+00+x(1),1.0d+00+x(2))+1.0d+00/
*(1.0d+00+2.0d+00*x(2))-(1.0d+00/(1.0d+00+x(1))-1.0d+00/
*(1.0d+00+x(2)))*2.0d+00)*3.0d+00)

```

C If active evaluate the constraint
C gradient at X.

```

IF (ACTIVE(1)) THEN
  DG(1,1) = X(2)
  DG(1,2) = X(1)
END IF

```

C

END IF

C Call N0ONF for the next update.

C

```
CALL dN0ONF (IDO, M, ME, N, IBTYPE, XLB, XUB, IPRINT, MAXITN, X,
&          FVALUE, G, DF, DG, LDDG, U, C, LDC, D, ACC, SCBOU,
&          MAXFUN, ACTIVE, MODE, WK, IWK, CONWK)
```

```
C          If IDO does not equal 1 or 2, exit.
```

```
IF (IDO.EQ.1 .OR. IDO.EQ.2) GO TO 10
```

```
C          solve for lambda1 and lambda2
```

```
twolam = dsqrt((1.0d+00/(2.0d+00*x(1)+1.0d+00)-2.0d+00*dbeta
*(x(1)+1.0d+00, x(2)+1.0d+00)+1.0d+00/(2.0d+00*x(2)+1.0d+00))-
*(1.0d+00/(x(1)+1.0d+00)-1.0d+00/(x(2)+1.0d+00))**2.0d+00)
```

```
if (twolam.lt.0.0d+00) then
    onelam=(1.0d+00/(x(1)+1.0d+00)-1.0d+00/(x(2)+1.0d+00))/twolam
end if
```

```
if (twolam.gt.0.0d+00) then
    onelam=-((1.0d+00/(x(1)+1.0d+00)-1.0d+00/(x(2)+1.0d+00))/twolam)
end if
```

```
C          If lambda3 and lambda4 less than zero, switch values
```

```
if (x(1).lt.0.0d+00) then
    twolam= -1.0d+00*twolam
    thrlam = x(2)
    fourlam = x(1)
end if
```

```
if (x(1).gt.0.0d+00) then
    thrlam = x(1)
    fourlam = x(2)
end if
```

```
return
```

```
END
```

Appendix G

- c Program to calculate average difference
- c between generated and desired parameters using
- c the Fleishman Power Method

use numerical_libraries

C Declare variables

```
implicit REAL*8 (a-h, o-z)
```

```
PARAMETER (K=10000, M=1000, N=3, NROOT=2)
```

```
data skew1/0.0d+00/
```

```
data skurt1/2.0d+00/
```

```
data skew2/0.0d+00/
```

```
data skurt2/2.0d+00/
```

```
data rho/0.1d+00/
```

```
REAL*8 FNORM, C(N), CGUESS(N), G, R(NROOT), RGUESS(NROOT), Z(M),  
& Eone(M), E2(M), Y1(M), Y2(M), X1(M), X2(M),  
& prodY1Y2(M)
```

```
common skew, skurt, a1, b1, c1, d1, a2, b2, c2, d2, rho
```

```
EXTERNAL FCN, G
```

```
call RNSET(12345)
```

```
open (unit=9, file='c:\ave FPM.out')
```

C Set values of initial guesses to find Fleishman constants and
intermediate correlations

```
time = CPSEC()
```

```
DATA CGUESS/3*0.50d+00/
```

```
DATA RGUESS/2*0.10d+00/
```

```
EPS = 0.000001d+00
```

```
ERRABS = 0.000001d+00
```

```
ERRREL = 0.0000001d+00
```

```
ETA = 0.0000001d+00
```

```
ITMAX = 10000
```

```
call UMACH (2, NOUT)
```

C Find the Fleishman constants (a, b, c, d)

```
skew = skew1
skurt = skurt1
CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
a1=-C(2)
b1=C(1)
c1=C(2)
d1=C(3)
```

```
write (*,*) 'a1 =', a1, ' b1 =', b1, ' c1 =', c1,
& ' d1 =', d1
```

```
skew = skew2
skurt = skurt2
CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
a2=-C(2)
b2=C(1)
c2=C(2)
d2=C(3)
```

C Find intermediate correlation (R)

```
CALL DZREAL (G, ERRABS, ERRREL, EPS, ETA, NROOT, ITMAX,
RGUESS,
& R, INFO)
```

c write (*,*) 'R =', R(1)

```
sumrho=0.0d+00
sum1=0.0d+00
sum2=0.0d+00
sumvar1=0.0d+00
sumvar2=0.0d+00
sumsk1=0.0d+00
sumsk2=0.0d+00
sumku1=0.0d+00
sumku2=0.0d+00
```

```
sumdmY1=0.0d+00
sumdmY2=0.0d+00
sumdvY1=0.0d+00
```

```

sumdvY2=0.0d+00
sumdsY1=0.0d+00
sumdsY2=0.0d+00
sumdkY1=0.0d+00
sumdkY2=0.0d+00
sumdr=0.0d+00

do 1 i=1,k

c   Generate random variables using FPM
c   Generate X1 and X2 with correlations of R-squared
c   Generate Y1 and Y2 by substituting into the Fleishman equations to generate
nonnormal distributions
c   with the desired post-correlation and the specified skew and kurtosis

do 10 j=1,M
Z(j)=drnnoF()
Eone(j)=drnnoF()
E2(j)=drnnoF()
X1(j)=R(1)*Z(j)+dsqrt(1.0d+00-R(1)**2.0d+00)*Eone(j)
X2(j)=R(1)*Z(j)+dsqrt(1.0d+00-R(1)**2.0d+00)*E2(j)
Y1(j)=a1+b1*X1(j)+c1*X1(j)**2.0d+00+d1*X1(j)**3.0d+00
Y2(j)=a2+b2*X2(j)+c2*X2(j)**2.0d+00+d2*X2(j)**3.0d+00
10 continue
c   write (9,*) 'Y1= ', Y1, ' Y2= ', Y2, ' Z= ', Z, ' E1 = ', Eone,
c   & ' E2= ', E2

call outsum(Y1, Y2, M, sumY1, sumY2,
& aveY1, aveY2, sumprodY1Y2, aveprodY1Y2,
& sumsqY1, sumsqY2, SY1, SY2, corrY, sumcubeY1, sumcubeY2,
& sumquY1, sumquY2, skewY1, skewY2, skurtY1, skurtY2)

c   Find absolute differences between specified and generated skews, kurtoses, and
rhos

sumrho=corrY+sumrho
sum1=aveY1+sum1
sum2=aveY2+sum2
sumvar1=sy1**2.0d+00+sumvar1
sumvar2=sy2**2.0d+00+sumvar2
sumsk1=skewY1+sumsk1
sumsk2=skewY2+sumsk2
sumku1=skurty1+sumku1
sumku2=skurty2+sumku2

```



```

dmY1=dabs(aveY1)
dmY2=dabs(aveY2)
dvarY1=dabs(sy1**2.0d+00-1.0d+00)
dvarY2=dabs(sy2**2.0d+00-1.0d+00)
dskY1=dabs(skewY1-skew1)
dskY2=dabs(skewY2-skew2)
dkuY1=dabs(skurtY1-skurt1)
dkuY2=dabs(skurtY2-skurt2)
drho=dabs(corrY-rho)

```

```

sumdmY1=sumdmY1+dmY1
sumdmY2=sumdmY2+dmY2
sumdvY1=sumdvY1+dvarY1
sumdvY2=sumdvY2+dvarY2
sumdsY1=sumdsY1+dskY1
sumdsY2=sumdsY2+dskY2
sumdkY1=sumdkY1+dkuY1
sumdkY2=sumdkY2+dkuY2
sumdr=sumdr+drho

```

1 continue

```

averho=sumrho/float(k)
avemean1=sum1/float(k)
avemean2=sum2/float(k)
avevar1=sumvar1/float(k)
avevar2=sumvar2/float(k)
avesk1=sumsk1/float(k)
avesk2=sumsk2/float(k)
aveku1=sumku1/float(k)
aveku2=sumku2/float(k)

```

```

avedmY1=sumdmY1/float(k)
avedmY2=sumdmY2/float(k)
avedvY1=sumdvY1/float(k)
avedvY2=sumdvY2/float(k)
avedsY1=sumdsY1/float(K)
avedsY2=sumdsY2/float(K)
avedkY1=sumdkY1/float(K)
avedkY2=sumdkY2/float(K)
avedr=sumdr/float(K)

```

```

write(9,*) averho
write(9,*) ''
write(9,*) avemean1
write(9,*) avevar1

```

```

write(9,*) avesk1
write(9,*) aveku1
write(9,*) ''
write(9,*) avemean2
write(9,*) avevar2
write(9,*) avesk2
write(9,*) aveku2
write(9,*) ''
write(9,*) ''
write(9,*) avedr
write(9,*) ''
write(9,*) avedmY1
write(9,*) avedvY1
write(9,*) avedsY1
write(9,*) avedkY1
write(9,*) ''
write(9,*) avedmY2
write(9,*) avedvY2
write(9,*) avedsY2
write(9,*) avedkY2
print*, k
END

```

C ***** Subroutine *****

C User-defined subroutine (find a, b, c, d)

SUBROUTINE FCN (C, F, N)

implicit REAL*8 (a-h, o-z)

REAL*8 F(N), C(N)

common skew, skurt,a1, b1, c1, d1, a2, b2, c2, d2, rho

$F(1) = C(1)**2.0D+00+6.0D+00*C(1)*C(3)+2.0D+00*C(2)**2.0D+00$
 $\&+15.0d+00*C(3)**2.0D+00-1.0D+00$

$F(2) = 2.0d+00* C(2)*(C(1)**2.0D+00+24.0D+00*C(1)*C(3)+105.0D+00$
 $\&*C(3)**2.0D+00+2.0D+00)-skew$

$F(3) = 24.0D+00*(C(1)*C(3)+C(2)**2.0D+00*(1.0D+00+C(1)**2.0D+00$
 $\&+28.0D+00*C(1)*C(3))+C(3)**2.0D+00*(12.0D+00+48.0D+00*C(1)*C(3)$
 $\&+141.0D+00*C(2)**2.0D+00+225.0D+00*C(3)**2.0D+00))-skurt$

RETURN

END


```

scux2=0.0d+00
sumqux1=0.0d+00
sumqux2=0.0d+00

do 20 i=1,L
sumx1=sumx1+x1(i)
sumx2=sumx2+x2(i)
spx1x2 = spx1x2 + x1(i)*x2(i)
20 continue

avex1 = sumx1/float(L)
avex2 = sumx2/float(L)
apx1x2 = spx1x2/float(L)

do 30 i30=1,L
ssqx1 = ssqx1 + (x1(i30)-avex1)**2.0d+00
ssqx2 = ssqx2 + (x2(i30)-avex2)**2.0d+00
scux1 = scux1 + (x1(i30)-avex1)**3.0d+00
scux2 = scux2 + (x2(i30)-avex2)**3.0d+00
sumqux1 = sumqux1 + (x1(i30)-avex1)**4.0d+00
sumqux2 = sumqux2 + (x2(i30)-avex2)**4.0d+00
30 continue

Sx1 = dsqrt(ssqx1/(float(L)))
Sx2 = dsqrt(ssqx2/(float(L)))
corr = (apx1x2-avex1*avex2)/(Sx1*Sx2)
skewx1=scux1/(float(L)*Sx1**3.0d+00)
skewx2=scux2/(float(L)*Sx2**3.0d+00)
skurtx1=(sumqux1/(float(L)*Sx1**4.0d+00))-3.0d+00
skurtx2=(sumqux2/(float(L)*Sx2**4.0d+00))-3.0d+00

RETURN
END

```

Appendix H

c Program to calculate average difference
c between generated and desired parameters using
c the Fifth-Order Polynomial Transformation Method

use numerical_libraries

C Declare variables

```
implicit REAL*8 (a-h, o-z)
```

```
PARAMETER (K=10000, M=1000, N=5, NROOT=3)
```

```
data skew1/0.0d+00/  
data skurt1/2.0d+00/  
data fifth1/0.0d+00/  
data sixth1/80.0d+00/
```

```
data skew2/0.0d+00/  
data skurt2/2.0d+00/  
data fifth2/0.0d+00/  
data sixth2/80.0d+00/
```

```
data rho/0.1d+00/
```

```
REAL*8 FNORM, C(N), CGUESS(N), G, R(NROOT), RGUESS(NROOT), Z(M),  
& Eone(M), E2(M), Y1(M), Y2(M), X1(M), X2(M),  
& pY1Y2(M)
```

```
common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,  
*c1b,c2b,c3b,c4b,c5b,rho
```

```
EXTERNAL FCN, G
```

```
call RNSET(12345)
```

```
open (unit=9,file='c:\avefifth.out')
```

C Set values of initial guesses to find Fleishman constants and
intermediate correlations

```
DATA CGUESS/5*0.01d+00/
```

```
DATA RGUESS/3*0.500d+00/
```

```

EPS = 0.000001d+00
ERRABS = 0.000001d+00
ERRREL = 0.0000001d+00
ETA = 0.0000001d+00
ITMAX = 100000
call UMACH (2, NOUT)

```

C Find the values of the constants (c(0), c(1), c(2), c(3), c(4), c(5))

```

skew = skew1
skurt = skurt1
fifth = fifth1
sixth = sixth1

```

```

CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
c0a= -c(2)-3.0d+00*c(4)
c1a=c(1)
c2a=c(2)
c3a=c(3)
c4a=c(4)
c5a=c(5)

```

```

skew = skew2
skurt = skurt2
fifth = fifth2
sixth = sixth2

```

```

CALL DNEQNF (FCN, ERRREL, N, ITMAX, CGUESS, C, FNORM)
c0b= -c(2)-3.0d+00*c(4)
c1b=c(1)
c2b=c(2)
c3b=c(3)
c4b=c(4)
c5b=c(5)

```

C Find intermediate correlation (R)

```

CALL DZREAL (G, ERRABS, ERRREL, EPS, ETA, NROOT, ITMAX,
RGUESS,
& R, INFO)

sumrho=0.0d+00
suml=0.0d+00

```

```

sum2=0.0d+00
sumvar1=0.0d+00
sumvar2=0.0d+00
sumsk1=0.0d+00
sumsk2=0.0d+00
sumku1=0.0d+00
sumku2=0.0d+00

```

```

sumdmY1=0.0d+00
sumdmY2=0.0d+00
sumdvY1=0.0d+00
sumdvY2=0.0d+00
sumdsY1=0.0d+00
sumdsY2=0.0d+00
sumdkY1=0.0d+00
sumdkY2=0.0d+00
sumdr=0.0d+00

```

```
do 1 i=1,k
```

- c Generate random variables
- c Generate X1 and X2 with correlations of R-squared
- c Generate Y1 and Y2 by substituting into the Fleishman equations to generate nonnormal distributions
- c with the desired post-correlation and the specified skew and kurtosis

```

do 10 j=1,M
Z(j)=drnnof()
Eone(j)=drnnof()
E2(j)=drnnof()
X1(j)=dsqrt(R(1))*Z(j)+dsqrt(1.0d+00-R(1))*Eone(j)
X2(j)=dsqrt(R(1))*Z(j)+dsqrt(1.0d+00-R(1))*E2(j)
Y1(j)=c0a+c1a*X1(j)+c2a*X1(j)**2.0d+00+c3a*X1(j)**3.0d+00+c4a*
*X1(j)**4.0d+00+c5a*X1(j)**5.0d+00
Y2(j)=c0b+c1b*X2(j)+c2b*X2(j)**2.0d+00+c3b*X2(j)**3.0d+00+c4b*
*X2(j)**4.0d+00+c5b*X2(j)**5.0d+00

```

```
10     continue
```

```

call outsum(Y1, Y2, M, sumY1, sumY2,
& aveY1, aveY2, spY1Y2, apY1Y2,
& sumsqY1, sumsqY2, SY1, SY2, corrY, scY1, scY2,
& sumquY1, sumquY2, skewY1, skewY2, skurtY1, skurtY2)

```

```

sumrho=corrY+sumrho
sum1=avey1+sum1
sum2=avey2+sum2
sumvar1=sy1**2.0d+00+sumvar1
sumvar2=sy2**2.0d+00+sumvar2
sumsk1=skewy1+sumsk1
sumsk2=skewy2+sumsk2
sumku1=skurty1+sumku1
sumku2=skurty2+sumku2

```

```

dmY1=dabs(aveY1)
dmY2=dabs(aveY2)
dvarY1=dabs(sy1**2.0d+00-1.0d+00)
dvarY2=dabs(sy2**2.0d+00-1.0d+00)
dskY1=dabs(skewY1-skew1)
dskY2=dabs(skewY2-skew2)
dkuY1=dabs(skurtY1-skurt1)
dkuY2=dabs(skurtY2-skurt2)
drho=dabs(corrY-rho)

```

```

sumdmY1=sumdmY1+dmY1
sumdmY2=sumdmY2+dmY2
sumdvY1=sumdvY1+dvarY1
sumdvY2=sumdvY2+dvarY2
sumdsY1=sumdsY1+dskY1
sumdsY2=sumdsY2+dskY2
sumdkY1=sumdkY1+dkuY1
sumdkY2=sumdkY2+dkuY2
sumdr=sumdr+drho

```

1 continue

```

averho=sumrho/float(k)
avemean1=sum1/float(k)
avemean2=sum2/float(k)
avevar1=sumvar1/float(k)
avevar2=sumvar2/float(k)
avesk1=sumsk1/float(k)
avesk2=sumsk2/float(k)
aveku1=sumku1/float(k)
aveku2=sumku2/float(k)

```

```

avedmY1=sumdmY1/float(k)
avedmY2=sumdmY2/float(k)
avedvY1=sumdvY1/float(k)
avedvY2=sumdvY2/float(k)

```



```

avedsY1=sumdsY1/float(K)
avedsY2=sumdsY2/float(K)
avedkY1=sumdkY1/float(K)
avedkY2=sumdkY2/float(K)
avedr=sumdr/float(K)

```

```

write(9,*) averho
write(9,*) ''
write(9,*) avemean1
write(9,*) avevar1
write(9,*) aveskl
write(9,*) aveku1
write(9,*) ''
write(9,*) avemean2
write(9,*) avevar2
write(9,*) avesk2
write(9,*) aveku2
write(9,*) ''
write(9,*) ''
write(9,*) avedr
write(9,*) ''
write(9,*) avedmY1
write(9,*) avedvY1
write(9,*) avedsY1
write(9,*) avedkY1
write(9,*) ''
write(9,*) avedmY2
write(9,*) avedvY2
write(9,*) avedsY2
write(9,*) avedkY2
END

```

C ***** Subroutine

C User-defined subroutine (find c(1) - c(5))
SUBROUTINE FCN (C, F, N)
implicit REAL*8 (a-h, o-z)
REAL*8 F(N), C(N)
common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,
*c1b,c2b,c3b,c4b,c5b,rho

$$F(1) = (c(1)**2.0d+00+2.0d+00*c(2)**2.0d+00+24.0d+00*c(2)*c(4)+$$

$$*6.0d+00*c(1)*(c(3)+5.0d+00*c(5))+ 3.0d+00*(5.0d+00*c(3)**2.0d+00$$

$$*+32.0d+00*c(4)**2.0d+00+70.0d+00*c(3)*c(5)+315.0d+00*$$

$$*c(5)**2.0d+00))-1.0d+00$$

$$F(2) = (2.0d+00*(4.0d+00*c(2)**3.0d+00+108.0d+00*c(2)**2.0d+00*
*c(4)+3.0d+00*c(1)**2.0d+00*(c(2)+6.0d+00*c(4))+18.0d+00*c(1)*
*(2.0d+00*c(2)*c(3)+16.0d+00*c(3)*c(4)+15.0d+00*c(2)*c(5)+
*150.0d+00*c(4)*c(5))+9.0d+00*c(2)*(15.0d+00*c(3)**2.0d+00+
*128.0d+00*c(4)**2.0d+00+280.0d+00*c(3)*c(5)+1575.0d+00*c(5)**
*2.0d+00)+54.0d+00*c(4)*(25.0d+00*c(3)**2.0d+00+88.0d+00*c(4)**
*2.0d+00+560.0d+00*c(3)*c(5)+3675.0d+00*c(5)**2.0d+00)))-skew$$

$$F(3) = (24.0d+00*(2.0d+00*c(2)**4.0d+00+96.0d+00*c(2)**3.0d+00*
*c(4)+c(1)**3.0d+00*(c(3)+10.0d+00*c(5))+30.0d+00*c(2)**2.0d+00*
*(6.0d+00*c(3)**2.0d+00+64.0d+00*c(4)**2.0d+00+140.0d+00*c(3)*c(5))+
*945.0d+00*c(5)**2.0d+00)+c(1)**2.0d+00*(2.0d+00*c(2)**2.0d+00+
*18.0d+00*c(3)**2.0d+00+36.0d+00*c(2)*c(4)+192.0d+00*c(4)**
*2.0d+00+375.0d+00*c(3)*c(5)+2250.0d+00*c(5)**2.0d+00)+36.0d+00*
*c(2)*c(4)*(125.0d+00*c(3)**2.0d+00+528.0d+00*c(4)**2.0d+00+
*3360.0d+00*c(3)*c(5)+25725.0d+00*c(5)**2.0d+00)+3.0d+00*c(1)*
*(45.0d+00*c(3)**3.0d+00+1584.0d+00*c(3)*c(4)**2.0d+00+1590.0d+00*
*c(3)**2.0d+00*c(5)+21360.0d+00*c(4)**2.0d+00*c(5)+21525.0d+00*
*c(3)*c(5)**2.0d+00+110250.0d+00*c(5)**3.0d+00+12.0d+00*c(2)**
2.0d+00(c(3)+10.0d+00*c(5))+8.0d+00*c(2)*c(4)*(32.0d+00*c(3)+
*375.0d+00*c(5))+9.0d+00*(45.0d+00*c(3)**4.0d+00+8704.0d+00*c(4)**
*4.0d+00+2415.0d+00*c(3)**3.0d+00*c(5)+932400.0d+00*c(4)**2.0d+00*
*c(5)**2.0d+00+3018750.0d+00*c(5)**4.0d+00+20.0d+00*c(3)**2.0d+00*
*(178.0d+00*c(4)**2.0d+00+2765.0d+00*c(5)**2.0d+00)+35.0d+00*c(3)*
*(3104.0d+00*c(4)**2.0d+00*c(5)+18075.0d+00*c(5)**3.0d+00)))-skurt$$

$$F(4) = (24.0d+00*(16.0d+00*c(2)**5.0d+00+5.0d+00*c(1)**4.0d+00*
*c(4)+1200.0d+00*c(2)**4.0d+00*c(4)+10.0d+00*c(1)**3.0d+00*
(3.0d+00
*c(2)*c(3)+42.0d+00*c(3)*c(4)+40.0d+00*c(2)*c(5)+570.0d+00*c(4)*
*c(5))+300.0d+00*c(2)**3.0d+00*(10.0d+00*c(3)**2.0d+00+128.0d+00*
*c(4)**2.0d+00+280.0d+00*c(3)*c(5)+2205.0d+00*c(5)**2.0d+00)+
*1080.0d+00*c(2)**2.0d+00*c(4)*(125.0d+00*c(3)**2.0d+00+3920.0d+00*
*c(3)*c(5)+28.0d+00*(22.0d+00*c(4)**2.0d+00+1225.0d+00*
*c(5)**2.0d+00))+10.0d+00*c(1)**2.0d+00*(2.0d+00*c(2)**
*3.0d+00+72.0d+00*c(2)**2.0d+00*c(4)+3.0d+00*c(2)*(24.0d+00*c(3)**
*2.0d+00+320.0d+00*c(4)**2.0d+00+625.0d+00*c(3)*c(5)+4500.0d+00*
*c(5)**2.0d+00)+9.0d+00*c(4)*(109.0d+00*c(3)**2.0d+00+528.0d+00*
*c(4)**2.0d+00+3130.0d+00*c(3)*c(5)+24975.0d+00*c(5)**2.0d+00))+
*30.0d+00*c(1)*(8.0d+00*c(2)**3.0d+00*(2.0d+00*c(3)+25.0d+00*c(5))+
*40.0d+00*c(2)**2.0d+00*c(4)*(16.0d+00*c(3)+225.0d+00*c(5))+
*3.0d+00*c(2)*(75.0d+00*c(3)**3.0d+00+3168.0d+00*c(3)*c(4)**$$

$*2.0d+00+3180.0d+00*c(3)**2.0d+00*c(5)+49840.0d+00*c(4)**2.0d+00*$
 $*c(5)+$
 $*50225.0d+00*c(3)*c(5)**2.0d+00+294000.0d+00*c(5)**3.0d+00)+$
 $*6.0d+00*c(4)*(555.0d+00*c(3)**3.0d+00+8704.0d+00*c(3)*c(4)**$
 $*2.0d+00+26225.0d+00*c(3)**2.0d+00*c(5)+152160.0d+00*c(4)**2.0d+00*$
 $*c(5)+459375.0d+00*c(3)*c(5)**2.0d+00+2963625.0d+00*c(5)**$
 $*3.0d+00))+90.0d+00*c(2)*(270.0d+00*c(3)**4.0d+00+16905.0d+00*$
 $*c(3)**3.0d+00*c(5)+280.0d+00*c(3)**2.0d+00*(89.0d+00*c(4)**$
 $*2.0d+00+1580.0d+00*c(5)**2.0d+00)+35.0d+00*c(3)*(24832.0d+00*$
 $*c(4)**2.0d+00*c(5)+162675.0d+00*c(5)**3.0d+00)+4.0d+00*$
 $*(17408.0d+00*c(4)**4.0d+00+2097900.0d+00*c(4)**2.0d+00*c(5)**$
 $*2.0d+00+7546875.0d+00*c(5)**4.0d+00))+27.0d+00*c(4)*(14775.0d+00*$
 $*c(3)**4.0d+00+1028300.0d+00*c(3)**3.0d+00*c(5)+50.0d+00*c(3)**$
 $*2.0d+00*(10144.0d+00*c(4)**2.0d+00+594055.0d+00*c(5)**2.0d+00)+$
 $*700.0d+00*c(3)*(27904.0d+00*c(4)**2.0d+00*c(5)+598575.0d+00*c(5)**$
 $*3.0d+00)+3.0d+00*(316928.0d+00*c(4)**4.0d+00+68908000.0d+00*c(4)**$
 $*2.0d+00*c(5)**2.0d+00+806378125.0d+00*c(5)**4.0d+00))))- fifth$

$F(5) = (120.0d+00*(32.0d+00*c(2)**6.0d+00+3456.0d+00*c(2)**$
 $*5.0d+00*c(4)+6.0d+00*c(1)**5.0d+00*c(5)+3.0d+00*c(1)**4.0d+00*$
 $*(9.0d+00*c(3)**2.0d+00+16.0d+00*c(2)*c(4)+168.0d+00*c(4)**2.0d+00+$
 $*330.0d+00*c(3)*c(5)+2850.0d+00*c(5)**2.0d+00)+720.0d+00*c(2)**$
 $*4.0d+00*(15.0d+00*c(3)**2.0d+00+224.0d+00*c(4)**2.0d+00+490.0d+00*$
 $*c(3)*c(5)+4410.0d+00*c(5)**2.0d+00)+6048.0d+00*c(2)**3.0d+00*c(4)*$
 $*(125.0d+00*c(3)**2.0d+00+704.0d+00*c(4)**2.0d+00+4480.0d+00*c(3)*$
 $*c(5)+44100.0d+00*c(5)**2.0d+00)+12.0d+00*c(1)**3.0d+00*(4.0d+00*$
 $*c(2)**2.0d+00*(3.0d+00*c(3)+50.0d+00*c(5))+60.0d+00*c(2)*c(4)*$
 $*(7.0d+00*c(3)+114.0d+00*c(5))+3.0d+00*(24.0d+00*c(3)**3.0d+00+$
 $*1192.0d+00*c(3)*c(4)**2.0d+00+1170.0d+00*c(3)**2.0d+00*c(5)+$
 $*20440.0d+00*c(4)**2.0d+00*c(5)+20150.0d+00*c(3)*c(5)**2.0d+00+$
 $*124875.0d+00*c(5)**3.0d+00))+216.0d+00*c(2)**2.0d+00*(945.0d+00*$
 $*c(3)**4.0d+00+67620.0d+00*c(3)**3.0d+00*c(5)+560.0d+00*c(3)**$
 $*2.0d+00*(178.0d+00*c(4)**2.0d+00+3555.0d+00*c(5)**2.0d+00)+$
 $*315.0d+00*c(3)*(12416.0d+00*c(4)**2.0d+00*c(5)+90375.0d+00*c(5)**$
 $*3.0d+00)+6.0d+00*(52224.0d+00*c(4)**4.0d+00+6993000.0d+00*c(4)**$
 $*2.0d+00*c(5)**2.0d+00+27671875.0d+00*c(5)**4.0d+00))+6.0d+00*$
 $*c(1)**2.0d+00*(8.0d+00*c(2)**4.0d+00+480.0d+00*c(2)**3.0d+00*c(4)+$
 $*180.0d+00*c(2)**2.0d+00*(4.0d+00*c(3)**2.0d+00+64.0d+00*c(4)**$
 $*2.0d+00+125.0d+00*c(3)*c(5)+1050.0d+00*c(5)**2.0d+00)+72.0d+00*$
 $*c(2)*c(4)*(327.0d+00*c(3)**2.0d+00+1848.0d+00*c(4)**2.0d+00+$
 $*10955.0d+00*c(3)*c(5)+99900.0d+00*c(5)**2.0d+00)+9.0d+00*$
 $*(225.0d+00*c(3)**4.0d+00+22824.0d+00*c(3)**2.0d+00*c(4)**2.0d+00+$
 $*69632.0d+00*c(4)**4.0d+00+15090.0d+00*c(3)**3.0d+00*c(5)+$
 $*830240.0d+00*c(3)*c(4)**2.0d+00*c(5)+412925.0d+00*c(3)**2.0d+00*$
 $*c(5)**2.0d+00+$
 $*8239800.0d+00*c(4)**2.0d+00*c(5)**2.0d+00+5475750.0d+00*c(3)*$

```

*c(5)**3.0d+00+29636250.0d+00*c(5)**4.0d+00))+1296.0d+00*c(2)*c(4)*
*(5910.0d+00*c(3)**4.0d+00+462735.0d+00*c(3)**3.0d+00*c(5)+c(3)**
*2.0d+00*(228240.0d+00*c(4)**2.0d+00+14851375.0d+00*c(5)**2.0d+00)+
*175.0d+00*c(3)*(55808.0d+00*c(4)**2.0d+00*c(5)+1316865.0d+00*
*c(5)**3.0d+00)+3.0d+00*(158464.0d+00*c(4)**4.0d+00+37899400.0d+00*
*c(4)**2.0d+00*c(5)**2.0d+00+483826875.0d+00*c(5)**4.0d+00))+
*27.0d+00*(9945.0d+00*c(3)**6.0d+00+92930048.0d+00*c(4)**6.0d+00+
*1166130.0d+00*c(3)**5.0d+00*c(5)+35724729600.0d+00*c(4)**4.0d+00*
*c(5)**2.0d+00+977816385000.0d+00*c(4)**2.0d+00*c(5)**4.0d+00+
*1907724656250.0d+00*c(5)**6.0d+00+180.0d+00*c(3)**4.0d+00*
*(16082.0d+00*c(4)**2.0d+00+345905.0d+00*c(5)**2.0d+00)+140.0d+00*
*c(3)**3.0d+00*(1765608.0d+00*c(4)**2.0d+00*c(5)+13775375.0d+00*
*c(5)**3.0d+00)+15.0d+00*c(3)**2.0d+00*(4076032.0d+00*c(4)**
*4.0d+00+574146160.0d+00*c(4)**2.0d+00*c(5)**2.0d+00+
*2424667875.0d+00*c(5)**4.0d+00)+210.0d+00*c(3)*(13526272.0d+00*
*c(4)**4.0d+00*c(5)+687499200.0d+00*c(4)**2.0d+00*c(5)**3.0d+00+
*1876468125.0d+00*c(5)**5.0d+00))+18.0d+00*c(1)*(80.0d+00*c(2)**
*4.0d+00*(c(3)+15.0d+00*c(5))+160.0d+00*c(2)**3.0d+00*c(4)**
*(32.0d+00*c(3)+525.0d+00*c(5))+12.0d+00*c(2)**2.0d+00*
*(225.0d+00*c(3)**3.0d+00+11088.0d+00*c(3)*c(4)**2.0d+00+
*11130.0d+00*c(3)**2.0d+00*c(5)+199360.0d+00*c(4)**2.0d+00*c(5)+
*200900.0d+00*c(3)*c(5)**2.0d+00+1323000.0d+00*c(5)**3.0d+00)+
*24.0d+00*c(2)*c(4)*(3885.0d+00*c(3)**3.0d+00+69632.0d+00*c(3)*
*c(4)**2.0d+00+209800.0d+00*c(3)**2.0d+00*c(5)+1369440.0d+00*c(4)**
*2.0d+00*c(5)+4134375.0d+00*c(3)*c(5)**2.0d+00+29636250.0d+00*
*c(5)**3.0d+00)+9.0d+00*(540.0d+00*c(3)**5.0d+00+48585.0d+00*
*c(3)**4.0d+00*c(5)+20.0d+00*c(3)**3.0d+00*(4856.0d+00*c(4)**
*2.0d+00+95655.0d+00*c(5)**2.0d+00)+80.0d+00*c(3)**2.0d+00*
*(71597.0d+00*c(4)**2.0d+00*c(5)+513625.0d+00*c(5)**3.0d+00)+
*4.0d+00*c(3)*(237696.0d+00*c(4)**4.0d+00+30726500.0d+00*c(4)**
*2.0d+00*c(5)**2.0d+00+119844375.0d+00*c(5)**4.0d+00)+5.0d+00*c(5)*
*(4076032.0d+00*c(4)**4.0d+00+191074800.0d+00*c(4)**2.0d+00*
*c(5)**2.0d+00+483826875.0d+00*c(5)**4.0d+00))))-sixth

```

```

c      write (*,*) c

```

```

RETURN
END

```

```

c      Function to determine R

```

```

double precision FUNCTION G (R)
implicit REAL*8 (a-h, o-z)

```

```

      REAL*8    R
      common skew,skurt,fifth,sixth,c0a,c1a,c2a,c3a,c4a,c5a,c0b,
      *c1b,c2b,c3b,c4b,c5b,rho

```

```

      G = (3.0d+00*c0b*c4a+3.0d+00*c2b*c4a+9.0d+00*c4a*c4b+
      *c0a*(c0b+c2b+3.0d+00*c4b)+c1a* c1b*R+
      *3.0d+00*c1b*c3a*R+3.0d+00*c1a*c3b*R+9.0d+00*c3a*c3b*R+
      *15.0d+00*c1b*c5a*R+45.0d+00*c3b*c5a*R+
      * 15.0d+00*c1a*c5b*R+
      *45.0d+00*c3a*c5b*R+225.0d+00*c5a*c5b*R+
      * 12.0d+00*c2b*c4a*R**2.0d+00+
      *72.0d+00*c4a*c4b*R**2.0d+00+6.0d+00*c3a*c3b*R**3.0d+00+
      *60.0d+00*c3b*c5a*R**3.0d+00+60.0d+00*c3a*c5b*R**3.0d+00+
      *600.0d+00*c5a*c5b*R**3.0d+00+24.0d+00*c4a*c4b*R**4.0d+00+
      *120.0d+00*c5a*c5b*R**5.0d+00+
      *c2a*(c0b+c2b+3.0d+00*c4b+2.0d+00*c2b*R**2.0d+00+
      *12.0d+00*c4b*R**2.0d+00))-rho

```

```

      RETURN
      END

```

```

C      ***** Subroutine *****

```

```

C      Used to find E[Y1], E[Y2], E[Y1Y2], standard deviations for Y1 and Y2,
      correlation between Y1 and Y2, skew and kurtosis

```

```

      subroutine outsum(x1, x2, L, sumx1, sumx2,
      & avex1, avex2, spx1x2, apx1x2,
      & ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,
      & sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2)

```

```

      implicit REAL*8 (a-h, o-z)
      integer L
      REAL*8  x1(L), x2(L), sumx1, sumx2,
      & avex1, avex2, spx1x2, apx1x2,
      & ssqx1, ssqx2, Sx1, Sx2, corrx, scux1, scux2,
      & sumqux1, sumqux2, skewx1, skewx2, skurtx1, skurtx2

```

```

      sumx1=0.0d+00
      sumx2=0.0d+00
      spx1x2 = 0.0d+00

```

```
ssqx1=0.0d+00
ssqx2=0.0d+00
scux1=0.0d+00
scux2=0.0d+00
sumqux1=0.0d+00
sumqux2=0.0d+00
```

```
do 20 i=1,L
sumx1=sumx1+x1(i)
sumx2=sumx2+x2(i)
spx1x2 = spx1x2 + x1(i)*x2(i)
20 continue
```

```
avex1 = sumx1/float(L)
avex2 = sumx2/float(L)
apx1x2 = spx1x2/float(L)
```

```
do 30 i30=1,L
ssqx1 = ssqx1 + (x1(i30)-avex1)**2.0d+00
ssqx2 = ssqx2 + (x2(i30)-avex2)**2.0d+00
scux1 = scux1 + (x1(i30)-avex1)**3.0d+00
scux2 = scux2 + (x2(i30)-avex2)**3.0d+00
sumqux1 = sumqux1 + (x1(i30)-avex1)**4.0d+00
sumqux2 = sumqux2 + (x2(i30)-avex2)**4.0d+00
30 continue
```

```
Sx1 = dsqrt(ssqx1/(float(L)))
Sx2 = dsqrt(ssqx2/(float(L)))
corr = (apx1x2-avex1*avex2)/(Sx1*Sx2)
skewx1=scux1/(float(L)*Sx1**3.0d+00)
skewx2=scux2/(float(L)*Sx2**3.0d+00)
skurtx1=(sumqux1/(float(L)*Sx1**4.0d+00))-3.0d+00
skurtx2=(sumqux2/(float(L)*Sx2**4.0d+00))-3.0d+00
```

```
RETURN
END
```

Appendix I

n=1, 000, 000		Correlation = 0.1			Correlation = 0.5			Correlation = 0.9			
Method:		GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	GLD	FPM	Fifth-Order	
Gaussian	0.0000	ρ	0.2007**	0.0998	0.0998	0.6009**	0.4997	0.4992	1.000**	0.8997	0.8997
	1.0000	μ	-0.0006	0.0000	0.0000	0.0007	0.0018	-0.0001	-0.0007	0.0020	-0.0001
	0.0000	σ^2	0.9971	1.0001	1.0001	1.0028	1.0012	0.9987	1.0006	1.0003	0.9975
	0.0000	γ_1	0.0001	-0.0017	-0.0017	-0.0013	0.0013	0.0004	0.0038	0.0030	0.0027
Gaussian	0.0000	γ_2	0.0011	0.0034	0.0034	0.0003	0.0028	0.0068	-0.0021	0.0093	0.0074
	0.0000	μ	0.0004	-0.0003	-0.0003	0.0012	0.0042	-0.0003	-0.0007	0.0031	-0.0002
	1.0000	σ^2	0.9994	1.0005	1.0005	0.9999	1.0061	0.9993	1.0006	1.0027	0.9978
	0.0000	γ_1	0.0013	-0.0032	-0.0032	-0.0024	0.0162	-0.0009	0.0038	0.0119	0.0016
Gaussian	0.0000	γ_2	-0.0023	0.0003	0.0003	0.0011	0.0014	-0.0005	-0.0021	0.0060	0.0008
	0.0000	ρ	0.1004*	0.0990	0.0990	0.4999	0.4996	0.4995	0.8993	0.8998	0.9000
	1.0000	μ	0.0008	0.0006	0.0006	0.0008	0.0018	0.0012	0.0008	0.0020	0.0014
	0.0000	σ^2	1.0031	1.0020	1.0020	1.0031	1.0012	1.0017	1.0031	1.0003	1.0007
Logistic	0.0000	γ_1	-0.0037	0.0023	0.0023	-0.0037	0.0013	0.0025	-0.0037	0.0030	0.0079
	0.0000	γ_2	0.0000	-0.0043	-0.0043	0.0000	0.0028	0.0082	0.0000	0.0093	0.0104
	0.0000	μ	-0.0002	0.0003	0.0003	0.0001	0.0043	0.0010	0.0005	0.0031	0.0015
	1.0000	σ^2	1.0002	0.9994	0.9994	1.0018	1.0075	0.9994	1.0038	1.0038	0.9999
Logistic	0.0000	γ_1	-0.0061	0.0026	0.0022	-0.0070	0.0280	0.0086	-0.0045	0.0215	0.0169
	1.2000	γ_2	1.5987	1.1770	1.1799	1.6382	1.1773	1.1740	1.6231	1.2078	1.1911
Gaussian	0.0000	ρ	0.0995	0.0995	0.0996	0.4998	0.4991	0.4991	0.8995	0.8995	0.9000
	1.0000	μ	0.0008	unable to calculate constants for uniform distribution	0.0003	0.0008	unable to calculate constants for uniform distribution	0.0001	0.0008	unable to calculate constants for uniform distribution	-0.0002
	0.0000	σ^2	1.0031	unable to calculate constants for uniform distribution	1.0011	1.0031	unable to calculate constants for uniform distribution	1.0000	1.0031	unable to calculate constants for uniform distribution	0.9991
	0.0000	γ_1	-0.0037	unable to calculate constants for uniform distribution	0.0033	-0.0037	unable to calculate constants for uniform distribution	0.0008	-0.0037	unable to calculate constants for uniform distribution	-0.0067
Uniform	0.0000	γ_2	0.0000	unable to calculate constants for uniform distribution	-0.0030	0.0000	unable to calculate constants for uniform distribution	0.0026	0.0000	unable to calculate constants for uniform distribution	0.0029
	0.0000	μ	0.0000	unable to calculate constants for uniform distribution	-0.0003	0.0005	unable to calculate constants for uniform distribution	0.0001	0.0010	unable to calculate constants for uniform distribution	0.0002
	1.0000	σ^2	1.0006	unable to calculate constants for uniform distribution	0.9998	1.0008	unable to calculate constants for uniform distribution	0.9988	1.0015	unable to calculate constants for uniform distribution	0.9987
	0.0000	γ_1	-0.0002	unable to calculate constants for uniform distribution	0.0010	-0.0013	unable to calculate constants for uniform distribution	0.0000	-0.0023	unable to calculate constants for uniform distribution	-0.0042

	-1.2000	γ_2	-1.2010		-1.1994	-1.2009		-1.1990	-1.2015		-1.1986	
Gaussian	0.0000	ρ	0.0999	0.0995	0.1007	0.4988	0.4965	0.5000	0.8989	0.8993	0.8999	
	0.0000	μ	0.0008	0.0003	-0.0016	0.0005	0.0017	-0.0018	0.0002	0.0021	-0.0015	
	1.0000	σ^2	1.0031	1.0011	1.0005	1.0006	1.0046	0.9999	0.9994	0.9975	0.9997	
Laplace	0.0000	γ_1	-0.0037	0.0033	0.0015	-0.0020	0.0006	0.0044	0.0021	0.0027	-0.0046	
	0.0000	γ_2	0.0000	-0.0031	-0.0072	0.0041	0.0438	-0.0036	-0.0005	0.0088	-0.0025	
	0.0000	μ	-0.0002	-0.0004	-0.0003	-0.0006	-0.0014	-0.0009	0.0007	0.0008	-0.0014	
	1.0000	σ^2	1.0001	1.0002	1.0053	1.0036	1.0071	1.0049	0.9973	0.9941	1.0014	
	0.0000	γ_1	-0.0115	-0.0023	0.0013	-0.0016	0.0044	-0.0071	-0.0013	0.0107	-0.0177	
	3.0000	γ_2	2.9644	2.9971	3.0442	3.1071	3.0738	3.0343	2.9067	2.8461	2.9944	
		ρ	0.1001*	0.1008	0.1000	0.5003	0.5040	0.5002	0.8991	0.9008	0.8999	
Gaussian	0.0000	μ	0.0008	-0.0016	0.0012	-0.0023	0.0008	0.0010	0.0000	-0.0021	0.0006	
	1.0000	σ^2	1.0031	1.0005	0.9983	1.0019	1.0052	0.9994	0.9987	1.0043	1.0009	
	0.0000	γ_1	-0.0037	0.0015	0.0014	0.0029	0.0043	0.0026	-0.0015	-0.0046	0.0096	
Triangular	0.0000	γ_2	0.0000	-0.0072	0.0125	-0.0011	0.0096	0.0171	0.0013	0.0007	0.0141	
	0.0000	μ	0.0000	-0.0003	0.0002	-0.0021	-0.0052	0.0001	-0.0003	-0.0047	0.0001	
	1.0000	σ^2	1.0006	1.0015	1.0012	1.0007	0.9994	1.0013	1.0003	1.0017	1.0012	
	0.0000	γ_1	-0.0007	-0.0006	-0.0001	-0.0002	-0.0128	0.0008	-0.0005	-0.0201	0.0058	
	-0.6000	γ_2	-0.6021	-0.5981	-0.5865	-0.6002	-0.6035	-0.5866	-0.5996	-0.6008	-0.5869	
	Gaussian	0.0000	ρ	0.1000	0.1001	0.1003	0.4992	0.4972	0.5006	0.8991	0.8992	0.9002
		0.0000	μ	0.0007	0.0012	-0.0002	0.0003	-0.0040	0.0000	0.0014	-0.0044	0.0002
1.0000		σ^2	1.0037	0.9983	0.9995	0.9978	0.9999	0.9998	1.0019	0.9962	1.0002	
t(7df)	0.0000	γ_1	0.1663	0.0014	-0.0002	-0.0006	-0.0069	0.0005	0.0005	0.0016	-0.0005	
	0.0000	γ_2	1.2367	0.0125	-0.0009	0.0026	-0.0098	0.0091	-0.0006	-0.0272	0.0094	
	0.0000	μ	-0.0002	0.0001	0.0017	0.0016	-0.0031	0.0015	0.0014	-0.0039	0.0009	
	1.0000	σ^2	1.0002	1.0027	0.9984	0.9994	1.0002	0.9994	1.0021	0.9930	1.0007	
	0.0000	γ_1	-0.0076	-0.0021	-0.0034	0.0118	0.0200	0.0060	-0.0043	0.0168	0.0042	
	2.0000	γ_2	1.9871	1.9984	1.9159	1.9523	1.9659	1.8936	1.9923	1.9404	2.0081	
		ρ	0.1003*	0.1003	0.0992	0.5002	0.5012	0.4996	0.8990	0.9003	0.9000	

Gaussian	0.0000	μ	0.0008	-0.0002	0.0012	0.0002	0.0017	0.0007	0.0000	-0.0005	0.0000
	1.0000	σ^2	1.0031	0.9995	0.9997	0.9990	0.9999	0.9999	1.0002	0.9997	1.0000
	0.0000	γ_1	-0.0037	-0.0002	-0.0010	-0.0012	0.0190	-0.0001	-0.0013	0.0145	-0.0007
	0.0000	γ_2	0.0000	-0.0009	0.0023	0.0003	0.0047	0.0001	0.0045	-0.0031	-0.0023
t(10df)	0.0000	μ	-0.0001	0.0018	0.0009	-0.0002	-0.0019	0.0004	-0.0001	-0.0020	-0.0002
	1.0000	σ^2	1.0003	0.9991	0.9995	0.9986	1.0023	0.9992	1.0001	0.9988	0.9991
	0.0000	γ_1	-0.0041	-0.0029	0.0082	0.0014	-0.0036	0.0020	0.0027	0.0097	-0.0066
	1.0000	γ_2	0.9942	0.9801	0.9846	0.9966	0.9658	0.9828	1.0187	0.9380	0.9636
Gaussian		ρ	0.1014	0.0998	0.0996	0.5086	0.5018	0.4994	0.8735		
	0.0000	μ	0.0008	0.0012	0.0007	0.0008	0.0017	0.0009	-0.0008		
	1.0000	σ^2	1.0031	0.9997	0.9993	1.0000	0.9967	0.9990	0.9991		
	0.0000	γ_1	-0.0037	-0.0009	-0.0005	-0.0024	-0.0031	-0.0018	0.0007	unable to calculate intermediate correlation	unable to calculate intermediate correlation
$\chi^2_{(1)}$	0.0000	γ_2	0.0000	0.0018	-0.0097	0.0020	0.0186	-0.0031	0.0163		
	0.0000	μ	0.0000	0.0004	0.0009	0.0010	-0.0022	0.0005	-0.0008		
	1.0000	σ^2	0.9976	1.0027	1.0026	1.0009	0.9911	1.0011	1.0025		
	2.8284	γ_1	2.5777	2.8379	2.8274	2.5905	2.7811	2.8283	2.6308		
	12.0000	γ_2	11.6070	12.1106	11.9865	11.8973	11.3773	11.9646	12.3417		
Gaussian		ρ	0.1011	0.0995	0.1003	0.5079	0.4986	0.4997	0.9040		0.9000
	0.0000	μ	0.0008	0.0006	-0.0013	0.0000	0.0022	-0.0011	-0.0004		0.0008
	1.0000	σ^2	1.0031	0.9993	0.9980	0.9994	1.0015	0.9976	1.0000		0.9986
	0.0000	γ_1	-0.0037	-0.0005	0.0044	-0.0026	0.0008	0.0010	0.0001	unable to calculate intermediate correlation	0.0024
$\chi^2_{(2)}$	0.0000	γ_2	0.0000	-0.0097	0.0062	-0.0024	-0.0006	0.0097	-0.0012		0.0039
	0.0000	μ	0.0000	0.0008	-0.0005	0.0010	0.0015	-0.0009	-0.0004		0.0003
	1.0000	σ^2	0.9988	1.0021	0.9978	1.0009	1.0089	0.9971	0.9990		0.9997
	2.0000	γ_1	1.9872	1.9997	1.9955	2.0008	2.0033	2.0020	1.9837		2.0011
	6.0000	γ_2	5.9132	5.9962	5.9656	6.0536	5.8442	6.0226	5.8434		5.9877
Gaussian		ρ	0.1007	0.1003	0.1013	0.5048	0.5001	0.5007	0.9098	0.9003	0.8999
	0.0000	μ	0.0008	-0.0013	0.0012	-0.0010	0.0035	0.0008	-0.0003	0.0026	-0.0006
	1.0000	σ^2	1.0031	0.9980	0.9984	0.9990	0.9934	0.9994	1.0008	0.9944	0.9980
	0.0000	γ_1	-0.0037	0.0044	-0.0030	0.0004	-0.0001	-0.0010	0.0040	0.0093	-0.0064

$\chi^2_{(3)}$	0.0000	γ_2	0.0000	0.0061	-0.0071	0.0019	-0.0165	0.0044	0.0031	0.0122	0.0064
	0.0000	μ	0.0000	-0.0005	-0.0006	-0.0033	0.0045	-0.0003	0.0001	0.0010	-0.0010
	1.0000	σ^2	0.9992	0.9982	1.0031	0.9931	1.0103	1.0019	1.0049	0.9944	0.9965
	1.6330	γ_1	1.6261	1.6290	1.6440	1.5698	1.6610	1.6379	1.5945	1.6261	1.6260
	4.0000	γ_2	3.9460	3.9726	4.0895	4.0520	4.1832	4.0323	4.2108	3.8348	3.9400
Gaussian		ρ	0.1006	0.1015	0.1026	0.5058	0.5035	0.5011	0.9092	0.8998	0.9001
	0.0000	μ	0.0008	0.0012	0.0008	-0.0005	0.0013	0.0007	0.0002	0.0001	0.0001
	1.0000	σ^2	1.0031	0.9984	1.0013	0.9993	0.9959	1.0008	1.0006	0.9980	1.0012
	0.0000	γ_1	-0.0037	-0.0030	-0.0028	-0.0047	0.0009	-0.0025	0.0002	-0.0029	0.0011
	0.0000	γ_2	0.0000	-0.0071	-0.0004	0.0074	-0.0040	0.0038	-0.0062	0.0086	0.0020
$\chi^2_{(4)}$	0.0000	μ	0.0000	-0.0007	0.0006	0.0001	0.0011	0.0008	0.0002	-0.0008	0.0000
	1.0000	σ^2	0.9994	1.0028	1.0030	1.0013	1.0040	1.0046	1.0029	1.0000	1.0008
	1.4142	γ_1	1.4085	1.4240	1.4197	1.4196	1.4057	1.4217	1.4210	1.4150	1.4162
	3.0000	γ_2	2.9610	3.0708	3.0177	3.0336	3.0238	3.0287	3.0540	3.0249	2.9954
	Gaussian		ρ	0.1003	0.1025	0.1013	0.5034	0.4983	0.5009	0.9065	0.8997
0.0000		μ	0.0008	0.0008	0.0013	-0.0024	-0.0006	0.0005	-0.0009	0.0030	0.0004
1.0000		σ^2	1.0031	1.0013	0.9992	0.9994	1.0045	0.9998	0.9994	0.9939	0.9993
0.0000		γ_1	-0.0037	-0.0028	0.0032	0.0000	-0.0054	-0.0013	-0.0028	0.0090	0.0014
0.0000		γ_2	0.0000	-0.0004	-0.0020	0.0058	-0.0102	0.0007	0.0004	0.0002	0.0031
$\chi^2_{(6)}$	0.0000	μ	0.0000	0.0006	-0.0012	-0.0002	-0.0015	-0.0013	-0.0009	0.0027	0.0003
	1.0000	σ^2	0.9998	1.0023	1.0000	1.0021	1.0014	1.0001	0.9991	1.0005	1.0014
	1.0000	γ_1	0.9959	1.0049	1.0001	1.0020	0.9893	0.9990	0.9959	1.0159	1.0056
	1.5000	γ_2	1.4803	1.5144	1.5016	1.5179	1.4606	1.4862	1.4897	1.5494	1.5173
	Gaussian		ρ	0.1001	0.1013	0.1014	0.5027	0.5027	0.5012	0.9046	0.9005
0.0000		μ	0.0008	0.0013	0.0006	0.0008	0.0019	-0.0001	0.0008	0.0015	-0.0005
1.0000		σ^2	1.0031	0.9992	1.0019	1.0031	1.0056	1.0020	1.0031	0.9977	1.0008
0.0000		γ_1	-0.0037	0.0033	-0.0022	-0.0037	-0.0016	-0.0017	-0.0037	0.0021	-0.0054
0.0000		γ_2	0.0000	-0.0020	-0.0088	0.0000	0.0024	-0.0062	0.0000	-0.0017	0.0054
$\chi^2_{(16)}$	0.0000	μ	0.0000	-0.0012	-0.0001	0.0004	0.0042	-0.0005	0.0009	0.0015	-0.0011

	1.0000	σ^2	1.0001	1.0003	0.9993	1.0008	1.0154	1.0007	1.0026	1.0036	1.0007
	0.7071	γ_1	0.7040	0.7066	0.7026	0.7039	0.7337	0.7059	0.7045	0.7129	0.7025
	0.7500	γ_2	0.7385	0.7512	0.7407	0.7473	0.7611	0.7438	0.7504	0.7591	0.7430
Gaussian		ρ	0.1000	0.1014	0.0990	0.5015	0.4985	0.4997	0.9031	0.8995	0.9001
	0.0000	μ	0.0008	0.0006	0.0004	-0.0012	-0.0009	0.0002	0.0010	-0.0003	-0.0008
	1.0000	σ^2	1.0031	1.0019	0.9998	0.9992	0.9969	1.0002	1.0016	1.0018	1.0008
$\chi^2_{(32)}$	0.0000	γ_1	-0.0037	-0.0022	-0.0019	0.0009	-0.0192	-0.0031	0.0019	-0.0019	0.0000
	0.0000	γ_2	0.0000	-0.0088	0.0005	-0.0017	-0.0162	0.0063	0.0061	-0.0085	-0.0013
	0.0000	μ	0.0000	-0.0001	0.0000	-0.0008	-0.0052	0.0000	0.0006	-0.0008	-0.0009
	1.0000	σ^2	1.0002	0.9996	0.9975	0.9987	0.9989	0.9980	1.0029	1.0009	1.0006
	0.5000	γ_1	0.4975	0.4956	0.4971	0.4984	0.4645	0.4950	0.5039	0.4984	0.5022
	0.3750	γ_2	0.3673	0.3675	0.3785	0.3753	0.3610	0.3642	0.3847	0.3971	0.3726
Gaussian		ρ	0.1001*	0.0990	0.1002	0.5004	0.5031	0.5001	0.8989	0.9006	0.8999
	0.0000	μ	0.0008	0.0004	0.0000	-0.0006	0.0028	0.0007	0.0016	0.0023	0.0001
	1.0000	σ^2	1.0031	0.9998	0.9990	1.0010	1.0048	0.9992	0.9999	1.0060	1.0002
Beta ($\alpha=4, \beta=4$)	0.0000	γ_1	-0.0037	-0.0019	0.0024	0.0032	0.0069	0.0028	-0.0010	-0.0061	0.0000
	0.0000	γ_2	0.0000	0.0005	0.0079	0.0031	-0.0266	0.0070	-0.0007	-0.0092	0.0035
	0.0000	μ	0.0000	0.0002	0.0013	-0.0008	0.0031	0.0015	0.0010	0.0028	0.0001
	1.0000	σ^2	1.0005	0.9982	0.9988	1.0024	1.0064	0.9987	0.9989	1.0084	1.0000
	0.0000	γ_1	-0.0008	-0.0028	-0.0015	0.0015	0.0143	0.0012	-0.0006	0.0094	0.0016
	-0.5455	γ_2	-0.5477	-0.5456	-0.5444	-0.5451	-0.5599	-0.5461	-0.5420	-0.5541	-0.5486
Gaussian		ρ	0.1005*	0.0999	0.1009	0.4993	0.4993	0.5012	0.8982	0.9001	0.8999
	0.0000	μ	0.0008	0.0000	0.0007	0.0008	0.0018	0.0003	0.0008	0.0001	0.0012
	1.0000	σ^2	1.0031	0.9990	0.9998	1.0031	0.9960	1.0011	1.0031	0.9951	0.9984
Beta ($\alpha=4, \beta=2$)	0.0000	γ_1	-0.0037	0.0024	-0.0011	-0.0037	0.0064	0.0010	-0.0037	-0.0180	0.0075
	0.0000	γ_2	0.0000	0.0079	0.0060	0.0000	-0.0058	0.0081	0.0000	-0.0200	0.0007
	0.0000	μ	-0.0001	0.0014	-0.0008	0.0001	0.0025	-0.0010	0.0005	-0.0011	0.0016
	1.0000	σ^2	1.0004	0.9986	1.0017	1.0018	1.0010	1.0030	1.0037	1.0002	0.9973
	-0.4677	γ_1	-0.1905	-0.4693	-0.4667	-0.1924	-0.4815	-0.4670	-0.1910	-0.4825	-0.4604
	-0.3750	γ_2	1.2403	-0.3727	-0.3769	1.2668	-0.3609	-0.3767	1.2549	-0.3644	-0.3837

Gaussian	0.0000	ρ	0.0992	0.1007	0.0991	0.4966	0.4985	0.4996	0.8934	0.9003	0.9002
	1.0000	μ	0.0008	0.0007	0.0017	0.0008	0.0009	0.0013	0.0008	0.0025	-0.0002
	0.0000	σ^2	1.0031	0.9998	0.9995	1.0031	0.9983	1.0000	1.0031	1.0045	1.0024
	0.0000	γ_1	-0.0037	-0.0010	-0.0028	-0.0037	-0.0005	-0.0003	-0.0037	0.0075	-0.0034
Beta ($\alpha=4, \beta=3/2$)	0.0000	γ_2	0.0000	0.0060	-0.0005	0.0000	0.0018	0.0107	0.0000	-0.0188	0.0070
	0.0000	μ	-0.0001	-0.0010	-0.0004	0.0002	0.0025	-0.0003	0.0004	0.0017	-0.0010
	1.0000	σ^2	1.0007	1.0014	0.9996	1.0014	0.9936	0.9999	1.0027	1.0037	1.0033
	-0.6939	γ_1	-0.6939	-0.6932	-0.6937	-0.6955	-0.6870	-0.6944	-0.6967	-0.6837	-0.7016
	-0.0686	γ_2	-0.0703	-0.0713	-0.0700	-0.0669	-0.0883	-0.0673	-0.0649	-0.0914	-0.0639
Gaussian	0.0000	ρ	0.0991	0.0992	0.0989	0.4958	0.5030	0.4996	0.8919	0.8996	0.8998
	1.0000	μ	0.0008	0.0017	-0.0005	0.0008	0.0005	0.0000	0.0008	0.0011	0.0006
	0.0000	σ^2	1.0031	0.9995	0.9969	1.0031	0.9980	0.9971	1.0031	1.0010	1.0010
	0.0000	γ_1	-0.0037	-0.0028	0.0005	-0.0037	-0.0003	-0.0015	-0.0037	-0.0020	0.0055
Beta ($\alpha=4, \beta=5/4$)	0.0000	γ_2	0.0000	-0.0004	-0.0006	0.0000	0.0042	-0.0002	0.0000	-0.0012	0.0101
	0.0000	μ	-0.0002	-0.0003	0.0006	0.0001	-0.0027	0.0005	0.0003	0.0012	-0.0003
	1.0000	σ^2	1.0008	0.9998	0.9997	1.0015	1.0048	1.0003	1.0029	1.0031	1.0000
	-0.8482	γ_1	-0.8480	-0.8476	-0.8495	-0.8498	-0.8604	-0.8485	-0.8512	-0.8573	-0.8437
	0.2210	γ_2	0.2193	0.2189	0.2263	0.2238	0.2283	0.2228	0.2262	0.2306	0.2136
Gaussian	0.0000	ρ	0.1005*	0.0988	0.0977	0.5001*	0.4963	0.4985	0.8966	0.9002	0.9000
	1.0000	μ	0.0008	-0.0005	-0.0006	-0.0001	0.0053	-0.0005	0.0008	0.0004	0.0005
	0.0000	σ^2	1.0031	0.9969	1.0014	1.0014	1.0000	1.0005	1.0031	0.9987	0.9988
	0.0000	γ_1	-0.0037	0.0005	0.0035	-0.0002	0.0075	0.0030	-0.0037	-0.0039	-0.0015
Weibull ($\alpha=6, \beta=10$)	0.0000	γ_2	0.0000	-0.0006	0.0051	-0.0027	-0.0029	0.0061	0.0000	-0.0230	0.0004
	0.0000	μ	-0.0001	0.0007	0.0012	-0.0001	-0.0009	0.0009	0.0005	0.0012	0.0008
	1.0000	σ^2	1.0006	1.0002	0.9983	1.0018	0.9959	0.9982	1.0028	0.9970	1.0009
	-0.3733	γ_1	-0.3743	-0.3746	-0.3729	-0.3740	-0.3577	-0.3730	-0.3767	-0.3642	-0.3688
	0.0355	γ_2	0.0333	0.0422	0.0407	0.0287	0.0144	0.0383	0.0397	0.0050	0.0355
Gaussian	0.0000	ρ	0.1002	0.0977	0.0986	0.5035	0.5030	0.4992	0.9059	0.9004	0.8997
	0.0000	μ	0.0008	-0.0006	0.0016	0.0008	-0.0037	0.0014	0.0008	-0.0012	-0.0002

Gamma ($\alpha=\beta=10$)	1.0000	σ^2	1.0031	1.0014	1.0007	1.0031	1.0031	1.0001	1.0031	1.0004	0.9992
	0.0000	γ_1	-0.0037	0.0035	-0.0018	-0.0037	-0.0290	0.0039	-0.0037	-0.0027	-0.0026
	0.0000	γ_2	0.0000	0.0051	0.0014	0.0000	-0.0198	0.0037	0.0000	0.0036	-0.0013
	0.0000	μ	0.0000	0.0009	0.0010	0.0005	0.0019	0.0008	0.0011	-0.0014	0.0001
	1.0000	σ^2	1.0001	0.9997	0.9999	1.0007	1.0040	0.9994	1.0023	1.0020	0.9985
	0.8222	γ_1	0.8196	0.8225	0.8260	0.8193	0.8312	0.8292	0.8196	0.8125	0.8187
	0.6000	γ_2	0.5905	0.5942	0.6026	0.5949	0.6086	0.6017	0.5978	0.5795	0.5728
Gaussian		ρ	0.1000	0.0987	0.1004	0.5026	0.5002	0.5001	0.9044	0.8988	0.9000
	0.0000	μ	0.0008	0.0017	-0.0004	0.0008	-0.0001	-0.0002	0.0008	0.0045	0.0009
	1.0000	σ^2	1.0031	1.0007	0.9992	1.0031	1.0020	0.9991	1.0031	0.9947	0.9994
	0.0000	γ_1	-0.0037	-0.0018	0.0011	-0.0037	0.0122	0.0033	-0.0037	0.0121	0.0092
Rayleigh ($\alpha=1/2$, $\mu=\sqrt{(\pi/2)}$)	0.0000	γ_2	0.0000	0.0013	-0.0008	0.0000	0.0365	0.0084	0.0000	0.0009	-0.0042
	0.0000	μ	0.0000	0.0011	-0.0009	0.0004	-0.0003	-0.0007	0.0010	0.0008	0.0005
	1.0000	σ^2	1.0002	0.9997	0.9976	1.0008	0.9939	0.9967	1.0023	0.9933	1.0001
	0.6311	γ_1	0.6290	0.6340	0.6298	0.6282	0.6204	0.6303	0.6282	0.6331	0.6340
	0.2451	γ_2	0.2382	0.2564	0.2461	0.2409	0.2046	0.2506	0.2429	0.2308	0.2469
Gaussian		ρ	0.1015	0.1004	0.0993	0.5095	0.4980		0.8650		
	0.0000	μ	0.0008	-0.0004	0.0005	0.0008	-0.0011		0.0008		
	1.0000	σ^2	1.0031	0.9992	1.0012	1.0031	1.0103		1.0031		
Pareto ($\theta=10$, $\alpha=1$)	0.0000	γ_1	-0.0037	0.0012	-0.0039	-0.0037	0.0007	unable to calculate intermediate correlation	-0.0037	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	0.0000	γ_2	0.0000	-0.0004	0.0005	0.0000	-0.0099		0.0000		
	0.0000	μ	-0.0001	-0.0013	0.0017	0.0006	0.0025		0.0015		
	1.0000	σ^2	0.9972	0.9950	1.0066	1.0014	1.0050		1.0038		
	2.8111	γ_1	2.7810	2.8207	2.8238	2.8226	2.8050		2.8210		
	14.8286	γ_2	14.1284	15.0376	14.7733	14.9674	14.1133	15.1357			
Logistic		ρ	0.0998	0.1000	0.1005	0.4984	0.4999	0.5001	0.8992	0.8999	0.9001
	0.0000	μ	0.0006	0.0000	0.0003	-0.0022	0.0017	0.0007	0.0010	0.0019	0.0001
	1.0000	σ^2	0.9980	1.0009	0.9994	1.0006	1.0018	1.0019	0.9999	1.0013	0.9996
	0.0000	γ_1	0.0083	-0.0025	-0.0048	-0.0029	0.0016	-0.0063	-0.0039	0.0089	0.0031

Logistic	1.2000	γ_2	1.2185	1.1786	1.1906	1.2059	1.2330	1.2306	1.1974	1.2824	1.2680
	0.0000	μ	0.0009	0.0013	-0.0006	-0.0007	0.0043	0.0018	0.0004	0.0031	-0.0001
	1.0000	σ^2	1.0002	1.0014	1.0006	1.0007	1.0075	1.0026	1.0020	1.0038	0.9982
	0.0000	γ_1	-0.0019	-0.0042	-0.0053	-0.0015	0.0280	0.0043	-0.0060	0.0217	-0.0017
	1.2000	γ_2	1.1961	1.2057	1.2209	1.2007	1.1772	1.2252	1.2118	1.2073	1.2201
Logistic		ρ	0.1003*		0.0999	0.4999		0.5006	0.8994		0.8996
	0.0000	μ	0.0006	unable to calculate constants for uniform distribution	-0.0007	0.0006		-0.0002	0.0006		0.0014
	1.0000	σ^2	1.0040		0.9995	1.0040		1.0009	1.0040		1.0010
	0.0000	γ_1	-0.0043		-0.0038	-0.0043	unable to calculate constants for uniform distribution	-0.0065	-0.0043	unable to calculate constants for uniform distribution	0.0023
	1.2000	γ_2	1.6076		1.2106	1.6078		1.2167	1.6078		1.2403
Uniform	0.0000	μ	-0.0001		-0.0003	0.0003		-0.0008	0.0007		0.0019
	1.0000	σ^2	1.0005		0.9999	1.0011		1.0006	1.0025		1.0006
	0.0000	γ_1	-0.0013		-0.0003	-0.0026		0.0011	-0.0033		0.0025
	-1.2000	γ_2	-0.2030		-1.1999	-0.2003		-1.2023	-0.1990		-1.2017
		ρ	0.0990	0.1004	0.1018	0.4991	0.4993	0.4996	0.8991	0.9005	0.9002
Logistic	0.0000	μ	-0.0012	0.0003	0.0000	0.0002	0.0012	-0.0011	-0.0020	-0.0005	-0.0005
	1.0000	σ^2	1.0004	0.9995	1.0003	1.0010	0.9942	0.9980	1.0010	1.0041	1.0019
	0.0000	γ_1	-0.0101	-0.0049	0.0000	-0.0052	-0.0111	0.0004	-0.0057	-0.0351	-0.0066
	1.2000	γ_2	4.6028	1.1941	1.1883	1.6395	1.2682	1.2200	1.6118	1.0899	1.2317
	0.0000	μ	0.0005	-0.0007	0.0024	-0.0001	-0.0008	-0.0013	-0.0024	0.0015	-0.0006
Laplace	1.0000	σ^2	0.9996	1.0012	1.0000	0.9998	0.9994	1.0007	1.0018	1.0057	1.0026
	0.0000	γ_1	-0.0110	-0.0103	0.0042	-0.0066	0.0156	-0.0061	-0.0019	-0.0276	-0.0003
	3.0000	γ_2	2.9483	3.0591	2.9910	3.0396	2.9894	3.0537	3.0321	2.7840	3.0162
		ρ	0.1008	0.1000	0.1007	0.4988	0.4997	0.5010	0.8994	0.8995	0.8998
	0.0000	μ	-0.0011	-0.0007	-0.0009	0.0009	0.0008	0.0001	-0.0007	0.0006	-0.0014
Triangular	1.0000	σ^2	0.9994	0.9995	0.9988	0.9996	1.0030	1.0008	0.9988	0.9984	0.9985
	0.0000	γ_1	-0.0032	-0.0037	-0.0031	-0.0112	0.0063	-0.0021	-0.0062	0.0257	-0.0024
	1.2000	γ_2	1.6023	1.2115	1.1893	1.6274	1.1776	1.2238	1.5449	1.2798	1.2342
	0.0000	μ	-0.0002	-0.0006	-0.0001	-0.0016	0.0043	0.0020	-0.0010	0.0000	-0.0014
	1.0000	σ^2	1.0022	1.0000	0.9997	1.0012	1.0077	1.0003	1.0000	0.9955	0.9992
0.0000	γ_1	-0.0014	-0.0016	-0.0006	-0.0001	0.0067	0.0001	-0.0008	0.0037	-0.0030	

	-0.6000	γ_2	-0.6015	-0.5993	-0.5836	-0.6016	-0.5965	-0.5869	-0.6029	-0.5955	-0.5826
Logistic		ρ	0.1007	0.1018	0.1007	0.4985	0.5013	0.5001	0.8989	0.8994	0.9003
	0.0000	μ	0.0007	0.0000	-0.0011	-0.0011	0.0044	-0.0009	-0.0003	-0.0004	0.0000
	1.0000	σ^2	0.9991	1.0002	1.0015	0.9975	0.9984	0.9983	0.9997	1.0073	1.0001
	0.0000	γ_1	-0.0041	-0.0002	0.0006	-0.0049	-0.0090	0.0019	0.0061	-0.0069	-0.0007
t(7df)	1.2000	γ_2	1.6024	1.1824	1.2370	1.5678	1.1690	1.2163	1.6049	1.2225	1.2385
	0.0000	μ	0.0007	0.0025	0.0001	-0.0003	0.0008	-0.0004	-0.0005	0.0019	0.0009
	1.0000	σ^2	0.9975	1.0000	0.9966	1.0005	1.0034	0.9987	0.9994	0.9991	0.9994
	0.0000	γ_1	-0.0047	0.0032	0.0025	-0.0001	-0.0248	-0.0040	0.0018	0.0000	0.0017
	2.0000	γ_2	1.9884	1.9900	2.2370	2.0489	2.0319	1.9858	2.0005	1.9695	1.9461
Logistic		ρ	0.0986	0.1006	0.1011	0.4994	0.4997	0.5010	0.8990	0.8996	0.9000
	0.0000	μ	-0.0004	-0.0009	-0.0013	0.0003	-0.0039	-0.0010	0.0007	0.0032	-0.0009
	1.0000	σ^2	0.9985	0.9988	0.9979	0.9994	1.0066	1.0026	1.0004	0.9905	0.9977
	0.0000	γ_1	0.0059	-0.0035	-0.0035	-0.0015	-0.0028	0.0019	-0.0006	0.0066	-0.0043
t(10df)	1.2000	γ_2	1.5850	1.1904	1.1809	1.6003	1.1800	1.2542	1.5979	1.2518	1.2287
	0.0000	μ	-0.0023	0.0000	0.0008	0.0005	-0.0036	-0.0002	0.0005	0.0022	-0.0007
	1.0000	σ^2	1.0004	0.9999	0.9987	1.0005	1.0050	0.9981	0.9993	0.9935	0.9976
	0.0000	γ_1	-0.0007	0.0030	-0.0020	-0.0014	-0.0253	-0.0041	-0.0032	0.0147	-0.0102
	1.0000	γ_2	0.9939	1.0141	0.9930	0.9999	0.9587	0.9960	0.9898	1.0118	0.9748
Logistic		ρ	0.1015	0.1003	0.1007	0.5097	0.5040	0.5005	0.8846		
	0.0000	μ	0.0001	-0.0011	-0.0004	-0.0001	-0.0012	-0.0018	0.0012		
	1.0000	σ^2	0.9997	1.0017	0.9992	1.0049	1.0023	0.9987	1.0013		
	0.0000	γ_1	0.0059	0.0005	0.0027	-0.0074	0.0133	-0.0033	0.0012	unable to calculate intermediate correlation	unable to calculate intermediate correlation
$\chi^2_{(1)}$	1.2000	γ_2	1.6060	1.2425	1.1828	1.6438	1.1586	1.2402	1.6228		
	0.0000	μ	-0.0007	-0.0016	0.0001	0.0009	0.0046	-0.0003	0.0014		
	1.0000	σ^2	0.9949	0.9934	1.0005	1.0005	1.0158	0.9982	1.0033		
	2.8284	γ_1	2.5891	2.8344	2.8258	2.5626	2.8684	2.7976	2.6238		
	12.0000	γ_2	11.8038	12.2804	11.9772	11.3016	12.4131	11.4630	12.8543		
		ρ	0.1011	0.1012	0.1004	0.5086	0.4987	0.5008	0.9073		0.9002

Logistic	0.0000	μ	0.0007	-0.0014	-0.0007	0.0007	-0.0047	-0.0004	0.0007		-0.0005
	1.0000	σ^2	1.0038	0.9979	0.9997	1.0038	0.9946	0.9999	1.0038		1.0025
$\chi^2_{(2)}$	0.0000	γ_1	-0.0045	-0.0035	-0.0023	-0.0045	0.0080	0.0046	-0.0045	unable to calculate intermediate correlation	0.0042
	1.2000	γ_2	1.1966	1.1789	1.1955	1.1966	1.1916	1.2067	1.1966		1.2125
	0.0000	μ	0.0000	0.0003	0.0019	0.0006	-0.0024	0.0000	0.0015		0.0002
	1.0000	σ^2	0.9988	0.9997	1.0034	1.0006	0.9999	1.0013	1.0030		1.0026
	2.0000	γ_1	1.9872	1.9929	1.9965	1.9974	2.0191	1.9973	1.9963		1.9998
	6.0000	γ_2	5.9133	5.8855	5.9289	6.0398	6.2104	5.9353	6.0211		5.9701
Logistic		ρ	0.1013	0.1005	0.0993	0.5064	0.5013	0.5011	0.9111	0.9004	0.8998
	0.0000	μ	0.0009	-0.0004	0.0015	0.0007	0.0023	0.0000	-0.0001	0.0031	-0.0016
$\chi^2_{(3)}$	1.0000	σ^2	0.9977	0.9992	0.9998	1.0029	1.0097	1.0015	1.0016	1.0069	0.9993
	0.0000	γ_1	0.0009	0.0031	0.0026	-0.0011	0.0112	0.0004	-0.0018	0.0047	-0.0071
	1.2000	γ_2	1.6279	1.1802	1.2326	1.6439	1.2663	1.2262	1.6662	1.1613	1.2341
	0.0000	μ	-0.0014	0.0003	0.0002	0.0024	0.0011	0.0021	0.0000	0.0057	-0.0012
	1.0000	σ^2	0.9981	1.0001	1.0009	1.0048	0.9898	1.0040	1.0024	1.0123	0.9984
	1.6330	γ_1	1.5850	1.6324	1.6350	1.5894	1.6200	1.6268	1.5953	1.6363	1.6282
4.0000	γ_2	4.1817	3.9946	3.9895	4.1726	3.9244	3.9358	4.2919	3.8917	3.9899	
Logistic		ρ	0.1003	0.1005	0.0980	0.5045	0.4983	0.5000	0.9106	0.8996	0.9002
	0.0000	μ	-0.0004	-0.0007	-0.0017	-0.0011	0.0007	0.0016	-0.0003	0.0029	-0.0004
$\chi^2_{(4)}$	1.0000	σ^2	0.9977	0.9997	0.9997	1.0011	1.0077	1.0020	1.0008	1.0003	0.9996
	0.0000	γ_1	-0.0057	-0.0018	-0.0037	-0.0051	-0.0066	0.0016	0.0019	-0.0112	0.0081
	1.2000	γ_2	1.5814	1.1928	1.1545	1.2018	1.2902	1.2721	1.5919	1.2140	1.1878
	0.0000	μ	-0.0003	0.0019	-0.0002	0.0001	-0.0011	0.0009	0.0000	0.0023	-0.0003
	1.0000	σ^2	0.9968	1.0027	1.0007	1.0011	0.9987	1.0033	0.9998	1.0039	1.0005
	1.4142	γ_1	1.4083	1.4131	1.4127	1.4224	1.4164	1.4250	1.4167	1.4000	1.4165
3.0000	γ_2	2.9784	2.9730	2.9889	3.0620	2.9707	3.0774	3.0238	2.9175	2.9808	
Logistic		ρ	0.1000	0.0993	0.1005	0.5044	0.5033	0.4986	0.9072	0.8993	0.9002
	0.0000	μ	0.0010	0.0015	0.0015	0.0013	0.0053	-0.0011	0.0001	-0.0056	0.0006
	1.0000	σ^2	0.9990	0.9999	1.0020	1.0036	1.0002	0.9993	1.0002	1.0011	1.0020
	0.0000	γ_1	-0.0020	0.0027	0.0061	0.0030	0.0197	-0.0004	0.0016	-0.0350	0.0061

$\chi^2_{(8)}$	1.2000	γ_2	1.2192	1.2435	1.2191	1.2424	1.1752	1.1784	1.2063	1.1691	1.2383
	0.0000	μ	-0.0004	0.0002	0.0000	0.0005	0.0022	-0.0003	0.0005	-0.0050	0.0015
	1.0000	σ^2	1.0012	1.0005	1.0013	0.9999	1.0053	0.9998	1.0007	0.9938	1.0034
	1.0000	γ_1	1.0061	1.0016	0.9989	0.9957	1.0165	0.9967	1.0010	0.9736	1.0060
	1.5000	γ_2	1.5267	1.5009	1.4736	1.4857	1.5078	1.4788	1.5072	1.4703	1.5042
Logistic		ρ	0.0993	0.0979	0.1000	0.5027	0.5009	0.5008	0.9049	0.9004	0.8999
	0.0000	μ	0.0006	-0.0017	0.0007	0.0001	-0.0028	0.0015	-0.0006	0.0007	0.0013
	1.0000	σ^2	1.0002	0.9996	1.0002	0.9988	1.0003	1.0039	0.9990	1.0010	1.0031
	0.0000	γ_1	0.0081	-0.0037	0.0006	0.0023	-0.0366	0.0019	-0.0077	0.0307	-0.0022
$\chi^2_{(16)}$	1.2000	γ_2	1.2244	1.1466	1.2024	1.1684	1.1714	1.2714	1.6008	1.1393	1.2543
	0.0000	μ	-0.0023	-0.0004	0.0008	0.0004	0.0019	0.0005	-0.0006	0.0041	0.0012
	1.0000	σ^2	0.9996	1.0012	1.0012	0.9991	1.0044	1.0023	0.9971	1.0058	1.0027
	0.7071	γ_1	0.7101	0.7058	0.7078	0.7041	0.7041	0.7088	0.7005	0.7418	0.7066
	0.7500	γ_2	0.7619	0.7449	0.7415	0.7461	0.7250	0.7509	0.7275	0.7882	0.7581
Logistic		ρ	0.0988	0.1007	0.0992	0.5000	0.4975	0.5006	0.9034	0.8994	0.8997
	0.0000	μ	0.0005	0.0015	0.0005	0.0016	-0.0012	0.0001	0.0001	-0.0029	-0.0003
	1.0000	σ^2	1.0005	1.0020	0.9993	0.9971	1.0030	1.0024	1.0025	0.9898	0.9987
	0.0000	γ_1	-0.0002	0.0056	-0.0064	0.0021	0.0253	-0.0039	0.0022	0.0058	-0.0036
$\chi^2_{(32)}$	1.2000	γ_2	1.2056	1.2279	1.2216	1.1932	1.2915	1.2278	1.2313	1.2659	1.1918
	0.0000	μ	-0.0021	-0.0001	-0.0005	0.0011	0.0029	0.0003	-0.0001	-0.0018	0.0000
	1.0000	σ^2	0.9993	1.0011	0.9994	0.9981	1.0059	1.0015	1.0032	0.9905	0.9991
	0.5000	γ_1	0.4991	0.5011	0.4998	0.4991	0.5117	0.5029	0.4999	0.4866	0.4951
	0.3750	γ_2	0.3751	0.3654	0.3797	0.3742	0.3964	0.3844	0.3804	0.4039	0.3682
Logistic		ρ	0.0993	0.1002	0.1004	0.4983	0.4969	0.4993	0.8994	0.8996	0.8998
	0.0000	μ	0.0001	0.0008	0.0001	0.0020	0.0042	-0.0009	-0.0012	0.0039	0.0012
	1.0000	σ^2	1.0022	1.0002	1.0010	0.9966	0.9947	0.9995	1.0060	1.0042	1.0039
	0.0000	γ_1	0.0072	0.0009	0.0033	0.0042	0.0238	-0.0034	0.0000	0.0307	0.0073
Beta ($\alpha=4, \beta=4$)	1.2000	γ_2	1.5959	1.2028	1.1909	1.5866	1.1524	1.2413	1.6325	1.2262	1.2860
	0.0000	μ	-0.0002	0.0007	0.0016	0.0003	0.0026	-0.0016	-0.0018	0.0040	0.0007
	1.0000	σ^2	0.9994	1.0005	1.0006	1.0001	1.0101	1.0000	1.0041	0.9987	1.0016

	0.0000	γ_1	-0.0003	0.0019	-0.0043	0.0010	-0.0051	-0.0001	0.0003	0.0104	0.0015
	-0.5455	γ_2	-0.5463	-0.5474	-0.5485	-0.5441	-0.5575	-0.5472	-0.5503	-0.5397	-0.5464
Logistic		ρ	0.0970	0.0986	0.0996	0.4956	0.4965	0.5003	0.8946	0.8994	0.9001
	0.0000	μ	0.0000	0.0005	0.0009	0.0006	-0.0011	0.0004	-0.0009	0.0005	-0.0005
	1.0000	σ^2	1.0002	0.9994	1.0011	0.9995	1.0065	1.0014	0.9982	1.0008	1.0041
	0.0000	γ_1	-0.0090	-0.0059	0.0033	-0.0001	0.0000	0.0051	0.0039	-0.0004	-0.0128
Beta ($\alpha=4, \beta=2$)	1.2000	γ_2	1.6319	1.2249	1.2099	1.6263	1.3783	1.2116	1.6187	1.2399	1.2474
	0.0000	μ	0.0002	-0.0004	0.0001	0.0013	-0.0008	0.0011	-0.0003	0.0001	-0.0006
	1.0000	σ^2	0.9991	1.0000	0.9988	1.0003	0.9956	1.0015	0.9996	0.9988	1.0016
	-0.4677	γ_1	-0.4679	-0.4671	-0.4681	-0.4699	-0.4714	-0.4681	-0.4662	-0.4826	-0.4722
	-0.3750	γ_2	-0.3698	-0.3767	-0.3718	-0.3717	-0.3811	-0.3794	-0.3776	-0.3585	-0.3689
Logistic		ρ	0.0982	0.1005	0.0995	0.4963	0.4991	0.4995	0.8927	0.9000	0.9000
	0.0000	μ	0.0017	0.0001	0.0000	0.0004	0.0013	0.0014	0.0001	0.0052	-0.0023
	1.0000	σ^2	1.0004	1.0010	0.9998	1.0032	0.9969	1.0001	1.0007	1.0001	1.0008
	0.0000	γ_1	0.0071	0.0030	0.0002	-0.0095	0.0009	0.0066	0.0017	0.0261	-0.0076
Beta ($\alpha=4, \beta=3/2$)	1.2000	γ_2	1.6479	1.1891	1.2073	1.5920	1.2356	1.2233	1.6076	1.1674	1.2273
	0.0000	μ	0.0015	0.0015	0.0005	0.0001	-0.0007	0.0008	0.0000	0.0034	-0.0026
	1.0000	σ^2	1.0000	1.0012	0.9981	0.9994	0.9991	0.9984	1.0004	0.9976	1.0026
	-0.6939	γ_1	-0.6948	-0.6967	-0.6916	-0.6911	-0.6995	-0.6949	-0.6963	-0.6787	-0.6997
	-0.0686	γ_2	-0.0697	-0.0669	-0.0686	-0.0742	-0.0416	-0.0664	-0.0619	-0.1056	-0.0668
Logistic		ρ	0.0978	0.0996	0.1001	0.4960	0.5035	0.4998	0.8906	0.9001	0.8998
	0.0000	μ	-0.0008	0.0009	-0.0018	-0.0015	-0.0032	0.0000	0.0012	-0.0026	0.0006
	1.0000	σ^2	0.9970	1.0011	1.0016	1.0013	1.0024	1.0003	0.9998	1.0027	1.0022
	0.0000	γ_1	-0.0016	0.0036	0.0002	-0.0050	0.0004	-0.0009	0.0119	-0.0249	0.0126
Beta ($\alpha=4, \beta=5/4$)	1.2000	γ_2	1.6395	1.2123	1.2104	1.6324	1.2716	1.2380	1.6029	1.1930	1.2130
	0.0000	μ	0.0003	0.0002	0.0011	-0.0005	-0.0033	0.0004	0.0012	-0.0017	0.0002
	1.0000	σ^2	0.9988	0.9988	0.9995	0.9998	1.0019	0.9985	0.9977	1.0031	1.0013
	-0.8482	γ_1	-0.8460	-0.8492	-0.8516	-0.8497	-0.8435	-0.8452	-0.8472	-0.8591	-0.8369
	0.2210	γ_2	0.2139	0.2264	0.2330	0.2271	0.1902	0.2132	0.2217	0.2249	0.2012

Logistic	0.0000	ρ	0.0999	0.0996	0.1021	0.4990	0.4994	0.4995	0.8963	0.8997	0.8997	
	1.0000	μ	0.0000	0.0000	0.0008	-0.0006	0.0020	-0.0019	0.0001	-0.0002	0.0015	
	0.0000	σ^2	1.0007	0.9998	0.9995	1.0005	1.0023	1.0012	0.9998	1.0078	0.9998	
	0.0000	γ_1	-0.0030	0.0003	0.0002	-0.0034	0.0100	0.0020	-0.0005	-0.0058	0.0127	
Weibull ($\alpha=6, \beta=10$)	1.2000	γ_2	1.6507	1.2086	1.1998	1.6107	1.4182	1.2167	1.5976	1.2796	1.2607	
	0.0000	μ	-0.0003	0.0006	-0.0010	0.0002	0.0012	0.0002	-0.0003	0.0014	0.0012	
	1.0000	σ^2	1.0014	0.9989	1.0003	0.9992	0.9978	0.9990	1.0000	1.0046	0.9985	
	-0.3733	γ_1	-0.3703	-0.3699	-0.3686	-0.3736	-0.3623	-0.3739	-0.3728	-0.3685	-0.3704	
Logistic	0.0355	γ_2	0.0290	0.0388	0.0219	0.0392	0.0265	0.0347	0.0299	0.0293	0.0386	
	Gamma ($\alpha=\beta=10$)	0.0000	ρ	0.0994	0.0999	0.1006	0.5039	0.5012	0.5016	0.9063	0.8994	0.8999
		1.0000	μ	0.0020	-0.0018	-0.0003	0.0016	0.0061	-0.0004	-0.0003	0.0028	0.0001
		0.0000	σ^2	0.9983	1.0016	0.9991	1.0021	1.0009	1.0016	1.0006	0.9960	1.0002
0.0000		γ_1	0.0053	0.0001	-0.0075	0.0085	0.0356	-0.0001	0.0044	0.0049	-0.0061	
Logistic	1.2000	γ_2	1.5965	1.2090	1.1904	1.6287	1.2810	1.2445	1.6171	1.1725	1.2267	
	0.0000	μ	-0.0001	0.0010	0.0001	0.0012	0.0027	-0.0014	-0.0001	0.0031	0.0004	
	1.0000	σ^2	0.9989	0.9995	1.0007	1.0024	0.9994	1.0012	1.0005	1.0021	1.0004	
	0.8222	γ_1	0.8192	0.8195	0.8278	0.8180	0.8247	0.8248	0.8212	0.8067	0.8172	
Rayleigh ($\alpha=1/2, \mu=\sqrt{\pi/2}$)	0.6000	γ_2	0.5916	0.5876	0.5942	0.5857	0.5739	0.6052	0.5948	0.5264	0.5834	
	Logistic	0.0000	ρ	0.0982	0.1021	0.0994	0.5028	0.5031	0.5008	0.9045	0.8994	0.9000
		1.0000	μ	-0.0003	0.0008	0.0007	0.0009	0.0043	0.0001	0.0000	-0.0007	-0.0016
		0.0000	σ^2	1.0002	0.9995	0.9989	1.0013	1.0010	1.0010	0.9963	1.0013	0.9994
0.0000		γ_1	-0.0005	0.0000	0.0034	-0.0002	0.0129	-0.0099	0.0005	0.0305	-0.0016	
Logistic	1.2000	γ_2	1.6043	1.1988	1.2051	1.6096	1.3598	1.2038	1.6352	1.4729	1.2103	
	0.0000	μ	0.0000	-0.0010	0.0003	-0.0003	0.0050	0.0006	-0.0002	-0.0014	-0.0010	
	1.0000	σ^2	1.0004	1.0009	0.9967	1.0017	1.0045	1.0020	0.9979	1.0000	0.9987	
	0.6311	γ_1	0.6285	0.6330	0.6280	0.6321	0.6369	0.6357	0.6292	0.6345	0.6261	
Logistic	0.2451	γ_2	0.2411	0.2479	0.2396	0.2469	0.2334	0.2574	0.2439	0.3059	0.2363	
	Logistic	0.0000	ρ	0.1011	0.1002	0.1014	0.5097	0.4978	0.8783	unable to calculate	unable to calculate	unable to calculate
		1.0000	μ	-0.0009	-0.0003	-0.0001	0.0003	0.0007	0.0012	0.0012	0.0012	0.0012
		0.0000	σ^2	1.0003	0.9991	0.9992	0.9988	0.9997	0.9991	0.9991	0.9991	0.9991
1.0000		γ_1	0.0003	0.9991	0.9992	0.9988	0.9997	0.9991	0.9991	0.9991	0.9991	

Pareto ($\theta=10, \alpha=1$)	0.0000	γ_1	0.0070	-0.0083	-0.0007	-0.0034	-0.0198	intermediate correlation	0.0119	intermediate correlation	intermediate correlation
	1.2000	γ_2	1.5640	1.1870	1.2098	1.6458	1.1748		1.5764		
	0.0000	μ	0.0007	0.0005	0.0019	0.0001	0.0047		0.0009		
	1.0000	σ^2	0.9981	1.0051	1.0002	1.0011	1.0085		1.0039		
	2.8111	γ_1	2.7873	2.8426	2.8095	2.8399	2.7989		2.7955		
	14.8286	γ_2	14.2396	15.2124	15.3696	15.4310	14.3730		14.2291		
Uniform		ρ	0.0983		0.0988	0.5001		0.4991	0.8996		0.9004
	0.0000	μ	0.0004		-0.0007	-0.0007		0.0000	-0.0006		-0.0015
Uniform	1.0000	σ^2	1.0008	unable to calculate constants for uniform distribution	0.9998	0.9998	unable to calculate constants for uniform distribution	0.9999	1.0002	unable to calculate intermediate correlation	1.0006
	0.0000	γ_1	0.0005		0.0014	-0.0002		-0.0041	-0.0002		0.0032
	-1.2000	γ_2	-1.2013		-1.2001	-1.2004		-1.1993	-1.2006		-1.1974
	0.0000	μ	-0.0025		-0.0007	-0.0009		0.0001	-0.0010		-0.0021
	1.0000	σ^2	1.0007		0.9996	1.0003		0.9988	0.9992		1.0008
	0.0000	γ_1	0.0028		0.0007	0.0004		-0.0029	0.0012		-0.0005
	-1.2000	γ_2	-1.1994		-1.2004	-1.1996		-1.1990	-1.1992		-1.2006
Uniform		ρ	0.1007		0.0995	0.4987		0.5019	0.8994		0.8997
	0.0000	μ	-0.0007		-0.0023	0.0021		-0.0006	0.0002		0.0004
Laplace	1.0000	σ^2	0.9991	unable to calculate constants for uniform distribution	0.9989	1.0006	unable to calculate constants for uniform distribution	1.0005	1.0006	unable to calculate constants for uniform distribution	0.9993
	0.0000	γ_1	0.0037		0.0041	-0.0021		0.0068	-0.0007		0.0063
	-1.2000	γ_2	-1.1973		-1.1981	-1.2005		-1.1983	-1.2000		-1.1988
	0.0000	μ	0.0007		0.0007	-0.0007		-0.0014	0.0004		0.0014
	1.0000	σ^2	0.9982		0.9958	1.0005		1.0012	1.0013		1.0018
	0.0000	γ_1	-0.0191		0.0089	-0.0060		0.0018	-0.0087		0.0247
	3.0000	γ_2	2.8840		2.9511	3.0232		2.9685	2.9886		3.0152
Uniform		ρ	0.0990		0.0994	0.4985		0.5009	0.8997		0.9003
	0.0000	μ	0.0014	unable to calculate constants for uniform distribution	-0.0003	-0.0012	unable to calculate constants for uniform distribution	0.0008	0.0021	unable to calculate constants for uniform distribution	0.0004
Triangular	1.0000	σ^2	1.0009		1.0005	0.9989		1.0007	0.9992		1.0003
	0.0000	γ_1	-0.0017		-0.0014	0.0002		0.0031	-0.0036		-0.0005
	-1.2000	γ_2	-1.2005		-1.1965	-1.1987		-1.2009	-1.1987		-1.1988
	0.0000	μ	0.0016		-0.0002	-0.0021		0.0004	0.0015		0.0002
	1.0000	σ^2	0.9994		0.9985	1.0002		1.0006	0.9997		1.0003
	0.0000	γ_1	-0.0019		-0.0005	-0.0001		0.0074	-0.0030		-0.0054

	-0.6000	γ_2	-0.5988		-0.5853	-0.5999		-0.5844	-0.5992		-0.5879
Uniform	0.0000	ρ	0.1001		0.1003	0.4991		0.5001	0.8998		0.9003
		μ	0.0014		0.0009	-0.0004		0.0008	-0.0003		0.0006
	1.0000	σ^2	0.9999	unable to	1.0013	0.9994		0.9990	1.0025	unable to	1.0010
t(7df)	0.0000	γ_1	-0.0022	calculate	-0.0015	-0.0004	calculate	-0.0018	0.0000	calculate	0.0011
	-1.2000	γ_2	-1.1991	constants	-1.2022	-1.1991	constants	-1.1977	-1.2026	constants	-1.1916
	0.0000	μ	0.0026	for	-0.0005	-0.0004	for uniform	0.0002	-0.0002	for uniform	0.0008
	1.0000	σ^2	1.0004	uniform	0.9987	1.0009	distribution	0.9983	1.0031	distribution	1.0017
	0.0000	γ_1	0.0069	distribution	0.0034	-0.0034		-0.0096	0.0014		0.0036
	2.0000	γ_2	1.9997		1.9734	2.0092		1.9923	1.9891		2.0918
Uniform	0.0000	ρ	0.0999		0.1000	0.4993		0.5020	0.8991		0.8999
		μ	-0.0009		-0.0012	0.0004		0.0003	-0.0002		-0.0006
	1.0000	σ^2	0.9997	unable to	0.9995	0.9998		1.0006	1.0001	unable to	1.0003
t(10df)	0.0000	γ_1	0.0025	calculate	0.0013	-0.0007	calculate	0.0000	0.0003	calculate	-0.0022
	-1.2000	γ_2	-1.2003	constants	-1.1998	-1.2001	constants	-1.1715	-1.2008	constants	-1.2008
	0.0000	μ	0.0000	for	-0.0007	0.0006	for uniform	0.0012	-0.0004	for uniform	-0.0006
	1.0000	σ^2	0.9986	uniform	0.9998	0.9994	distribution	1.0021	1.0013	distribution	0.9996
	0.0000	γ_1	0.0079	distribution	-0.0088	-0.0011		0.0052	0.0026		-0.0114
	1.0000	γ_2	1.0005		1.0280	0.9884		0.9953	1.0289		1.0134
Uniform	0.0000	ρ	0.1008		0.0999	0.5066		0.5002	0.8274		
		μ	0.0011		-0.0007	0.0020		-0.0010	-0.0003		
	1.0000	σ^2	0.9985	unable to	0.9994	1.0000		0.9999	1.0009	unable to	
$\chi^2_{(1)}$	0.0000	γ_1	-0.0022	calculate	0.0000	-0.0031	calculate	-0.0013	0.0003	calculate	unable to
	-1.2000	γ_2	-1.1982	constants	-1.2006	-1.1999	constants	-1.2006	-1.2010	constants	unable to
	0.0000	μ	0.0001	for	-0.0011	0.0016	for uniform	-0.0010	-0.0001	for uniform	intermediate
	1.0000	σ^2	1.0004	uniform	0.9958	1.0028	distribution	0.9979	0.9991	distribution	correlation
	2.8284	γ_1	2.5948	distribution	2.8223	2.6333		2.8279	2.5886		
	12.0000	γ_2	11.8239		11.9648	13.0059		12.0717	11.7699		
Uniform	0.0000	ρ	0.1018		0.1000	0.5056		0.4997	0.9015		
		μ	0.0004	unable to	0.0011	0.0010	calculate	-0.0011	0.0008	calculate	unable to
	1.0000	σ^2	1.0004	constants	0.9998	0.9990	constants	0.9981	1.0029	constants	calculate
0.0000	γ_1	-0.0018		-0.0004	-0.0027		0.0012	-0.0035		intermediate	

$\chi^2_{(2)}$	-1.2000	γ_2	-1.1998	for uniform distribution	-1.2003	-1.1993	for uniform distribution	-1.1985	-0.2006	for uniform distribution	correlation
	0.0000	μ	0.0001		-0.0015	-0.0004		-0.0001	0.0015		
	1.0000	σ^2	1.0004		0.9937	0.9964		0.9963	1.0030		
	2.0000	γ_1	1.9943		1.9961	1.9956		2.0035	1.9963		
	6.0000	γ_2	6.0021		5.9609	5.9962		6.0674	6.0209		
Uniform		ρ	0.1012		0.0993	0.5056		0.4988	0.8982		
	0.0000	μ	-0.0002		0.0005	0.0013		0.0001	-0.0012		
	1.0000	σ^2	1.0001	unable to calculate constants	1.0018	1.0008	unable to calculate constants for uniform distribution	1.0000	0.9994	unable to calculate constants for uniform distribution	unable to calculate intermediate correlation
	0.0000	γ_1	0.0007		-0.0015	-0.0027		0.0018	0.0030		
	-1.2000	γ_2	-1.2006		-1.2023	-1.2000		-1.2006	-1.1996		
$\chi^2_{(3)}$	0.0000	μ	-0.0009	for uniform distribution	-0.0016	0.0004	for uniform distribution	-0.0001	-0.0005	for uniform distribution	correlation
	1.0000	σ^2	0.9983		0.9980	0.9997		1.0005	1.0038		
	1.6330	γ_1	1.6299		1.6309	1.6216		1.6495	1.6491		
	4.0000	γ_2	3.9802		3.9570	3.9010		4.1872	4.0981		
	Uniform		ρ	0.0978		0.1007	0.5045		0.5004	0.9077	
0.0000		μ	-0.0006		-0.0018	0.0005		0.0001	-0.0009		0.0003
1.0000		σ^2	1.0000	unable to calculate constants	1.0025	0.9999	unable to calculate constants for uniform distribution	1.0011	1.0007	unable to calculate constants for uniform distribution	1.0000
0.0000		γ_1	-0.0006		0.0021	-0.0002		-0.0004	0.0004		0.0029
-1.2000		γ_2	-1.1995		-1.2023	-1.2017		-1.2012	-1.1998		-1.2000
$\chi^2_{(4)}$	0.0000	μ	0.0020	for uniform distribution	-0.0005	-0.0001	for uniform distribution	-0.0008	-0.0010	for uniform distribution	0.0007
	1.0000	σ^2	1.0014		1.0011	0.9999		0.9999	0.9995		1.0009
	1.4142	γ_1	1.4080		1.4090	1.4170		1.4122	1.4101		1.4196
	3.0000	γ_2	2.9614		2.9345	3.0176		2.9834	2.9721		3.0194
	Uniform		ρ	0.1005		0.1011	0.5017		0.5002	0.9051	
0.0000		μ	-0.0008		0.0002	-0.0018		-0.0009	0.0002		0.0003
1.0000		σ^2	0.9984	unable to calculate constants	1.0020	0.9994	unable to calculate constants for uniform distribution	0.9996	0.9987	unable to calculate constants for uniform distribution	1.0003
0.0000		γ_1	0.0006		-0.0002	0.0016		-0.0035	-0.0002		-0.0002
-1.2000		γ_2	-1.1985		-1.2022	-1.1995		-1.1980	-1.1994		-1.1998
$\chi^2_{(8)}$	0.0000	μ	0.0010	for uniform distribution	0.0002	-0.0005	for uniform distribution	-0.0007	0.0006	for uniform distribution	0.0004
	1.0000	σ^2	0.9995		1.0030	0.9995		0.9976	0.9989		1.0008
	1.0000	γ_1	0.9948		1.0049	0.9957		1.0007	1.0015		1.0022
	1.5000	γ_2	1.4709		1.5185	1.4915		1.5229	1.5141		1.5203

		ρ	0.1025		0.1015	0.5017		0.4993	0.9034		0.9005
Uniform	0.0000	μ	-0.0001		0.0003	-0.0017		-0.0013	0.0014		-0.0005
	1.0000	σ^2	1.0002	unable to	0.9994	1.0003		0.9989	1.0001	unable to	1.0011
	0.0000	γ_1	-0.0007	calculate	0.0002	0.0018	unable to	-0.0014	-0.0016	calculate	-0.0015
	-1.2000	γ_2	-1.2007	constants	-1.2000	-1.2003	constants	-1.1981	-1.1993	constants	-1.2012
$\chi^2(16)$	0.0000	μ	0.0017	for	-0.0020	-0.0008	uniform	-0.0014	0.0012	uniform	-0.0007
	1.0000	σ^2	1.0022	uniform	0.9967	0.9997	distribution	0.9970	1.0029	distribution	1.0001
	0.7071	γ_1	0.7115	distribution	0.7091	0.7075		0.7023	0.7059		0.7007
	0.7500	γ_2	0.7603		0.7599	0.7578		0.7342	0.7487		0.7212
		ρ	0.0999		0.0999	0.4999		0.5006	0.9025		0.8999
Uniform	0.0000	μ	0.0004		0.0001	0.0001		0.0008	-0.0015		-0.0005
	1.0000	σ^2	0.9996	unable to	1.0009	0.9996		1.0003	1.0013	unable to	0.9994
	0.0000	γ_1	0.0002	calculate	0.0004	0.0000	unable to	0.0054	0.0008	calculate	-0.0001
	-1.2000	γ_2	-1.1992	constants	-1.2015	-1.2010	constants	-1.1955	-1.2013	constants	-1.1982
$\chi^2(32)$	0.0000	μ	0.0016	for	0.0007	-0.0003	uniform	-0.0003	-0.0013	uniform	-0.0003
	1.0000	σ^2	1.0000	uniform	1.0017	0.9984	distribution	0.9982	1.0024	distribution	0.9994
	0.5000	γ_1	0.4978	distribution	0.5027	0.5048		0.5005	0.4979		0.5026
	0.3750	γ_2	0.3651		0.3770	0.3886		0.3746	0.3834		0.3820
		ρ	0.0976		0.0994	0.4984		0.4990	0.8994		0.9001
Uniform	0.0000	μ	0.0005		-0.0014	-0.0001		-0.0003	0.0001		-0.0017
	1.0000	σ^2	1.0009	unable to	0.9996	0.9988		1.0005	1.0009	unable to	0.9998
	0.0000	γ_1	-0.0008	calculate	0.0023	0.0008	unable to	-0.0032	0.0011	calculate	-0.0019
	-1.2000	γ_2	-1.2023	constants	-1.2004	-1.1983	constants	-1.2009	-1.2010	constants	-1.1995
Beta	0.0000	μ	0.0000	for	-0.0012	-0.0006	uniform	-0.0019	0.0001	uniform	-0.0017
$(\alpha=4, \beta=4)$	1.0000	σ^2	1.0023	uniform	0.9998	0.9983	distribution	0.9992	1.0008	distribution	0.9988
	0.0000	γ_1	0.0019	distribution	0.0014	-0.0004		-0.0015	0.0023		-0.0042
	-0.5455	γ_2	-0.5519		-0.5468	-0.5469		-0.5463	-0.5475		-0.5474
		ρ	0.0994		0.0986	0.4981		0.4999	0.8960		0.9003
Uniform	0.0000	μ	0.0023	unable to	-0.0001	0.0008	unable to	-0.0017	0.0005	unable to	0.0003
	1.0000	σ^2	1.0000	calculate	0.9996	1.0007	calculate	1.0015	0.9992	calculate	1.0003
	0.0000	γ_1	-0.0034	constants	-0.0006	-0.0012	constants	-0.0005	0.0010	constants	0.0051
	-1.2000	γ_2	-1.2007	for	-1.1989	-1.2001	uniform	-1.1995	-1.2004	uniform	-1.1956
Beta	0.0000	μ	0.0006	uniform	0.0020	0.0007	distribution	-0.0013	0.0003	distribution	0.0001

(α=4, β=2)	1.0000	σ^2	1.0015	distribution	0.9997	1.0005	1.0020	0.9984	0.9993
	-0.4677	γ_1	-0.4673		-0.4712	-0.4716	-0.4642	-0.4640	-0.4619
	-0.3750	γ_2	-0.3769		-0.3739	-0.3711	-0.3782	-0.3780	-0.3788
Uniform		ρ	0.0994		0.0989	0.4954	0.5005	0.8945	0.8998
	0.0000	μ	0.0008		-0.0006	-0.0003	-0.0004	0.0013	-0.0012
	1.0000	σ^2	1.0007	unable to	1.0005	1.0012	1.0018	1.0008	0.9996
Beta (α=4, β=3/2)	0.0000	γ_1	-0.0012	calculate	0.0022	0.0003	0.0052	-0.0007	calculate
	-1.2000	γ_2	-1.2002	constants	-1.2015	-1.2010	-1.2012	-1.2013	constants
	0.0000	μ	0.0004	for	-0.0006	0.0006	-0.0009	0.0015	for uniform
	1.0000	σ^2	0.9999	uniform	1.0009	1.0010	1.0005	0.9987	distribution
	-0.6939	γ_1	-0.6952	distribution	-0.6941	-0.6934	-0.6897	-0.6938	
	-0.0686	γ_2	-0.0672		-0.0676	-0.0715	-0.0748	-0.0677	-0.0662
Uniform		ρ	0.1004		0.0991	0.4953	0.5018	0.8935	0.8998
	0.0000	μ	-0.0014		-0.0003	-0.0003	-0.0001	0.0001	-0.0014
	1.0000	σ^2	0.9991	unable to	0.9981	0.9994	1.0003	1.0008	1.0003
Beta (α=4, β=5/4)	0.0000	γ_1	0.0019	calculate	0.0008	-0.0004	0.0022	-0.0004	calculate
	-1.2000	γ_2	-1.1990	constants	-1.1975	-1.1996	-1.1965	-1.2012	constants
	0.0000	μ	0.0008	for	-0.0001	0.0001	-0.0013	-0.0004	for uniform
	1.0000	σ^2	1.0001	uniform	1.0010	0.9996	1.0006	1.0002	distribution
	-0.8482	γ_1	-0.8479	distribution	-0.8501	-0.8513	-0.8499	-0.8464	
	0.2210	γ_2	0.2175		0.2241	0.2312	0.2257	0.2116	0.2141
Uniform		ρ	0.0980		0.1001	0.4985	0.5000	0.8970	0.9001
	0.0000	μ	0.0009		-0.0002	0.0017	0.0001	-0.0003	-0.0007
	1.0000	σ^2	0.9999	unable to	0.9995	1.0004	1.0009	0.9996	1.0011
Weibull (α=6, β=10)	0.0000	γ_1	-0.0016	calculate	0.0003	-0.0018	0.0008	0.0004	calculate
	-1.2000	γ_2	-1.1985	constants	-1.1990	-1.2003	-1.2016	-1.2003	constants
	0.0000	μ	0.0003	for	0.0006	0.0006	0.0004	0.0006	for uniform
	1.0000	σ^2	0.9996	uniform	1.0004	1.0009	1.0007	0.9990	distribution
	-0.3733	γ_1	-0.3707	distribution	-0.3686	-0.3773	-0.3689	-0.3727	
	0.0355	γ_2	0.0251		0.0268	0.0380	0.0364	0.0361	0.0323
Uniform		ρ	0.1017		0.0996	0.5023	0.4997	0.9045	0.9002
	0.0000	μ	0.0005	unable to	0.0013	-0.0001	-0.0007	0.0000	unable to

Gamma ($\alpha=\beta=10$)	1.0000	σ^2	0.9998	calculate constants for uniform distribution	1.0001	1.0001	calculate constants for uniform distribution	0.9997	0.9982	calculate constants for uniform distribution	1.0012
	0.0000	γ_1	0.0004		-0.0024	0.0006		-0.0016	-0.0002		0.0016
	-1.2000	γ_2	-1.1996		-1.2006	-1.2006		-1.2007	-1.1988		-1.1976
	0.0000	μ	-0.0005		-0.0003	0.0007		-0.0005	0.0001		-0.0008
	1.0000	σ^2	1.0003		1.0008	0.9990		0.9996	0.9975		1.0008
	0.8222	γ_1	0.8196		0.8245	0.8233		0.8229	0.8222		0.8168
Uniform	0.6000	γ_2	0.5916		0.5937	0.6057		0.5926	0.6003		0.6253
		ρ	0.1005		0.0994	0.5028		0.4998	0.9031		0.9002
	0.0000	μ	0.0005		0.0002	-0.0010		0.0009	0.0007		0.0002
	1.0000	σ^2	0.9995	unable to calculate constants for uniform distribution	1.0004	0.9999		0.9998	0.9992		1.0007
	0.0000	γ_1	-0.0001		0.0014	0.0000	unable to calculate constants for uniform distribution	0.0046	-0.0002	unable to calculate constants for uniform distribution	0.0012
	-1.2000	γ_2	-1.1991		-1.2005	-1.1999		-1.1982	-1.1988		-1.2005
Rayleigh ($\alpha=1/2$, $\mu=\sqrt{(\pi/2)}$)	0.0000	μ	-0.0002		-0.0009	0.0009		0.0027	0.0009		0.0002
	1.0000	σ^2	0.9987		1.0015	0.9982		1.0003	1.0002		1.0002
	0.6311	γ_1	0.6329		0.6319	0.6285		0.6354	0.6355		0.6317
	0.2451	γ_2	0.2581		0.2485	0.2353		0.2420	0.2568		0.2364
		ρ	0.1029		0.0976	0.5008			0.8151		
	0.0000	μ	-0.0002		-0.0005	0.0017			-0.0003		
Pareto ($\theta=10$, $\alpha=1$)	1.0000	σ^2	0.9998	unable to calculate constants for uniform distribution	0.9991	0.9999	unable to calculate constants for uniform distribution	unable to calculate intermediate correlation	1.0010	unable to calculate constants for uniform distribution	unable to calculate intermediate correlation
	0.0000	γ_1	0.0002		0.0014	-0.0009			0.0006		
	-1.2000	γ_2	-1.1992		-1.1988	-1.2001			-1.1998		
	0.0000	μ	0.0019		0.0009	0.0007			0.0007		
	1.0000	σ^2	1.0042		1.0007	1.0040			1.0041		
	2.8111	γ_1	2.8282		2.8129	2.8135			2.7910		
Laplace	14.8286	γ_2	14.9976		14.7191	14.6075			14.1378		
		ρ	0.0949	0.1008	0.1018	0.4981	0.5002	0.5000	0.8989	0.9001	0.9000
	0.0000	μ	-0.0017	0.0011	-0.0011	-0.0012	0.0015	-0.0015	0.0010	0.0018	0.0005
	1.0000	σ^2	1.0119	1.0024	1.0007	0.9982	1.0025	0.9992	0.9988	1.0025	1.0012
	0.0000	γ_1	0.0074	-0.0040	-0.0150	-0.0026	0.0056	0.0086	-0.0038	0.0241	0.0131
	3.0000	γ_2	3.0618	2.9290	2.9873	3.0055	3.1208	3.0029	2.9896	3.3118	3.0844
Laplace	0.0000	μ	0.0018	0.0044	0.0002	-0.0001	0.0044	-0.0009	0.0012	0.0032	0.0005

	1.0000	σ^2	1.0034	1.0122	0.9982	0.9997	1.0087	1.0049	0.9976	1.0047	0.9988
	0.0000	γ_1	-0.0438	0.0029	0.0055	-0.0069	0.0446	-0.0102	-0.0063	0.0360	0.0094
	3.0000	γ_2	2.6736	3.0361	3.0187	2.9544	2.8998	3.0342	2.9401	3.0025	2.9951
Laplace		ρ	0.0976	0.1026	0.1004	0.4983	0.4994	0.4996	0.8994	0.8993	0.8995
	0.0000	μ	-0.0004	-0.0002	0.0001	0.0009	0.0019	-0.0002	-0.0018	-0.0016	0.0024
	1.0000	σ^2	1.0015	0.9981	1.0000	1.0013	1.0080	0.9994	0.9998	1.0068	1.0021
Triangular	0.0000	γ_1	-0.0009	-0.0012	0.0020	0.0022	0.0054	-0.0018	-0.0050	-0.0157	0.0160
	3.0000	γ_2	2.9918	2.9851	3.0092	3.0828	3.4963	2.9793	2.8982	3.2129	3.0544
	0.0000	μ	0.0006	0.0012	-0.0002	-0.0004	-0.0017	-0.0003	-0.0020	-0.0018	0.0029
	1.0000	σ^2	0.9991	1.0023	1.0012	0.9982	0.9967	1.0009	1.0003	1.0013	1.0002
	0.0000	γ_1	0.0006	-0.0023	-0.0005	0.0005	-0.0190	-0.0080	0.0015	-0.0041	0.0048
	-0.6000	γ_2	-0.5994	-0.6042	-0.5846	-0.6000	-0.5960	-0.5882	-0.6029	-0.6006	-0.5866
Laplace		ρ	0.0996	0.0992	0.1008	0.4986	0.5018	0.5002	0.8988	0.9011	0.8998
	0.0000	μ	0.0008	-0.0006	0.0010	-0.0007	-0.0028	0.0002	0.0011	-0.0044	-0.0001
	1.0000	σ^2	1.0010	0.9990	1.0034	0.9974	1.0090	1.0007	0.9983	1.0068	0.9984
t(7df)	0.0000	γ_1	0.0082	-0.0017	-0.0082	0.0049	0.0122	0.0083	-0.0116	0.0067	-0.0053
	3.0000	γ_2	3.1191	2.9654	3.0233	3.0433	3.3132	3.0790	2.9841	3.2241	2.9987
	0.0000	μ	-0.0015	-0.0004	0.0004	-0.0005	0.0021	-0.0001	0.0008	-0.0036	0.0000
	1.0000	σ^2	0.9991	0.9987	1.0027	0.9980	1.0069	0.9996	0.9986	1.0078	1.0004
	0.0000	γ_1	-0.0074	0.0051	-0.0123	0.0124	0.0493	0.0019	-0.0035	0.0012	-0.0055
	2.0000	γ_2	1.9523	2.0360	2.1816	1.9995	1.9783	1.9555	1.9865	1.9904	1.9771
Laplace		ρ	0.0991	0.0998	0.1018	0.4993	0.5011	0.5001	0.8988	0.8989	0.9001
	0.0000	μ	0.0021	-0.0018	-0.0006	-0.0011	-0.0002	0.0001	-0.0014	0.0028	0.0002
	1.0000	σ^2	1.0029	0.9991	1.0020	1.0023	0.9998	1.0026	1.0000	0.9985	1.0023
t(10df)	0.0000	γ_1	0.0021	-0.0008	-0.0018	-0.0028	-0.0016	0.0070	-0.0140	0.0107	0.0052
	3.0000	γ_2	3.0730	3.0935	2.9846	3.0231	2.8943	3.0506	3.0071	4.0793	3.0607
	0.0000	μ	0.0006	0.0007	0.0002	-0.0015	0.0022	0.0009	-0.0006	0.0022	0.0002
	1.0000	σ^2	0.9984	0.9973	1.0001	1.0000	1.0015	1.0008	1.0000	0.9936	1.0012
	0.0000	γ_1	-0.0046	0.0054	0.0045	-0.0009	0.0183	0.0098	-0.0088	0.0320	-0.0030
	1.0000	γ_2	0.9892	0.9689	1.0177	0.9860	1.1038	0.9775	0.9796	1.0398	0.9976

Laplace	0.0000	ρ	0.1019	0.0994	0.0999	0.5101	0.4957	0.4993	0.8853		
	1.0000	μ	0.0001	-0.0006	0.0002	-0.0004	-0.0041	0.0009	0.0006		
	0.0000	σ^2	1.0026	0.9988	0.9967	0.9979	0.9982	0.9977	1.0044		
	3.0000	γ_1	0.0017	-0.0092	-0.0008	0.0039	-0.0142	-0.0003	-0.0022	unable to calculate intermediate correlation	unable to calculate intermediate correlation
$\chi^2_{(1)}$	0.0000	γ_2	3.0060	3.0513	2.9760	3.0080	2.9657	3.0071	2.9819		
	0.0000	μ	-0.0003	-0.0009	0.0006	0.0002	0.0004	0.0002	0.0015		
	1.0000	σ^2	0.9976	1.0004	1.0037	0.9965	0.9974	1.0019	1.0036		
	2.8284	γ_1	2.5791	2.8650	2.8382	2.5825	2.8318	2.8379	2.6059		
12.0000	γ_2	11.6117	12.5272	12.1225	11.7744	12.2249	12.1737	12.1886			
Laplace	0.0000	ρ	0.1000	0.1001	0.0981	0.5085	0.4973	0.4998	0.9047		0.8998
	1.0000	μ	-0.0015	0.0009	-0.0014	-0.0001	-0.0025	0.0002	-0.0013		0.0003
	0.0000	σ^2	1.0010	0.9994	1.0020	1.0024	0.9959	1.0006	0.9993		1.0033
	3.0000	γ_1	-0.0059	-0.0009	-0.0044	0.0118	0.0027	-0.0086	-0.0080	unable to calculate intermediate correlation	-0.0013
$\chi^2_{(2)}$	0.0000	γ_2	2.8978	2.9480	3.0082	3.0311	2.8405	3.0021	3.0577		3.0163
	0.0000	μ	-0.0004	-0.0008	0.0002	0.0009	-0.0042	-0.0003	-0.0016		0.0009
	1.0000	σ^2	1.0016	0.9998	1.0017	1.0046	0.9816	1.0003	0.9958		1.0033
	2.0000	γ_1	2.0008	2.0027	1.9871	2.0025	1.9586	1.9959	2.0033		2.0044
6.0000	γ_2	6.0085	6.0084	5.8024	6.0197	5.6765	5.9701	6.0787		6.0375	
Laplace	0.0000	ρ	0.0997	0.1006	0.1019	0.5073	0.5010	0.5001	0.9129	0.9008	0.9000
	1.0000	μ	0.0001	-0.0011	0.0005	-0.0006	-0.0031	-0.0001	-0.0007	0.0066	0.0002
	0.0000	σ^2	0.9990	0.9980	1.0012	1.0017	1.0051	1.0014	1.0000	1.0013	0.9969
	3.0000	γ_1	-0.0033	0.0016	-0.0014	0.0085	-0.0060	0.0003	0.0015	0.0530	0.0005
$\chi^2_{(3)}$	0.0000	γ_2	2.8808	2.9199	3.0198	3.1423	3.3680	3.0566	3.0969	3.3237	2.9631
	0.0000	μ	-0.0008	-0.0007	-0.0003	0.0005	-0.0006	-0.0007	-0.0005	0.0045	-0.0003
	1.0000	σ^2	0.9980	0.9962	0.9984	1.0042	0.9938	0.9974	0.9985	1.0040	0.9984
	1.6330	γ_1	1.6271	1.6318	1.6389	1.6337	1.6454	1.6305	1.6410	1.6544	1.6255
4.0000	γ_2	3.9316	4.0397	4.0760	3.9917	4.0548	4.0371	4.0877	4.1657	3.9520	
Laplace	0.0000	ρ	0.1007	0.0997	0.0997	0.5060	0.5032	0.5010	0.9114	0.9018	0.8998
	0.0000	μ	-0.0003	-0.0007	-0.0002	0.0009	0.0026	-0.0008	-0.0004	0.0072	0.0001

$\chi^2_{(4)}$	1.0000	σ^2	1.0026	1.0007	0.9988	1.0025	1.0117	1.0001	0.9999	1.0038	1.0006
	0.0000	γ_1	0.0059	0.0030	-0.0087	0.0101	0.0070	-0.0137	0.0059	0.0326	-0.0006
	3.0000	γ_2	3.0020	3.0180	3.0009	2.8654	3.1284	2.9774	2.9676	3.3428	3.0014
	0.0000	μ	-0.0019	-0.0008	-0.0008	0.0016	0.0019	-0.0004	-0.0004	0.0083	-0.0001
	1.0000	σ^2	0.9966	0.9973	0.9984	1.0043	0.9997	0.9996	1.0019	1.0134	1.0001
	1.4142	γ_1	1.4066	1.4113	1.4118	1.4165	1.4260	1.4183	1.4179	1.4081	1.4168
	3.0000	γ_2	2.9390	2.9844	2.9631	3.0066	3.0717	3.0381	3.0216	2.9574	3.0166
Laplace		ρ	0.1003	0.1005	0.1017	0.5037	0.4968	0.4994	0.9072	0.8999	0.8999
	0.0000	μ	-0.0017	0.0009	0.0006	0.0007	0.0017	0.0001	0.0000	0.0007	-0.0003
	1.0000	σ^2	1.0006	0.9985	1.0004	1.0026	0.9973	1.0016	1.0025	0.9959	1.0018
$\chi^2_{(8)}$	0.0000	γ_1	-0.0031	-0.0095	0.0101	0.0099	-0.0192	0.0015	-0.0138	-0.0215	-0.0032
	3.0000	γ_2	3.0981	2.9208	3.0214	2.9219	3.2560	3.0974	3.0186	2.9547	3.0555
	0.0000	μ	0.0003	-0.0010	0.0005	-0.0002	-0.0039	-0.0001	0.0003	0.0023	-0.0004
	1.0000	σ^2	1.0021	0.9952	0.9984	0.9993	0.9992	0.9992	1.0013	1.0002	0.9996
	1.0000	γ_1	0.9960	0.9968	1.0016	0.9981	0.9969	0.9976	0.9992	0.9856	1.0012
	1.5000	γ_2	1.4793	1.4896	1.5155	1.4843	1.5386	1.4883	1.4963	1.5159	1.5326
		ρ	0.1005	0.0996	0.0998	0.5020	0.5018	0.4999	0.9050	0.8987	0.9003
Laplace	0.0000	μ	0.0001	-0.0003	-0.0002	0.0009	-0.0048	0.0005	-0.0015	-0.0010	-0.0005
	1.0000	σ^2	1.0014	1.0024	1.0007	0.9998	1.0107	1.0027	0.9997	1.0055	0.9993
	0.0000	γ_1	0.0082	-0.0146	-0.0035	0.0082	-0.0589	-0.0089	0.0146	-0.0066	-0.0071
$\chi^2_{(16)}$	3.0000	γ_2	3.0051	2.9833	3.0362	3.1479	3.4474	3.0852	3.0884	3.9139	3.0051
	0.0000	μ	-0.0005	-0.0016	0.0007	0.0004	-0.0024	0.0004	-0.0010	-0.0031	-0.0003
	1.0000	σ^2	0.9992	0.9987	0.9998	1.0017	0.9956	0.9996	0.9996	0.9953	0.9995
	0.7071	γ_1	0.7030	0.7076	0.7050	0.7056	0.7099	0.7005	0.7073	0.7006	0.7054
	0.7500	γ_2	0.7342	0.7419	0.7436	0.7445	0.7798	0.7187	0.7454	0.7927	0.7456
		ρ	0.1002	0.1007	0.0994	0.5010	0.5012	0.5013	0.9034	0.8998	0.8996
	0.0000	μ	0.0002	-0.0016	0.0026	-0.0004	-0.0002	-0.0005	0.0016	-0.0002	0.0000
Laplace	1.0000	σ^2	1.0018	1.0049	0.9975	0.9994	1.0025	1.0019	1.0042	1.0097	0.9997
	0.0000	γ_1	0.0195	-0.0013	0.0150	0.0043	0.0200	-0.0002	0.0100	0.0239	-0.0052
	3.0000	γ_2	3.0226	3.0309	2.9472	2.9627	3.1071	2.9954	3.0745	3.4827	3.0414

$\chi^2_{(32)}$	0.0000	μ	0.0002	-0.0008	-0.0027	0.0011	0.0030	-0.0001	0.0015	0.0018	-0.0001
	1.0000	σ^2	1.0019	1.0016	0.9997	0.9980	0.9976	1.0009	1.0023	1.0052	0.9982
	0.5000	γ_1	0.5026	0.4990	0.4994	0.4990	0.4965	0.5010	0.5007	0.5289	0.4959
	0.3750	γ_2	0.3856	0.3624	0.3719	0.3661	0.3444	0.3802	0.3816	0.4380	0.3610
Laplace		ρ	0.0977	0.1012	0.1010	0.4987	0.5033	0.4994	0.8990	0.8992	0.8995
	0.0000	μ	0.0006	0.0005	0.0002	-0.0002	0.0018	-0.0003	-0.0004	0.0011	-0.0001
	1.0000	σ^2	1.0018	1.0023	0.9962	1.0026	0.9987	0.9972	0.9991	1.0062	1.0025
	0.0000	γ_1	0.0149	-0.0001	-0.0067	-0.0045	0.0018	-0.0012	-0.0051	0.0271	-0.0046
Beta ($\alpha=4, \beta=4$)	3.0000	γ_2	2.8850	2.9639	2.9965	3.0545	2.9069	2.9971	2.9428	3.3271	3.1198
	0.0000	μ	-0.0017	-0.0002	0.0014	-0.0002	0.0024	-0.0005	-0.0005	0.0013	0.0000
	1.0000	σ^2	0.9979	1.0014	0.9985	1.0011	0.9964	1.0010	0.9987	1.0027	1.0002
	0.0000	γ_1	0.0030	0.0030	0.0004	-0.0006	-0.0098	-0.0005	0.0010	-0.0041	-0.0012
Laplace	-0.5455	γ_2	-0.5455	-0.5463	-0.5425	-0.5491	-0.5445	-0.5452	-0.5420	-0.5500	-0.5448
		ρ	0.0983	0.1015	0.0988	0.4975	0.5004	0.4986	0.8946	0.9007	0.9003
	0.0000	μ	-0.0013	0.0003	-0.0007	-0.0003	0.0020	-0.0013	-0.0024	-0.0032	-0.0004
	1.0000	σ^2	1.0044	0.9973	1.0009	1.0014	0.9818	1.0016	1.0033	0.9962	0.9988
Beta ($\alpha=4, \beta=2$)	0.0000	γ_1	0.0042	-0.0003	0.0018	0.0122	-0.0019	-0.0016	-0.0093	-0.0241	0.0056
	3.0000	γ_2	2.9660	2.9898	3.0067	3.0208	2.7100	3.0332	3.0833	2.8807	2.9565
	0.0000	μ	-0.0012	-0.0014	0.0022	-0.0009	0.0002	-0.0007	-0.0022	-0.0013	-0.0005
	1.0000	σ^2	1.0007	0.9980	1.0013	1.0015	0.9928	0.9993	1.0013	1.0009	1.0003
Laplace	-0.4677	γ_1	-0.4643	-0.4659	-0.4696	-0.4666	-0.4710	-0.4665	-0.4671	-0.4751	-0.4667
	-0.3750	γ_2	-0.3782	-0.3739	-0.3761	-0.3822	-0.3678	-0.3742	-0.3732	-0.3698	-0.3834
		ρ	0.0990	0.0999	0.1012	0.4957	0.4991	0.5005	0.8918	0.9029	0.9002
	0.0000	μ	-0.0002	0.0005	0.0015	0.0015	-0.0053	-0.0001	0.0003	-0.0029	-0.0006
Beta ($\alpha=4, \beta=3/2$)	1.0000	σ^2	0.9998	1.0021	1.0015	1.0020	1.0057	1.0018	1.0045	0.9938	0.9993
	0.0000	γ_1	0.0006	0.0027	-0.0004	-0.0078	-0.0757	0.0008	-0.0019	-0.0261	-0.0001
	3.0000	γ_2	2.9301	3.1254	3.0138	3.1224	3.5216	3.0128	3.1322	2.8743	2.9865
	0.0000	μ	-0.0013	0.0003	0.0008	0.0006	-0.0031	0.0001	-0.0003	-0.0011	-0.0008
Laplace	1.0000	σ^2	1.0005	0.9997	1.0027	0.9999	1.0037	0.9986	1.0005	1.0007	1.0011
	-0.6939	γ_1	-0.6919	-0.6937	-0.6937	-0.6952	-0.6998	-0.6948	-0.6932	-0.6946	-0.6937

	-0.0686	γ_2	-0.0713	-0.0648	-0.0652	-0.0688	-0.0583	-0.0600	-0.0669	-0.0689	-0.0722
Laplace	0.0000	ρ	0.1013	0.0999	0.0978	0.4949	0.4978	0.5001	0.8906	0.8991	0.8998
	1.0000	μ	0.0012	-0.0010	-0.0022	0.0001	-0.0040	-0.0007	0.0002	-0.0023	-0.0009
	0.0000	σ^2	1.0043	1.0007	1.0010	1.0021	1.0175	1.0008	1.0029	0.9989	0.9997
	0.0000	γ_1	0.0088	0.0015	0.0014	0.0107	-0.0351	-0.0033	-0.0076	0.0226	0.0078
Beta ($\alpha=4, \beta=5/4$)	3.0000	γ_2	2.9845	3.0065	3.0499	2.9876	3.0038	3.0319	2.9756	2.9610	3.0521
	0.0000	μ	0.0001	-0.0011	0.0002	-0.0016	-0.0027	-0.0009	-0.0004	-0.0012	-0.0007
	1.0000	σ^2	0.9997	1.0000	0.9995	1.0024	0.9995	0.9984	1.0006	0.9966	0.9991
	-0.8482	γ_1	-0.8472	-0.8449	-0.8500	-0.8463	-0.8447	-0.8486	-0.8469	-0.8516	-0.8463
	0.2210	γ_2	0.2191	0.2117	0.2303	0.2163	0.2201	0.2176	0.2199	0.2403	0.2207
Laplace	0.0000	ρ	0.0988	0.0989	0.0988	0.4971	0.5047	0.5013	0.8958	0.9005	0.9002
	1.0000	μ	-0.0011	-0.0003	-0.0017	0.0006	-0.0026	-0.0004	-0.0003	0.0041	-0.0006
	0.0000	σ^2	1.0007	1.0007	0.9986	1.0003	0.9914	1.0007	0.9989	1.0124	1.0010
	0.0000	γ_1	-0.0087	0.0013	-0.0048	-0.0017	0.0171	0.0045	-0.0029	0.0200	-0.0052
Weibull ($\alpha=6, \beta=10$)	3.0000	γ_2	2.9148	3.0249	2.9598	3.0244	2.6832	3.0627	3.0807	3.2214	3.0225
	0.0000	μ	0.0001	0.0019	-0.0012	0.0004	-0.0027	-0.0001	0.0002	0.0038	-0.0006
	1.0000	σ^2	0.9990	0.9995	0.9978	0.9982	0.9909	0.9995	0.9999	1.0018	0.9994
	-0.3733	γ_1	-0.3744	-0.3759	-0.3721	-0.3725	-0.3681	-0.3695	-0.3723	-0.3675	-0.3744
	0.0355	γ_2	0.0433	0.0303	0.0360	0.0391	0.0297	0.0344	0.0278	0.0656	0.0429
Laplace	0.0000	ρ	0.0987	0.0992	0.1021	0.5030	0.4998	0.5000	0.9070	0.8996	0.8999
	1.0000	μ	0.0015	-0.0005	0.0007	0.0018	-0.0017	-0.0005	0.0009	0.0014	-0.0012
	0.0000	σ^2	0.9963	0.9996	0.9992	0.9964	1.0009	1.0015	0.9979	0.9947	1.0009
	0.0000	γ_1	0.0052	-0.0053	-0.0013	0.0044	0.0578	-0.0101	-0.0041	-0.0110	-0.0005
Gamma ($\alpha=\beta=10$)	3.0000	γ_2	2.8696	2.9324	2.9519	3.0181	2.8927	3.0677	2.9241	2.9688	3.0018
	0.0000	μ	0.0006	-0.0003	0.0013	0.0004	-0.0045	0.0003	0.0005	-0.0009	-0.0014
	1.0000	σ^2	0.9996	1.0008	0.9993	1.0005	0.9965	0.9998	1.0003	0.9990	0.9992
	0.8222	γ_1	0.8213	0.8240	0.8247	0.8228	0.8056	0.8233	0.8207	0.8063	0.8207
	0.6000	γ_2	0.5983	0.6043	0.6119	0.6045	0.5318	0.6005	0.5940	0.5627	0.5934
		ρ	0.1018	0.0996	0.0994	0.5020	0.4945	0.4997	0.9050	0.8992	0.9000

Laplace	0.0000	μ	-0.0008	0.0000	-0.0003	0.0005	-0.0032	0.0009	0.0008	-0.0037	-0.0013
	1.0000	σ^2	1.0016	0.9985	1.0002	0.9997	1.0058	0.9984	0.9975	1.0058	1.0001
	0.0000	γ_1	-0.0001	0.0063	-0.0037	0.0072	0.0279	0.0098	0.0094	-0.0474	-0.0019
	3.0000	γ_2	2.8997	3.0430	2.9388	2.9386	3.3511	2.9815	2.9396	3.5879	3.0331
Rayleigh ($\alpha=1/2$, $\mu=\sqrt{\pi/2}$)	0.0000	μ	0.0001	-0.0002	-0.0005	0.0009	0.0020	-0.0028	0.0007	-0.0027	-0.0011
	1.0000	σ^2	1.0000	0.9990	0.9997	1.0005	1.0061	0.9987	0.9996	0.9980	1.0006
	0.6311	γ_1	0.6315	0.6281	0.6317	0.6342	0.6609	0.6211	0.6320	0.6259	0.6331
	0.2451	γ_2	0.2440	0.2415	0.2454	0.2430	0.2867	0.2391	0.2445	0.2549	0.2500
Laplace		ρ	0.1026	0.1001	0.1020	0.5108	0.4971		0.8747	0.9009	
	0.0000	μ	0.0001	-0.0001	0.0008	0.0002	-0.0007		0.0007	-0.0018	
	1.0000	σ^2	1.0043	1.0018	1.0017	1.0017	0.9985	unable to calculate intermediate correlation	1.0037	1.0048	unable to calculate intermediate correlation
	0.0000	γ_1	-0.0048	-0.0026	-0.0020	-0.0047	-0.0259		-0.0044	-0.0885	
Pareto ($\theta=10$, $\alpha=1$)	3.0000	γ_2	3.1494	3.0642	2.9669	2.9556	3.0180		1.0020	3.2363	
	0.0000	μ	0.0006	0.0014	-0.0016	-0.0004	0.0016		0.0015	-0.0014	
	1.0000	σ^2	0.9965	1.0041	0.9918	0.9998	1.0038		1.0038	0.9830	
	2.8111	γ_1	2.8162	2.7947	2.7857	2.8297	2.8277		2.8211	2.6530	
	γ_2	15.2966	14.4497	14.4647	15.0719	14.7226		15.1360	12.8831		
Triangular		ρ	0.1002*	0.0994	0.0986	0.5000	0.4996	0.5001	0.8991	0.8996	0.8999
	0.0000	μ	0.0006	0.0014	-0.0003	-0.0006	0.0020	0.0009	0.0006	0.0021	-0.0004
	1.0000	σ^2	0.9996	1.0001	1.0011	1.0004	1.0009	0.9986	0.9985	0.9998	1.0000
	0.0000	γ_1	-0.0006	-0.0005	0.0002	-0.0001	0.0025	0.0081	-0.0021	0.0034	0.0031
Triangular	-0.6000	γ_2	-0.6035	-0.6005	-0.5876	-0.6020	-0.6008	-0.5790	-0.5963	-0.5976	-0.5836
	0.0000	μ	0.0007	-0.0004	0.0006	-0.0002	0.0040	0.0001	0.0000	0.0029	-0.0002
	1.0000	σ^2	0.9999	1.0006	1.0012	1.0019	1.0048	1.0013	0.9987	1.0018	0.9999
	0.0000	γ_1	-0.0005	0.0009	-0.0025	0.0020	0.0099	0.0039	0.0001	0.0076	0.0028
Triangular	-0.6000	γ_2	-0.6032	-0.5998	-0.5854	-0.6033	-0.5985	-0.5866	-0.5956	-0.5958	-0.5827
		ρ	0.1000	0.0998	0.1005	0.4998	0.4992	0.4997	0.8992	0.9011	0.8987
	0.0000	μ	0.0013	0.0003	0.0005	0.0003	0.0020	0.0001	0.0002	0.0029	-0.0008
	1.0000	σ^2	0.9999	1.0000	0.9996	1.0007	1.0009	0.9994	1.0008	0.9983	1.0004

	0.0000	γ_1	-0.0012	0.0014	-0.0010	-0.0044	0.0025	0.0006	-0.0012	0.0154	0.0003
	-0.6000	γ_2	-0.5999	-0.6019	-0.5835	-0.6019	-0.6007	-0.5827	-0.6005	-0.5985	-0.5613
t(7df)	0.0000	μ	-0.0007	-0.0012	0.0003	-0.0009	0.0043	0.0001	0.0003	0.0029	-0.0025
	1.0000	σ^2	0.9999	1.0028	0.9966	0.9973	1.0081	0.9999	1.0013	0.9969	1.0007
	0.0000	γ_1	-0.0076	0.0034	0.0143	-0.0020	0.0356	0.0024	0.0070	0.0023	-0.0223
	2.0000	γ_2	2.0470	1.9847	1.9389	1.9399	1.9476	1.9502	2.0215	1.8851	2.1805
		ρ	0.0979*	0.0996	0.1003	0.4993	0.5009	0.4988	0.8992	0.8999	0.9002
Triangular	0.0000	μ	-0.0005	-0.0003	0.0000	0.0016	0.0005	0.0001	0.0004	0.0010	0.0006
	1.0000	σ^2	0.9990	0.9992	1.0010	0.9984	1.0025	1.0006	1.0008	0.9922	1.0013
	0.0000	γ_1	-0.0011	0.0016	-0.0007	-0.0041	0.0094	-0.0004	-0.0003	-0.0038	0.0027
t(10df)	-0.6000	γ_2	-0.5998	-0.5965	-0.5878	-0.5957	-0.5992	-0.5863	-0.6039	-0.5910	-0.5849
	0.0000	μ	0.0005	-0.0014	-0.0012	0.0005	0.0039	0.0027	0.0007	0.0004	0.0009
	1.0000	σ^2	0.9980	1.0006	1.0018	1.0007	1.0028	1.0011	0.9999	0.9904	1.0036
	0.0000	γ_1	-0.0008	-0.0091	0.0009	-0.0024	0.0066	0.0105	-0.0016	0.0079	0.0132
	1.0000	γ_2	1.0058	0.9814	0.9968	1.0106	0.9949	0.9945	0.9859	1.0181	1.0057
		ρ	0.1013	0.1001	0.0985	0.5086	0.4996	0.4996	0.8607		
Triangular	0.0000	μ	0.0009	-0.0006	0.0011	0.0009	-0.0015	0.0012	0.0009		
	1.0000	σ^2	1.0025	1.0007	0.9983	1.0025	1.0009	1.0030	1.0025		
	0.0000	γ_1	-0.0030	-0.0022	-0.0016	-0.0030	-0.0003	-0.0267	-0.0030	unable to calculate intermediate correlation	unable to calculate intermediate correlation
$\chi^2_{(1)}$	-0.6000	γ_2	-0.6017	-0.6012	-0.5840	-0.6017	-0.5962	0.4317	-0.6017		
	0.0000	μ	0.0000	-0.0008	0.0010	0.0006	0.0019	0.0028	0.0015		
	1.0000	σ^2	0.9976	1.0009	1.0040	1.0012	1.0209	1.0129	1.0035		
	2.8284	γ_1	2.5777	2.8286	2.8206	2.6084	2.8453	2.8638	2.6055		
	12.0000	γ_2	11.6072	11.9327	11.8618	12.1523	11.9632	12.5773	12.1841		
		ρ	0.1010	0.1001	0.0998	0.5076	0.5007	0.4993	0.8944		
Triangular	0.0000	μ	0.0009	-0.0001	0.0004	0.0009	-0.0043	-0.0025	0.0009	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	1.0000	σ^2	1.0025	1.0002	1.0004	1.0025	1.0011	0.9990	1.0025		
	0.0000	γ_1	-0.0030	0.0019	-0.0011	-0.0030	-0.0055	-0.0085	-0.0030		
$\chi^2_{(2)}$	-0.6000	γ_2	-0.6017	-0.6016	-0.5824	-0.6017	-0.5971	-0.5853	-0.6017		
	0.0000	μ	0.0000	0.0004	0.0010	0.0006	-0.0065	-0.0006	0.0015		

	1.0000	σ^2	0.9988	1.0032	1.0035	1.0006	0.9902	0.9987	1.0030		
	2.0000	γ_1	1.9872	2.0145	1.9951	1.9974	1.9826	1.9862	1.9962		
	6.0000	γ_2	5.9133	6.1859	5.9109	6.0401	5.9121	5.9565	6.0208		
Triangular	0.0000	ρ	0.1007	0.1003	0.0985	0.5061	0.4993	0.4987	0.9101	0.8991	0.8995
		μ	0.0009	0.0013	-0.0013	0.0009	0.0001	-0.0014	0.0009	-0.0047	0.0014
	1.0000	σ^2	1.0025	1.0016	1.0011	1.0025	0.9989	0.9979	1.0025	0.9986	1.0007
$\chi^2_{(3)}$	0.0000	γ_1	-0.0030	-0.0017	0.0033	-0.0030	0.0071	0.0030	-0.0030	-0.0109	0.0035
	-0.6000	γ_2	-0.6017	-0.6006	-0.5878	-0.6017	-0.6080	-0.5823	-0.6017	-0.5954	-0.5853
	0.0000	μ	0.0000	0.0010	-0.0006	0.0005	-0.0007	-0.0021	0.0013	-0.0031	0.0022
	1.0000	σ^2	0.9992	1.0009	1.0024	1.0006	1.0042	0.9945	1.0030	1.0027	1.0056
	1.6330	γ_1	1.6261	1.6292	1.6337	1.6322	1.6581	1.6244	1.6330	1.6472	1.6440
	4.0000	γ_2	3.9461	3.9860	3.9904	4.0147	4.0882	3.9467	4.0147	4.2144	4.0441
Triangular	0.0000	ρ	0.1005	0.1016	0.1000	0.5052	0.5001	0.5016	0.9087	0.8998	0.8987
		μ	0.0009	-0.0006	-0.0005	0.0009	-0.0102	0.0007	0.0009	-0.0017	0.0008
	1.0000	σ^2	1.0025	1.0017	1.0005	1.0025	1.0012	1.0019	1.0025	1.0011	1.0025
$\chi^2_{(4)}$	0.0000	γ_1	-0.0030	0.0002	-0.0007	-0.0030	-0.0189	0.0037	-0.0030	0.0109	-0.0077
	-0.6000	γ_2	-0.6017	-0.6028	-0.5866	-0.6017	-0.6039	-0.5845	-0.6017	-0.5971	-0.4083
	0.0000	μ	0.0000	0.0001	-0.0008	0.0005	-0.0037	0.0012	0.0012	-0.0019	0.0018
	1.0000	σ^2	0.9994	1.0008	0.9994	1.0006	0.9999	1.0021	1.0029	0.9958	1.0077
	1.4142	γ_1	1.4084	1.4190	1.4129	1.4125	1.4024	1.4286	1.4137	1.4052	1.4298
	3.0000	γ_2	2.9611	3.0499	2.9723	3.0062	2.9616	3.0675	3.0099	2.8546	3.1351
Triangular	0.0000	ρ	0.1002	0.0994	0.0997	0.5037	0.5022	0.4993	0.9061	0.9006	0.8998
		μ	0.0009	0.0005	0.0009	0.0009	0.0033	-0.0011	0.0009	-0.0019	-0.0019
	1.0000	σ^2	1.0025	0.9985	1.0000	1.0025	1.0086	1.0008	1.0025	0.9998	0.9994
$\chi^2_{(8)}$	0.0000	γ_1	-0.0030	-0.0025	-0.0014	-0.0030	0.0349	-0.0068	-0.0030	0.0065	-0.0054
	-0.6000	γ_2	-0.6017	-0.5998	-0.5857	-0.6017	-0.6056	-0.5855	-0.6017	-0.6006	-0.5863
	0.0000	μ	0.0000	0.0001	0.0008	0.0004	-0.0001	-0.0013	0.0011	-0.0020	-0.0011
	1.0000	σ^2	0.9998	1.0023	1.0005	1.0007	0.9958	0.9988	1.0027	1.0011	0.9991
	1.0000	γ_1	0.9960	1.0025	1.0019	0.9972	1.0103	0.9889	0.9981	1.0145	0.9921
	1.5000	γ_2	1.4803	1.5136	1.5202	1.4982	1.5308	1.4741	1.5025	1.5309	1.4911

Triangular	0.0000	ρ	0.1001	0.0977	0.1014	0.5026	0.4991	0.4997	0.9042	0.9007	0.8998
	1.0000	μ	0.0009	-0.0012	-0.0003	0.0009	0.0032	0.0011	0.0009	-0.0027	-0.0010
	0.0000	σ^2	1.0025	1.0005	1.0003	1.0025	0.9987	1.0008	1.0025	0.9996	0.9969
	0.0000	γ_1	-0.0030	-0.0003	-0.0013	-0.0030	0.0038	0.0046	-0.0030	0.0052	0.0026
$\chi^2(16)$	-0.6000	γ_2	-0.6017	-0.5986	-0.5866	-0.6017	-0.6031	-0.5857	-0.6017	-0.5997	-0.5801
	0.0000	μ	0.0000	-0.0001	0.0007	0.0004	0.0005	0.0014	0.0009	-0.0033	-0.0014
	1.0000	σ^2	1.0001	1.0016	0.9990	1.0008	1.0041	0.9975	1.0026	0.9965	0.9954
	0.7071	γ_1	0.7040	0.7065	0.7029	0.7039	0.7218	0.7128	0.7045	0.6920	0.7040
Triangular	0.7500	γ_2	0.7385	0.7268	0.7365	0.7473	0.7806	0.7454	0.7503	0.6686	0.7410
	0.0000	ρ	0.0999	0.1016	0.1004	0.5018	0.5037	0.4989	0.9029	0.8989	0.9002
	1.0000	μ	0.0009	0.0006	0.0000	0.0009	0.0018	-0.0005	0.0009	-0.0003	0.0002
	0.0000	σ^2	1.0025	1.0000	1.0001	1.0025	1.0054	1.0008	1.0025	1.0012	1.0017
$\chi^2(32)$	0.0000	γ_1	-0.0030	0.0001	-0.0001	-0.0030	-0.0036	-0.0032	-0.0030	-0.0003	0.0009
	-0.6000	γ_2	-0.6017	-0.5990	-0.5894	-0.6017	-0.6043	-0.5833	-0.6017	-0.6091	-0.5797
	0.0000	μ	0.0000	0.0000	-0.0005	0.0004	-0.0034	0.0003	0.0009	0.0018	0.0005
	1.0000	σ^2	1.0002	0.9986	1.0001	1.0009	0.9911	1.0009	1.0026	1.0011	1.0024
Beta ($\alpha=4, \beta=4$)	0.5000	γ_1	0.4975	0.4994	0.5046	0.4968	0.4733	0.4953	0.4969	0.5153	0.5053
	0.3750	γ_2	0.3673	0.3870	0.3797	0.3730	0.3215	0.3768	0.3751	0.3625	0.3816
	0.0000	ρ	0.1001*	0.0999	0.0995	0.4999	0.4980	0.5001	0.8996	0.8997	0.8998
	1.0000	μ	0.0009	0.0000	-0.0007	0.0009	0.0017	0.0003	0.0009	0.0024	-0.0017
Triangular	0.0000	σ^2	1.0025	0.9997	1.0017	1.0025	1.0007	0.9992	1.0025	0.9987	1.0004
	0.0000	γ_1	-0.0030	-0.0003	0.0007	-0.0030	-0.0008	0.0032	-0.0030	0.0024	-0.0086
	-0.6000	γ_2	-0.6017	-0.6011	-0.5870	-0.6017	-0.6018	-0.5847	-0.6017	-0.5914	-0.5850
	0.0000	μ	0.0000	-0.0006	-0.0016	0.0004	0.0034	0.0000	0.0008	0.0039	-0.0019
Triangular	1.0000	σ^2	1.0005	0.9991	1.0006	1.0010	1.0002	0.9988	1.0022	0.9989	1.0007
	0.0000	γ_1	-0.0008	0.0010	-0.0010	-0.0020	0.0186	0.0013	-0.0029	-0.0003	-0.0088
	-0.5455	γ_2	-0.5477	-0.5449	-0.5502	-0.5467	-0.5467	-0.5484	-0.5461	-0.5371	-0.5494
	0.0000	ρ	0.0993	0.1011	0.0988	0.4978	0.4977	0.5001	0.8958	0.8996	0.8999
Triangular	0.0000	μ	0.0009	0.0005	0.0000	0.0009	0.0002	-0.0009	0.0009	-0.0018	0.0009

Beta ($\alpha=4, \beta=2$)	1.0000	σ^2	1.0025	1.0005	1.0015	1.0025	1.0003	1.0005	1.0025	1.0025	0.9992
	0.0000	γ_1	-0.0030	-0.0013	-0.0011	-0.0030	-0.0045	-0.0044	-0.0030	-0.0118	0.0061
	-0.6000	γ_2	-0.6017	-0.6024	-0.5869	-0.6017	-0.5942	-0.5872	-0.6017	-0.6072	-0.5835
	0.0000	μ	-0.0001	0.0011	-0.0014	0.0002	-0.0027	-0.0020	0.0005	-0.0028	0.0015
	1.0000	σ^2	1.0006	0.9974	1.0013	1.0012	0.9992	1.0019	1.0024	1.0037	0.9979
	-0.4677	γ_1	-0.4680	-0.4665	-0.4685	-0.4693	-0.4843	-0.4730	-0.4703	-0.4927	-0.4603
	-0.3750	γ_2	-0.3769	-0.3759	-0.3729	-0.3745	-0.3656	-0.3731	-0.3732	-0.3468	-0.3765
Triangular		ρ	0.0991	0.0993	0.1009	0.4968	0.4966	0.4994	0.8939	0.9000	0.8995
	0.0000	μ	0.0009	-0.0001	-0.0003	0.0009	-0.0022	0.0011	0.0009	0.0027	-0.0005
	1.0000	σ^2	1.0025	1.0001	0.9982	1.0025	0.9972	0.9995	1.0025	1.0007	1.0002
	0.0000	γ_1	-0.0030	-0.0004	0.0017	-0.0030	-0.0103	0.0041	-0.0030	0.0115	-0.0019
	-0.6000	γ_2	-0.6017	-0.5998	-0.5810	-0.6017	-0.6006	-0.5870	-0.6017	-0.5974	-0.5824
	0.0000	μ	-0.0001	0.0009	0.0000	0.0002	0.0021	-0.0001	0.0004	0.0040	-0.0001
	1.0000	σ^2	1.0007	0.9997	0.9982	1.0014	0.9984	1.0012	1.0027	1.0013	1.0003
Beta ($\alpha=4, \beta=3/2$)	-0.6939	γ_1	-0.6939	-0.6943	-0.6944	-0.6955	-0.6781	-0.6888	-0.6967	-0.6838	-0.6984
	-0.0686	γ_2	-0.0703	-0.0666	-0.0659	-0.0669	-0.0812	-0.0705	-0.0649	-0.0707	-0.0642
		ρ	0.0990	0.0991	0.1022	0.4960	0.5032	0.5001	0.8925	0.8990	0.9000
	0.0000	μ	0.0009	0.0023	-0.0010	0.0009	-0.0002	-0.0002	0.0009	-0.0002	0.0000
	1.0000	σ^2	1.0025	0.9997	0.9990	1.0025	0.9982	1.0009	1.0025	1.0004	0.9995
	0.0000	γ_1	-0.0030	0.0003	0.0012	-0.0030	0.0101	-0.0041	-0.0030	-0.0050	0.0027
	-0.6000	γ_2	-0.6017	-0.6014	-0.5858	-0.6017	-0.5967	-0.5851	-0.6017	-0.5944	-0.5871
Beta ($\alpha=4, \beta=5/4$)	0.0000	μ	-0.0002	-0.0027	0.0008	0.0001	-0.0014	-0.0006	0.0003	-0.0004	0.0000
	1.0000	σ^2	1.0008	1.0025	0.9998	1.0015	0.9976	1.0027	1.0029	0.9987	0.9982
	-0.8482	γ_1	-0.8480	-0.8444	-0.8499	-0.8498	-0.8597	-0.8526	-0.8512	-0.8649	-0.8444
	0.2210	γ_2	0.2193	0.2080	0.2241	0.2238	0.2489	0.2266	0.2262	0.2626	0.2144
		ρ	0.1004*	0.1012	0.0998	0.4984	0.4979	0.4993	0.8968	0.8997	0.8999
	0.0000	μ	0.0009	0.0003	-0.0007	0.0009	-0.0025	-0.0005	0.0009	-0.0059	-0.0016
	1.0000	σ^2	1.0025	0.9976	0.9985	1.0025	0.9988	1.0013	1.0025	0.9982	1.0001
0.0000	γ_1	-0.0030	-0.0005	0.0021	-0.0030	-0.0077	0.0004	-0.0030	-0.0145	-0.0043	
-0.6000	γ_2	-0.6017	-0.5982	-0.5839	-0.6017	-0.5942	-0.5847	-0.6017	-0.5925	-0.5850	

Weibull ($\alpha=6, \beta=10$)	0.0000	μ	-0.0001	0.0016	-0.0006	0.0002	0.0030	-0.0007	0.0005	-0.0066	-0.0021
	1.0000	σ^2	1.0006	0.9979	0.9995	1.0013	0.9857	1.0024	1.0028	0.9987	1.0016
	-0.3733	γ_1	-0.3743	-0.3729	-0.3735	-0.3758	-0.3612	-0.3752	-0.3767	-0.3868	-0.3797
	0.0355	γ_2	0.0333	0.0415	0.0336	0.0380	0.0477	0.0408	0.0397	0.0221	0.0358
Triangular		ρ	0.1001	0.0989	0.1007	0.5033	0.5033	0.4995	0.9055	0.8996	0.8998
	0.0000	μ	0.0009	-0.0008	-0.0007	0.0009	-0.0013	-0.0004	0.0009	0.0001	0.0010
	1.0000	σ^2	1.0025	1.0013	1.0001	1.0025	1.0004	1.0011	1.0025	0.9992	1.0000
	0.0000	γ_1	-0.0030	0.0007	0.0001	-0.0030	0.0046	-0.0099	-0.0030	0.0107	0.0066
Gamma ($\alpha=\beta=10$)	-0.6000	γ_2	-0.6017	-0.6039	-0.5847	-0.6017	-0.6010	-0.3894	-0.6017	-0.5999	-0.5866
	0.0000	μ	0.0000	0.0027	-0.0019	0.0005	-0.0008	-0.0007	0.0011	-0.0007	0.0006
	1.0000	σ^2	1.0001	1.0028	0.9965	1.0007	1.0012	0.9995	1.0023	0.9991	1.0008
	0.8222	γ_1	0.8196	0.8187	0.8281	0.8193	0.8117	0.8275	0.8196	0.8375	0.8309
Triangular	0.6000	γ_2	0.5905	0.5889	0.5868	0.5949	0.5956	0.5909	0.5977	0.6038	0.5989
		ρ	0.1000	0.1007	0.1010	0.5025	0.4962	0.4992	0.9041	0.8998	0.8997
	0.0000	μ	0.0009	0.0015	0.0004	0.0009	-0.0028	0.0016	0.0009	-0.0059	-0.0008
	1.0000	σ^2	1.0025	1.0007	1.0014	1.0025	0.9962	0.9991	1.0025	1.0012	1.0001
Rayleigh ($\alpha=1/2, \mu=\sqrt{(\pi/2)}$)	0.0000	γ_1	-0.0030	-0.0002	0.0012	-0.0030	-0.0050	0.0057	-0.0030	-0.0116	-0.0059
	-0.6000	γ_2	-0.6017	-0.6008	-0.5864	-0.6017	-0.5925	-0.5823	-0.6017	-0.6059	-0.5838
	0.0000	μ	0.0000	0.0018	-0.0016	0.0004	-0.0023	0.0014	0.0010	-0.0034	-0.0007
	1.0000	σ^2	1.0002	1.0042	0.9993	1.0008	0.9959	0.9997	1.0023	1.0017	0.9994
Triangular	0.6311	γ_1	0.6290	0.6321	0.6309	0.6282	0.6270	0.6398	0.6282	0.6242	0.6264
	0.2451	γ_2	0.2382	0.2510	0.2469	0.2410	0.2466	0.2607	0.2429	0.2348	0.2469
		ρ	0.1014	0.0991	0.1002	0.5089	0.5017	0.4995	0.8504		
	0.0000	μ	0.0009	-0.0026	0.0005	0.0009	0.0079	0.0001	0.0009	unable to calculate intermediate correlation	unable to calculate intermediate correlation
Pareto ($\theta=10, \alpha=1$)	1.0000	σ^2	1.0025	0.9998	1.0003	1.0025	1.0015	0.9994	1.0025		
	0.0000	γ_1	-0.0030	0.0037	-0.0005	-0.0030	0.0316	0.0006	-0.0030		
	-0.6000	γ_2	-0.6017	-0.5999	-0.5879	-0.6017	-0.6048	-0.5810	-0.6017		
	0.0000	μ	0.0000	0.0000	-0.0012	0.0006	-0.0002	0.0001	0.0015		
	σ^2	0.9972	0.9999	1.0019	1.0015	0.9926	1.0001	1.0038			

	2.8111	γ_1	2.7810	2.7782	2.8195	2.8229	2.8741	2.8090	2.8210		
	14.8286	γ_2	14.1287	14.3554	14.4380	14.9783	15.9805	14.6251	15.1354		
t(7df)		ρ	0.0999	0.0987	0.1012	0.4999	0.5001	0.5009	0.8992	0.8936	0.8999
	0.0000	μ	0.0006	-0.0016	0.0004	0.0006	0.0016	0.0001	0.0006	-0.0061	0.0005
	1.0000	σ^2	1.0041	0.9986	1.0030	1.0041	1.0021	1.0014	1.0041	0.9890	1.0028
	0.0000	γ_1	-0.0039	-0.0030	0.0022	-0.0039	0.0031	0.0033	-0.0039	-0.2344	-0.0113
t(7df)	2.0000	γ_2	1.9973	1.9499	2.1054	1.9973	2.0667	2.4001	1.9973	2.1678	2.6015
	0.0000	μ	-0.0002	-0.0013	0.0000	0.0001	0.0043	0.0015	0.0005	0.0019	0.0004
	1.0000	σ^2	1.0002	0.9969	1.0009	1.0020	1.0081	0.9994	1.0040	1.0173	1.0021
	0.0000	γ_1	-0.0076	-0.0022	-0.0100	-0.0082	0.0355	0.0102	-0.0045	-0.1860	-0.0038
	2.0000	γ_2	1.9871	2.0012	2.0002	2.0422	1.9477	1.8937	2.0192	2.4257	1.9509
t(7df)		ρ	0.0998	0.1023	0.1008	0.4999	0.4983	0.5015	0.8993	0.9009	0.8992
	0.0000	μ	0.0006	0.0008	-0.0006	0.0006	-0.0005	-0.0008	0.0006	0.0010	0.0000
	1.0000	σ^2	1.0041	0.9994	0.9976	1.0041	1.0056	1.0036	1.0041	1.0126	1.0063
	0.0000	γ_1	-0.0039	-0.0018	0.0117	-0.0039	-0.0326	-0.0086	-0.0039	-0.0036	0.0951
t(10df)	2.0000	γ_2	1.9973	1.9536	1.9549	1.9973	2.0949	2.1688	1.9973	2.0404	8.2258
	0.0000	μ	-0.0001	0.0016	-0.0002	0.0002	-0.0014	0.0000	0.0005	-0.0001	-0.0001
	1.0000	σ^2	1.0003	0.9987	0.9996	1.0016	1.0045	1.0002	1.0034	1.0050	0.9999
	0.0000	γ_1	-0.0041	0.0044	-0.0009	-0.0052	0.0014	-0.0148	-0.0043	-0.0132	0.0139
	1.0000	γ_2	0.9943	1.0055	1.0018	1.0156	1.0977	0.9974	1.0105	1.0493	1.0600
t(7df)		ρ	0.1015	0.0993	0.1002	0.5101	0.4985	0.5002	0.8848		
	0.0000	μ	0.0006	-0.0005	0.0014	0.0006	0.0008	0.0001	0.0006		
	1.0000	σ^2	1.0041	1.0006	0.9967	1.0041	0.9982	1.0009	1.0041		
	0.0000	γ_1	-0.0039	-0.0038	-0.0185	-0.0039	-0.0034	-0.0114	-0.0039	unable to calculate intermediate correlation	unable to calculate intermediate correlation
$\chi^2_{(1)}$	2.0000	γ_2	1.9973	1.9647	1.9739	1.9973	1.8576	2.1943	1.9973		
	0.0000	μ	0.0000	-0.0004	0.0008	0.0006	0.0031	0.0003	0.0015		
	1.0000	σ^2	0.9977	0.9994	1.0036	1.0012	1.0075	1.0046	1.0036		
	2.8284	γ_1	2.5777	2.8329	2.8398	2.6083	2.8400	2.8400	2.6059		
	12.0000	γ_2	11.6076	12.1251	12.1807	12.1493	11.8646	12.1067	12.1886		

t(7df)	0.0000	ρ	0.1011	0.1005	0.1018	0.5088	0.4969	0.5004	0.9067		0.9006
	1.0000	μ	0.0006	0.0008	-0.0004	0.0006	-0.0057	-0.0010	0.0006		0.0023
	0.0000	σ^2	1.0041	1.0027	1.0000	1.0041	0.9918	1.0041	1.0041		0.9973
	2.0000	γ_1	-0.0039	0.0014	0.0029	-0.0039	-0.0207	0.0024	-0.0039	unable to calculate intermediate correlation	0.0119
0.0000	γ_2	1.9973	1.9906	1.9764	1.9973	1.8585	2.1729	1.9973	1.8867		
1.0000	μ	0.0000	0.0010	0.0018	0.0006	-0.0068	-0.0012	0.0015	0.0016		
1.0000	σ^2	0.9988	1.0000	1.0059	1.0006	0.9942	0.9958	1.0030	1.0007		
$\chi^2_{(2)}$	2.0000	γ_1	1.9872	2.0012	2.0114	1.9974	1.9896	1.9762	1.9963		1.9989
	6.0000	γ_2	5.9134	6.0419	6.1263	6.0402	6.1692	5.8163	6.0210		5.9362
	0.0000	ρ	0.1008	0.0991	0.0987	0.5070	0.5013	0.5000	0.9126	0.9013	0.9002
	1.0000	μ	0.0006	-0.0004	0.0014	0.0006	0.0043	0.0007	0.0006	0.0040	-0.0008
t(7df)	1.0000	σ^2	1.0041	1.0008	1.0006	1.0041	1.0058	1.0032	1.0041	0.9935	1.0028
	0.0000	γ_1	-0.0039	-0.0030	-0.0012	-0.0039	0.0733	0.0227	-0.0039	0.0319	0.0047
	2.0000	γ_2	1.9973	1.9613	2.1329	1.9973	2.3910	2.0926	1.9973	1.9020	2.2825
	0.0000	μ	0.0000	0.0009	0.0010	0.0005	0.0064	-0.0002	0.0013	0.0019	-0.0005
$\chi^2_{(3)}$	1.0000	σ^2	0.9992	1.0020	1.0017	1.0006	1.0037	1.0008	1.0029	1.0000	1.0013
	1.6330	γ_1	1.6261	1.6375	1.6277	1.6322	1.6523	1.6363	1.6331	1.6373	1.6277
	4.0000	γ_2	3.9462	4.0425	3.9751	4.0149	3.8653	3.9876	4.0159	4.0045	3.9669
	0.0000	ρ	0.1007	0.1006	0.0983	0.5060	0.4959	0.5005	0.9107	0.9000	0.8994
t(7df)	1.0000	μ	0.0006	0.0004	-0.0017	0.0006	-0.0049	-0.0009	0.0006	0.0006	0.0003
	0.0000	σ^2	1.0041	1.0015	0.9981	1.0041	0.9987	0.9999	1.0041	1.0056	1.0013
	2.0000	γ_1	-0.0039	-0.0006	0.0062	-0.0039	-0.0332	-0.0333	-0.0039	-0.0144	-0.0066
	0.0000	γ_2	1.9973	2.0828	1.8916	1.9973	1.9427	3.1122	1.9973	2.3268	2.1243
$\chi^2_{(4)}$	0.0000	μ	0.0000	-0.0003	0.0000	0.0005	-0.0064	-0.0011	0.0012	-0.0014	0.0003
	1.0000	σ^2	0.9994	0.9986	0.9973	1.0006	0.9911	0.9990	1.0028	0.9959	1.0007
	1.4142	γ_1	1.4084	1.4143	1.4141	1.4125	1.3678	1.4149	1.4137	1.4086	1.4212
	3.0000	γ_2	2.9611	3.0142	3.0263	3.0063	2.8314	3.0218	3.0102	3.0264	3.0212
t(7df)	0.0000	ρ	0.1004	0.1003	0.1001	0.5042	0.5025	0.5012	0.9074	0.8995	0.9000
	1.0000	μ	0.0006	0.0000	-0.0001	0.0006	0.0007	-0.0001	0.0006	0.0014	-0.0011
	0.0000	σ^2	1.0041	1.0009	1.0019	1.0041	1.0051	0.9973	1.0041	0.9974	1.0022

$\chi^2_{(8)}$	0.0000	γ_1	-0.0039	0.0014	0.0128	-0.0039	-0.0001	-0.0051	-0.0039	-0.0032	0.0059
	2.0000	γ_2	1.9973	1.9709	2.1201	1.9973	1.9207	2.0115	1.9973	1.9806	2.1634
	0.0000	μ	0.0000	-0.0010	0.0005	0.0004	0.0011	0.0010	0.0010	0.0014	-0.0011
	1.0000	σ^2	0.9998	1.0013	1.0031	1.0007	1.0019	1.0000	1.0027	0.9991	1.0000
	1.0000	γ_1	0.9960	1.0035	1.0014	0.9972	1.0164	1.0028	0.9982	1.0051	0.9894
	1.5000	γ_2	1.4804	1.5089	1.5145	1.4983	1.5139	1.4872	1.5026	1.4650	1.4582
$t(7df)$		ρ	0.1002	0.0982	0.1005	0.5030	0.4992	0.5003	0.9051	0.8992	0.8996
	0.0000	μ	0.0006	0.0008	0.0010	0.0006	-0.0012	-0.0012	0.0006	-0.0021	0.0010
	1.0000	σ^2	1.0041	0.9962	0.9983	1.0041	1.0153	1.0014	1.0041	0.9996	1.0046
$\chi^2_{(16)}$	0.0000	γ_1	-0.0039	-0.0075	0.0011	-0.0039	-0.0155	0.0051	-0.0039	0.0004	0.0172
	2.0000	γ_2	1.9973	1.9595	1.9911	1.9973	1.9067	2.2799	1.9973	2.0691	2.3993
	0.0000	μ	0.0000	0.0001	-0.0002	0.0004	-0.0007	-0.0012	0.0009	-0.0038	0.0006
	1.0000	σ^2	1.0001	1.0030	1.0001	1.0008	0.9989	0.9994	1.0026	0.9953	1.0020
	0.7071	γ_1	0.7040	0.7078	0.7089	0.7040	0.7033	0.6997	0.7045	0.6949	0.7154
	0.7500	γ_2	0.7385	0.7425	0.7521	0.7474	0.7866	0.7401	0.7503	0.7582	0.7637
$t(7df)$		ρ	0.1000	0.0998	0.1002	0.5021	0.5003	0.4999	0.9034	0.9001	0.8994
	0.0000	μ	0.0006	0.0003	0.0001	0.0006	0.0033	0.0001	0.0006	0.0003	-0.0015
	1.0000	σ^2	1.0041	1.0023	0.9996	1.0041	1.0026	1.0012	1.0041	1.0003	1.0011
$\chi^2_{(32)}$	0.0000	γ_1	-0.0039	-0.0015	0.0079	-0.0039	0.0238	0.0029	-0.0039	0.0488	-0.0552
	2.0000	γ_2	1.9973	2.0001	1.9854	1.9973	1.9653	2.2485	1.9973	2.2677	3.2638
	0.0000	μ	0.0000	0.0005	-0.0008	0.0004	0.0053	-0.0008	0.0009	0.0012	-0.0015
	1.0000	σ^2	1.0002	1.0026	0.9981	1.0009	1.0092	0.9954	1.0026	0.9992	0.9995
	0.5000	γ_1	0.4975	0.5002	0.4974	0.4968	0.5163	0.4992	0.4969	0.5184	0.4937
	0.3750	γ_2	0.3673	0.3705	0.3779	0.3730	0.3531	0.3649	0.3750	0.4262	0.3696
$t(7df)$		ρ	0.0996	0.0989	0.1016	0.4998	0.5025	0.5001	0.8993	0.9005	0.9002
	0.0000	μ	0.0006	-0.0012	0.0009	0.0006	-0.0002	-0.0002	0.0006	-0.0028	0.0008
	1.0000	σ^2	1.0041	1.0000	0.9980	1.0041	0.9966	1.0026	1.0041	0.9978	0.9983
Beta	0.0000	γ_1	-0.0039	-0.0049	0.0031	-0.0039	0.0410	0.0010	-0.0039	-0.0181	0.0075
	2.0000	γ_2	1.9973	1.9708	1.9813	1.9973	2.0155	2.1984	1.9973	2.0427	2.1695
	0.0000	μ	0.0000	-0.0015	-0.0006	0.0004	-0.0001	-0.0018	0.0008	-0.0036	0.0012

(α=4, β=4)	1.0000	σ^2	1.0005	1.0025	1.0015	1.0010	1.0029	0.9997	1.0022	0.9945	0.9998
	0.0000	γ_1	-0.0008	0.0015	0.0007	-0.0020	-0.0159	-0.0021	-0.0029	-0.0191	0.0093
	-0.5455	γ_2	-0.5477	-0.5468	-0.5497	-0.5467	-0.5558	-0.5403	-0.5461	-0.5355	-0.5458
t(7df)	0.0000	ρ	0.0994	0.1000	0.1007	0.4975	0.4971	0.4999	0.8950	0.9006	0.8998
	1.0000	μ	0.0006	-0.0005	-0.0001	0.0006	-0.0005	0.0002	0.0006	-0.0021	-0.0014
	0.0000	σ^2	1.0041	1.0004	1.0024	1.0041	1.0012	1.0016	1.0041	0.9920	1.0038
Beta	0.0000	γ_1	-0.0039	0.0008	0.0050	-0.0039	0.0381	-0.0235	-0.0039	0.0111	0.0146
	2.0000	γ_2	1.9973	2.0340	1.9840	1.9973	2.1237	3.0760	1.9973	1.9108	2.4424
	0.0000	μ	-0.0001	-0.0008	-0.0010	0.0002	-0.0010	0.0002	0.0005	0.0007	-0.0017
(α=4, β=2)	1.0000	σ^2	1.0006	0.9995	0.9995	1.0012	0.9955	0.9975	1.0025	0.9923	1.0024
	-0.4677	γ_1	-0.4680	-0.4646	-0.4666	-0.4693	-0.4637	-0.4614	-0.4703	-0.4554	-0.4751
	-0.3750	γ_2	-0.3769	-0.3812	-0.3743	-0.3745	-0.3895	-0.3746	-0.3731	-0.3734	-0.3753
t(7df)	0.0000	ρ	0.0992	0.0995	0.1002	0.4963	0.5029	0.5003	0.8926	0.9002	0.8996
	1.0000	μ	0.0006	0.0007	0.0006	0.0006	-0.0032	0.0009	0.0006	-0.0030	0.0009
	0.0000	σ^2	1.0041	1.0002	0.9990	1.0041	0.9967	1.0035	1.0041	0.9997	1.0000
Beta	0.0000	γ_1	-0.0039	-0.0029	-0.0060	-0.0039	-0.0084	0.0155	-0.0039	0.0179	0.0063
	2.0000	γ_2	1.9973	1.9901	1.9268	1.9973	1.8004	2.1666	1.9973	1.9396	2.5670
	0.0000	μ	-0.0001	0.0009	-0.0013	0.0002	-0.0007	0.0007	0.0004	-0.0045	0.0012
(α=4, β=3/2)	1.0000	σ^2	1.0007	0.9991	1.0007	1.0014	1.0028	1.0001	1.0027	1.0044	0.9975
	-0.6939	γ_1	-0.6939	-0.6929	-0.6922	-0.6955	-0.6884	-0.6947	-0.6968	-0.6988	-0.6857
	-0.0686	γ_2	-0.0703	-0.0734	-0.0725	-0.0669	-0.0978	-0.0636	-0.0649	-0.0929	-0.0762
t(7df)	0.0000	ρ	0.0991	0.1015	0.1003	0.4955	0.4967	0.5005	0.8909	0.8991	0.8995
	1.0000	μ	0.0006	-0.0006	0.0010	0.0006	0.0049	-0.0002	0.0006	-0.0026	-0.0010
	0.0000	σ^2	1.0041	1.0007	0.9960	1.0041	1.0023	1.0006	1.0041	0.9982	1.0003
Beta	0.0000	γ_1	-0.0039	-0.0089	0.0070	-0.0039	0.0321	0.0022	-0.0039	0.0113	-0.0045
	2.0000	γ_2	1.9973	2.0201	1.9305	1.9973	2.0161	2.0555	1.9973	1.9726	2.3264
	0.0000	μ	-0.0002	0.0010	0.0004	0.0001	0.0026	0.0000	0.0003	-0.0026	-0.0015
(α=4, β=5/4)	1.0000	σ^2	1.0008	1.0001	0.9995	1.0015	0.9992	0.9987	1.0029	0.9987	0.9996
	-0.8482	γ_1	-0.8480	-0.8523	-0.8449	-0.8498	-0.8410	-0.8475	-0.8512	-0.8412	-0.8558
	0.2210	γ_2	0.2193	0.2313	0.2103	0.2238	0.2129	0.2199	0.2263	0.2189	0.2346

t(7df)	0.0000	ρ	0.0995	0.1011	0.1019	0.4982	0.4976	0.5007	0.8962	0.8999	0.8990
	1.0000	μ	0.0006	-0.0002	-0.0009	0.0006	-0.0079	0.0014	0.0006	0.0070	-0.0001
	0.0000	σ^2	1.0041	0.9988	0.9961	1.0041	1.0036	0.9997	1.0041	1.0006	1.0021
	2.0000	γ_1	-0.0039	-0.0084	0.0076	-0.0039	-0.0510	-0.0089	-0.0039	0.0623	-0.0234
Weibull ($\alpha=6, \beta=10$)	0.0000	γ_2	1.9973	2.0024	1.9888	1.9973	1.9957	2.4425	1.9973	2.1266	3.6283
	0.0000	μ	-0.0001	-0.0005	-0.0004	0.0002	-0.0035	0.0007	0.0005	0.0045	-0.0001
	1.0000	σ^2	1.0006	0.9990	1.0019	1.0013	1.0126	1.0001	1.0028	0.9937	0.9981
	-0.3733	γ_1	-0.3743	-0.3673	-0.3778	-0.3758	-0.3980	-0.3698	-0.3767	-0.3523	-0.3687
	0.0355	γ_2	0.0333	0.0223	0.0397	0.0381	0.0384	0.0350	0.0396	0.0342	0.0364
t(7df)	0.0000	ρ	0.1002	0.0997	0.1025	0.5037	0.4979	0.5012	0.9065	0.8988	0.8999
	1.0000	μ	0.0006	-0.0007	-0.0010	0.0006	0.0018	-0.0001	0.0006	-0.0016	0.0011
	0.0000	σ^2	1.0041	1.0035	0.9992	1.0041	1.0002	1.0018	1.0041	1.0069	1.0010
	2.0000	γ_1	-0.0039	0.0005	-0.0122	-0.0039	0.0208	-0.0145	-0.0039	-0.0332	0.0179
Gamma ($\alpha=\beta=10$)	0.0000	γ_2	1.9973	2.0768	1.8553	1.9973	2.0241	2.3334	1.9973	2.2630	2.2617
	0.0000	μ	0.0000	-0.0016	0.0001	0.0005	0.0038	0.0017	0.0011	-0.0020	0.0010
	1.0000	σ^2	1.0001	0.9984	1.0020	1.0007	1.0021	1.0047	1.0023	1.0007	0.9998
	0.8222	γ_1	0.8196	0.8165	0.8182	0.8193	0.8319	0.8249	0.8196	0.8137	0.8210
	0.6000	γ_2	0.5905	0.5745	0.6128	0.5950	0.5735	0.5819	0.5976	0.6187	0.5870
t(7df)	0.0000	ρ	0.1001	0.0990	0.0996	0.5028	0.4980	0.4988	0.9048	0.9003	0.8999
	1.0000	μ	0.0006	0.0001	-0.0008	0.0006	0.0016	0.0006	0.0006	0.0020	0.0003
	0.0000	σ^2	1.0041	1.0026	0.9981	1.0041	0.9934	1.0021	1.0041	0.9974	1.0026
	2.0000	γ_1	-0.0039	0.0105	0.0022	-0.0039	-0.0263	0.0266	-0.0039	0.0198	-0.0046
Rayleigh ($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	0.0000	γ_2	1.9973	1.9940	1.8611	1.9973	2.0296	3.2203	1.9973	1.8614	2.2059
	0.0000	μ	0.0000	-0.0013	-0.0002	0.0004	0.0002	0.0002	0.0010	0.0032	0.0004
	1.0000	σ^2	1.0002	0.9998	1.0008	1.0008	0.9995	1.0007	1.0023	1.0032	1.0018
	0.6311	γ_1	0.6290	0.6318	0.6316	0.6282	0.6230	0.6250	0.6282	0.6470	0.6335
	0.2451	γ_2	0.2382	0.2516	0.2468	0.2410	0.2447	0.2326	0.2428	0.2682	0.2467
t(7df)	0.0000	ρ	0.1016	0.1012	0.1012	0.5104	0.5029	0.5000	0.8781	0.8996	
		μ	0.0006	0.0000	-0.0006	0.0006	0.0062	0.0002	0.0006	-0.0065	unable to

Pareto ($\theta=10, \alpha=1$)	1.0000	σ^2	1.0041	0.9991	1.0000	1.0041	0.9985	1.0011	1.0041	0.9996	calculate intermediate correlation
	0.0000	γ_1	-0.0039	0.0079	0.0137	-0.0039	0.0264	-0.0055	-0.0039	-0.0154	
	2.0000	γ_2	1.9973	2.0170	2.3748	1.9973	1.8631	2.5975	1.9973	2.0029	
	0.0000	μ	0.0000	-0.0007	-0.0012	0.0006	0.0046	0.0001	0.0015	-0.0054	
	1.0000	σ^2	0.9972	0.9981	0.9949	1.0015	0.9921	1.0001	1.0038	0.9967	
	2.8111	γ_1	2.7810	2.8005	2.7957	2.8227	2.8163	2.8091	2.8211	2.8145	
	14.8286	γ_2	14.1292	14.5306	14.5818	14.9701	14.5607	14.6259	15.1361	15.0990	
t(10df)		ρ	0.1005*	0.1017	0.1002	0.4999	0.4999	0.4997	0.8993	0.4993	0.9002
	0.0000	μ	0.0007	-0.0010	-0.0012	0.0007	0.0017	0.0007	0.0007	0.0000	0.0010
	1.0000	σ^2	1.0037	0.9978	0.9986	1.0037	1.0017	0.9998	1.0037	0.9990	1.0010
t(10df)	0.0000	γ_1	-0.0044	-0.0008	0.0040	-0.0044	0.0014	0.0018	-0.0044	0.0021	0.0028
	1.0000	γ_2	1.0020	0.9766	1.0010	1.0020	1.0261	0.9955	1.0020	1.0236	1.0777
	0.0000	μ	-0.0001	0.0005	0.0013	0.0002	0.0043	0.0004	0.0005	-0.0003	0.0008
t(10df)	1.0000	σ^2	1.0003	0.9989	0.9971	1.0016	1.0074	0.9992	1.0034	0.9991	1.0009
	0.0000	γ_1	-0.0041	-0.0051	0.0022	-0.0052	0.0261	-0.0014	-0.0043	-0.0005	0.0044
	1.0000	γ_2	0.9943	0.9937	1.0003	1.0155	0.9831	0.9828	1.0106	0.9991	0.9964
t(10df)		ρ	0.1015	0.0993	0.1010	0.5097	0.5041	0.4992	0.8824		
	0.0000	μ	0.0007	-0.0006	-0.0007	0.0007	-0.0005	-0.0015	0.0007		
	1.0000	σ^2	1.0037	0.9986	0.9987	1.0037	0.9993	1.0000	1.0037		
$\chi^2_{(1)}$	0.0000	γ_1	-0.0044	0.0031	0.0023	-0.0044	-0.0057	-0.0024	-0.0044	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	1.0000	γ_2	1.0020	1.0043	0.9903	1.0020	1.0197	1.1221	1.0020		
	0.0000	μ	0.0000	-0.0009	0.0004	0.0006	-0.0057	-0.0012	0.0015		
	1.0000	σ^2	0.9976	0.9960	1.0015	1.0012	0.9869	0.9908	1.0036		
	2.8284	γ_1	2.5777	2.8153	2.8239	2.6082	2.7815	2.8108	2.6058		
12.0000	γ_2	11.6072	11.7896	11.9565	12.1469	11.5827	11.8622	12.1885			
t(10df)		ρ	0.1011	0.1006	0.0990	0.5085	0.4989	0.4994	0.9073		0.9000
	0.0000	μ	0.0007	-0.0006	-0.0011	0.0007	-0.0014	-0.0001	0.0007	unable to calculate intermediate correlation	0.0005
	1.0000	σ^2	1.0037	1.0008	1.0018	1.0037	1.0046	1.0010	1.0037		1.0028
	0.0000	γ_1	-0.0044	0.0037	0.0020	-0.0044	-0.0064	0.0040	-0.0044		-0.0006
	1.0000	γ_2	1.0020	1.0005	1.0160	1.0020	0.9861	1.0374	1.0020		1.0209

$\chi^2_{(2)}$	0.0000	μ	0.0000	-0.0022	0.0006	0.0006	0.0010	0.0007	0.0015	0.0014	
	1.0000	σ^2	0.9988	0.9961	0.9988	1.0006	1.0191	1.0033	1.0030	1.0045	
	2.0000	γ_1	1.9872	2.0041	1.9984	1.9973	2.0758	2.0132	1.9963	2.0016	
	6.0000	γ_2	5.9133	6.0577	6.0175	6.0397	6.5262	6.1053	6.0211	6.0012	
$t_{(10df)}$		ρ	0.1003	0.1011	0.1007	0.5041	0.5026	0.4999	0.9071	0.9010	0.8996
	0.0000	μ	0.0007	0.0005	-0.0005	0.0007	-0.0011	0.0012	0.0007	0.0047	-0.0005
	1.0000	σ^2	1.0037	1.0033	0.9978	1.0037	0.9915	0.9973	1.0037	0.9948	1.0010
	0.0000	γ_1	-0.0044	-0.0002	0.0003	-0.0044	0.0154	0.0013	-0.0044	0.0538	-0.0103
$\chi^2_{(3)}$	1.0000	γ_2	1.0020	1.0326	0.9774	1.0020	1.0414	1.0256	1.0020	1.0956	1.1125
	0.0000	μ	0.0000	-0.0015	0.0007	0.0004	0.0018	0.0004	0.0010	0.0033	-0.0005
	1.0000	σ^2	0.9998	0.9982	1.0034	1.0007	0.9965	1.0023	1.0026	1.0038	0.9986
	1.6330	γ_1	0.9960	1.6356	1.6408	0.9972	1.6003	1.6399	0.9982	1.6610	1.6206
$t_{(10df)}$	4.0000	γ_2	1.4803	4.0302	4.0879	1.4982	3.7395	4.0599	1.5027	3.9739	3.9474
		ρ	0.1006	0.1009	0.0990	0.5058	0.4991	0.5007	0.9102	0.8991	0.8997
	0.0000	μ	0.0007	0.0005	-0.0015	0.0007	-0.0005	-0.0005	0.0007	-0.0062	-0.0011
	1.0000	σ^2	1.0037	1.0000	0.9985	1.0037	1.0032	1.0006	1.0037	0.9922	1.0008
$\chi^2_{(4)}$	0.0000	γ_1	-0.0044	-0.0033	-0.0052	-0.0044	-0.0005	-0.0058	-0.0044	-0.0448	-0.0088
	1.0000	γ_2	1.0020	0.9939	0.9940	1.0020	1.0529	1.0517	1.0020	0.9698	1.1198
	0.0000	μ	0.0000	-0.0002	-0.0010	0.0005	-0.0061	-0.0012	0.0012	-0.0076	-0.0006
	1.0000	σ^2	0.9994	0.9996	1.0001	1.0006	0.9937	0.9974	1.0028	0.9852	0.9958
$t_{(10df)}$	1.4142	γ_1	1.4084	1.4219	1.4131	1.4125	1.3967	1.4099	1.4138	1.3595	1.4042
	3.0000	γ_2	2.9611	3.0526	2.9555	3.0061	3.0216	3.0058	3.0104	2.7352	2.9478
		ρ	0.1003	0.1008	0.0991	0.5041	0.4981	0.5019	0.9071	0.8999	0.8998
	0.0000	μ	0.0007	0.0004	0.0006	0.0007	-0.0013	0.0006	0.0007	0.0013	0.0000
$\chi^2_{(8)}$	1.0000	σ^2	1.0037	1.0020	1.0011	1.0037	0.9977	1.0022	1.0037	1.0095	0.9996
	0.0000	γ_1	-0.0044	-0.0007	-0.0001	-0.0044	0.0122	0.0110	-0.0044	0.0018	0.0053
	1.0000	γ_2	1.0020	1.0305	1.0052	1.0020	1.0644	1.0456	1.0020	1.0025	1.0589
	0.0000	μ	0.0000	0.0002	-0.0006	0.0004	-0.0040	-0.0001	0.0010	0.0028	0.0000
$t_{(10df)}$	1.0000	σ^2	0.9998	1.0003	1.0012	1.0007	1.0013	1.0016	1.0026	1.0076	1.0003
	1.0000	γ_1	0.9960	0.9973	1.0073	0.9972	1.0291	1.0009	0.9982	1.0259	1.0024

	1.5000	γ_2	1.4803	1.4788	1.5417	1.4982	1.6673	1.4875	1.5027	1.4998	1.5042
t(10df)	0.0000	ρ	0.1001	0.1010	0.1000	0.5029	0.5023	0.5002	0.9049	0.8995	0.9000
	1.0000	μ	0.0007	-0.0007	-0.0016	0.0007	0.0008	0.0007	0.0007	-0.0018	0.0012
	0.0000	σ^2	1.0037	0.9978	1.0001	1.0037	1.0019	1.0019	1.0037	1.0095	0.9981
	1.0000	γ_1	-0.0044	0.0026	0.0052	-0.0044	-0.0022	0.0014	-0.0044	-0.0209	0.0058
$\chi^2_{(16)}$	0.0000	γ_2	1.0020	0.9996	1.0061	1.0020	1.0291	1.0018	1.0020	0.9918	1.0204
	0.0000	μ	0.0000	-0.0003	-0.0005	0.0004	0.0060	0.0000	0.0009	-0.0010	0.0008
	1.0000	σ^2	1.0001	0.9997	1.0009	1.0008	1.0048	1.0000	1.0026	1.0023	1.0008
	0.7071	γ_1	0.7040	0.7072	0.7072	0.7039	0.7321	0.7097	0.7045	0.6900	0.7117
t(10df)	0.7500	γ_2	0.7385	0.7465	0.7487	0.7473	0.7663	0.7615	0.7504	0.7312	0.7581
	0.0000	ρ	0.1000	0.1004	0.0995	0.5020	0.5014	0.5005	0.9033	0.9002	0.8999
	1.0000	μ	0.0007	0.0014	-0.0012	0.0007	-0.0026	0.0001	0.0007	0.0034	-0.0009
	0.0000	σ^2	1.0037	0.9974	0.9998	1.0037	1.0036	1.0006	1.0037	1.0024	1.0015
$\chi^2_{(32)}$	0.0000	γ_1	-0.0044	-0.0053	0.0049	-0.0044	-0.0134	-0.0043	-0.0044	0.0178	-0.0092
	1.0000	γ_2	1.0020	0.9730	0.9918	1.0020	1.0929	0.9881	1.0020	0.9397	1.1148
	0.0000	μ	0.0000	0.0006	0.0003	0.0004	0.0003	-0.0012	0.0008	0.0048	-0.0011
	1.0000	σ^2	1.0002	1.0009	1.0013	1.0009	1.0087	1.0001	1.0026	1.0081	0.9992
t(10df)	0.5000	γ_1	0.4975	0.5023	0.5004	0.4968	0.4971	0.4976	0.4969	0.5102	0.4917
	0.3750	γ_2	0.3673	0.3825	0.3758	0.3730	0.4262	0.3758	0.3751	0.3629	0.3696
	0.0000	ρ	0.1002*	0.1015	0.1006	0.4999	0.5033	0.4999	0.8994	0.9002	0.9000
	1.0000	μ	0.0007	-0.0005	0.0003	0.0007	-0.0059	0.0015	0.0007	-0.0011	0.0002
Beta ($\alpha=4, \beta=4$)	0.0000	σ^2	1.0037	1.0001	1.0001	1.0037	1.0019	0.9966	1.0037	1.0006	1.0047
	0.0000	γ_1	-0.0044	0.0016	0.0070	-0.0044	-0.0117	0.0071	-0.0044	-0.0002	0.0085
	1.0000	γ_2	1.0020	0.9976	0.9890	1.0020	1.0668	1.0185	1.0020	1.0332	1.1300
	0.0000	μ	0.0000	0.0011	-0.0002	0.0004	-0.0009	0.0011	0.0008	-0.0009	-0.0002
Beta ($\alpha=4, \beta=4$)	1.0000	σ^2	1.0005	1.0021	1.0004	1.0010	0.9913	0.9996	1.0022	1.0006	1.0025
	0.0000	γ_1	-0.0008	-0.0010	0.0010	-0.0020	-0.0007	0.0086	-0.0029	-0.0173	0.0012
	-0.5455	γ_2	-0.5477	-0.5457	-0.5486	-0.5467	-0.5399	-0.5470	-0.5461	-0.5431	-0.5460
		ρ	0.0993	0.0991	0.0994	0.4976	0.5020	0.5011	0.8952	0.8983	0.9001

t(10df)	0.0000	μ	0.0007	0.0014	-0.0011	0.0007	-0.0004	-0.0005	0.0007	-0.0003	0.0014
	1.0000	σ^2	1.0037	0.9999	0.9990	1.0037	1.0019	0.9975	1.0037	0.9930	1.0006
	0.0000	γ_1	-0.0044	0.0000	0.0031	-0.0044	-0.0124	0.0010	-0.0044	0.0362	-0.0013
	1.0000	γ_2	1.0020	1.0229	1.0207	1.0020	1.0473	1.0685	1.0020	1.0624	0.9972
Beta ($\alpha=4, \beta=2$)	0.0000	μ	-0.0001	0.0005	0.0008	0.0002	-0.0025	-0.0001	0.0005	-0.0008	0.0012
	1.0000	σ^2	1.0006	1.0011	0.9996	1.0012	1.0049	1.0021	1.0024	0.9902	1.0001
	-0.4677	γ_1	-0.4680	-0.4668	-0.4673	-0.4693	-0.4818	-0.4741	-0.4703	-0.4592	-0.4645
	-0.3750	γ_2	-0.3769	-0.3769	-0.3748	-0.3745	-0.3487	-0.3722	-0.3732	-0.3805	-0.3781
t(10df)		ρ	0.0992	0.0980	0.1007	0.4964	0.4989	0.5015	0.8929	0.9011	0.8999
	0.0000	μ	0.0007	-0.0017	0.0007	0.0007	0.0017	-0.0013	0.0007	-0.0021	-0.0005
	1.0000	σ^2	1.0037	0.9988	0.9992	1.0037	1.0033	0.9997	1.0037	0.9999	1.0014
	0.0000	γ_1	-0.0044	0.0039	-0.0014	-0.0044	0.0087	-0.0103	-0.0044	-0.0114	-0.0085
Beta ($\alpha=4, \beta=3/2$)	1.0000	γ_2	1.0020	0.9790	0.9717	1.0020	1.0915	1.0423	1.0020	0.9465	1.0593
	0.0000	μ	-0.0001	0.0008	0.0011	0.0002	-0.0024	-0.0010	0.0004	-0.0012	-0.0010
	1.0000	σ^2	1.0007	0.9988	0.9991	1.0014	0.9998	1.0031	1.0027	1.0026	1.0003
	-0.6939	γ_1	-0.6939	-0.6956	-0.6943	-0.6955	-0.6956	-0.6978	-0.6967	-0.6943	-0.7002
t(10df)		ρ	0.0991	0.0995	0.1011	0.4956	0.5032	0.4994	0.8913	0.8987	0.8998
	0.0000	μ	0.0007	-0.0001	0.0006	0.0007	0.0015	-0.0009	0.0007	0.0039	0.0016
	1.0000	σ^2	1.0037	1.0016	0.9994	1.0037	1.0047	1.0001	1.0037	1.0003	0.9982
	0.0000	γ_1	-0.0044	0.0009	-0.0010	-0.0044	0.0167	0.0120	-0.0044	0.0268	0.0111
Beta ($\alpha=4, \beta=5/4$)	1.0000	γ_2	1.0020	1.0099	0.9679	1.0020	1.0428	1.0567	1.0020	1.0073	1.0586
	0.0000	μ	-0.0002	-0.0005	-0.0002	0.0001	-0.0072	-0.0008	0.0003	0.0029	0.0016
	1.0000	σ^2	1.0008	1.0023	1.0017	1.0015	1.0082	1.0008	1.0029	0.9968	0.9971
	-0.8482	γ_1	-0.8480	-0.8493	-0.8471	-0.8498	-0.8518	-0.8459	-0.8512	-0.8373	-0.8405
t(10df)	0.2210	γ_2	0.2193	0.2265	0.2172	0.2238	0.2014	0.2135	0.2263	0.2049	0.2173
		ρ	0.1006*	0.1003	0.1010	0.4983	0.4999	0.5014	0.8964	0.8996	0.9002
	0.0000	μ	0.0007	0.0010	-0.0006	0.0007	0.0059	-0.0003	0.0007	-0.0090	-0.0001
	1.0000	σ^2	1.0037	0.9981	1.0020	1.0037	0.9961	0.9999	1.0037	1.0038	1.0000
0.0000	γ_1	-0.0044	0.0007	0.0098	-0.0044	0.0266	0.0019	-0.0044	-0.0568	-0.0040	

Weibull ($\alpha=6, \beta=10$)	1.0000	γ_2	1.0020	1.0131	1.0309	1.0020	0.9370	1.0278	1.0020	0.9637	1.0683
	0.0000	μ	-0.0001	-0.0002	0.0010	0.0002	-0.0005	0.0012	0.0005	-0.0071	0.0001
	1.0000	σ^2	1.0006	0.9998	1.0014	1.0013	1.0012	0.9982	1.0028	1.0090	1.0018
	-0.3733	γ_1	-0.3743	-0.3730	-0.3751	-0.3758	-0.3476	-0.3662	-0.3767	-0.4042	-0.3810
	0.0355	γ_2	0.0333	0.0408	0.0371	0.0380	0.0108	0.0302	0.0397	0.0406	0.0379
t(10df)		ρ	0.1002	0.1002	0.1020	0.5036	0.4998	0.5005	0.9063	0.8991	0.9000
	0.0000	μ	0.0007	0.0001	-0.0003	0.0007	0.0077	-0.0010	0.0007	0.0006	-0.0012
	1.0000	σ^2	1.0037	0.9997	1.0008	1.0037	1.0054	1.0009	1.0037	0.9984	0.9990
	0.0000	γ_1	-0.0044	0.0002	-0.0012	-0.0044	0.0138	0.0111	-0.0044	0.0156	-0.0058
	1.0000	γ_2	1.0020	1.0008	0.9993	1.0020	1.0093	1.0967	1.0020	1.0744	1.0795
Gamma ($\alpha=\beta=10$)	0.0000	μ	0.0000	-0.0008	-0.0018	0.0005	0.0038	0.0004	0.0011	0.0016	-0.0009
	1.0000	σ^2	1.0001	0.9975	0.9962	1.0007	1.0047	0.9997	1.0023	0.9996	0.9991
	0.8222	γ_1	0.8196	0.8214	0.8189	0.8193	0.8393	0.8146	0.8196	0.8353	0.8179
	0.6000	γ_2	0.5905	0.6016	0.6170	0.5949	0.6290	0.6709	0.5977	0.5874	0.5977
		ρ	0.1001	0.1014	0.1018	0.5027	0.4996	0.5006	0.9047	0.8989	0.8999
t(10df)	0.0000	μ	0.0007	0.0009	0.0002	0.0007	-0.0027	-0.0007	0.0007	0.0000	-0.0010
	1.0000	σ^2	1.0037	0.9985	0.9989	1.0037	0.9989	0.9991	1.0037	0.9968	1.0019
	0.0000	γ_1	-0.0044	0.0037	0.0007	-0.0044	-0.0549	0.0015	-0.0044	-0.0206	0.0091
	1.0000	γ_2	1.0020	0.9803	1.0193	1.0020	1.1008	0.9956	1.0020	1.0856	1.1312
	0.0000	μ	0.0000	-0.0005	0.0014	0.0004	-0.0005	-0.0001	0.0010	-0.0007	-0.0008
Rayleigh ($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	1.0000	σ^2	1.0002	1.0013	1.0015	1.0008	1.0116	0.9990	1.0023	0.9988	1.0004
	0.6311	γ_1	0.6290	0.6300	0.6293	0.6282	0.6393	0.6311	0.6282	0.6307	0.6325
	0.2451	γ_2	0.2382	0.2368	0.2426	0.2410	0.2599	0.2511	0.2428	0.2503	0.2499
		ρ	0.1016	0.1006	0.0995	0.5100	0.4956	0.5000	0.8747		
	0.0000	μ	0.0007	-0.0001	0.0015	0.0007	-0.0011	0.0002	0.0007	unable to calculate intermediate correlation	unable to calculate intermediate correlation
Pareto	1.0000	σ^2	1.0037	1.0021	0.9986	1.0037	0.9942	1.0001	1.0037		
	0.0000	γ_1	-0.0044	0.0046	-0.0039	-0.0044	0.0061	0.0011	-0.0044		
	1.0000	γ_2	1.0020	1.0072	0.9913	1.0020	0.9542	1.0797	1.0020		
	0.0000	μ	0.0000	-0.0007	0.0017	0.0006	-0.0006	0.0001	0.0015		

$(\theta=10, \alpha=1)$	1.0000	σ^2	0.9972	1.0038	1.0033	1.0014	0.9832	1.0001	1.0038		
	2.8111	γ_1	2.7810	2.8414	2.7761	2.8225	2.7202	2.8091	2.8211		
	14.8286	γ_2	14.1287	15.1964	13.7962	14.9662	13.5747	14.6260	15.1360		
$\chi^2_{(1)}$		ρ	0.1036	0.1009	0.1008	0.5185	0.4987	0.5002	0.9317	0.8999	0.8998
	0.0000	μ	0.0015	0.0005	0.0010	0.0015	0.0017	0.0002	0.0015	-0.0014	0.0003
	1.0000	σ^2	1.0035	0.9981	1.0028	1.0035	1.0027	0.9973	1.0035	1.0007	0.9982
	2.8284	γ_1	2.6052	2.8125	2.8151	2.6052	2.8614	2.8291	2.6052	2.8645	2.8349
$\chi^2_{(1)}$	12.0000	γ_2	12.1789	11.7789	11.7268	12.1789	12.8720	12.2670	12.1789	12.6247	12.1919
	0.0000	μ	0.0000	-0.0007	-0.0010	0.0006	0.0055	0.0005	0.0014	-0.0013	0.0005
	1.0000	σ^2	0.9977	0.9987	0.9988	1.0011	1.0191	1.0012	1.0038	0.9985	0.9963
	2.8284	γ_1	2.5785	2.8292	2.8201	2.6080	2.8619	2.8287	2.6093	2.8361	2.8122
$\chi^2_{(1)}$	12.0000	γ_2	11.6173	11.9688	11.8322	12.1419	12.1614	11.9643	12.1908	12.0751	11.7822
		ρ	0.1032	0.1011	0.0998	0.5172	0.4954	0.4999	0.9292	0.9018	0.8996
	0.0000	μ	0.0015	-0.0010	-0.0006	0.0015	-0.0016	-0.0004	0.0015	0.0030	-0.0001
	1.0000	σ^2	1.0035	0.9969	0.9989	1.0035	0.9902	1.0023	1.0035	1.0100	1.0048
$\chi^2_{(2)}$	2.8284	γ_1	2.6052	2.8320	2.8277	2.6052	2.8186	2.8315	2.6052	2.8405	2.8721
	12.0000	γ_2	12.1789	12.0760	12.0212	12.1789	11.7959	11.9765	12.1789	11.9821	12.8839
	0.0000	μ	0.0000	0.0010	0.0011	0.0006	0.0008	0.0005	0.0014	0.0049	0.0000
	1.0000	σ^2	0.9988	1.0041	1.0019	1.0007	1.0174	1.0009	1.0030	1.0024	1.0002
$\chi^2_{(2)}$	2.0000	γ_1	1.9875	2.0056	1.9900	1.9975	2.0700	2.0049	1.9987	2.0099	2.0043
	6.0000	γ_2	5.9170	6.0396	5.9077	6.0412	6.7673	6.0327	6.0402	5.9185	6.0882
		ρ	0.1029	0.0999	0.0986	0.5158	0.5020	0.5002	0.9270		0.8999
	0.0000	μ	0.0015	-0.0025	0.0026	0.0015	0.0035	-0.0010	0.0015		-0.0013
$\chi^2_{(3)}$	1.0000	σ^2	1.0035	0.9955	1.0052	1.0035	1.0330	0.9983	1.0035		0.9941
	2.8284	γ_1	2.6052	2.8570	2.8037	2.6052	2.9404	2.8282	2.6052	unable to calculate intermediate correlation	2.8426
	12.0000	γ_2	12.1789	12.3417	11.6130	12.1789	13.1244	12.0769	12.1789		12.4473
	0.0000	μ	0.0000	0.0000	0.0016	0.0006	-0.0005	0.0005	0.0013		-0.0009
$\chi^2_{(3)}$	1.0000	σ^2	0.9992	0.9987	1.0013	1.0007	1.0079	1.0026	1.0028		0.9958
	1.6330	γ_1	1.6263	1.6257	1.6358	1.6324	1.6363	1.6266	1.6336		1.6314

	4.0000	γ_2	3.9482	3.9279	4.0497	4.0170	3.9312	3.9318	4.0188		4.0419
$\chi^2_{(1)}$		ρ	0.1027	0.1012	0.0998	0.5149	0.4968	0.4993	0.9256		0.8999
	0.0000	μ	0.0015	-0.0010	0.0008	0.0015	0.0042	-0.0014	0.0015		-0.0004
	1.0000	σ^2	1.0035	0.9961	1.0000	1.0035	1.0055	0.9976	1.0035		1.0008
	2.8284	γ_1	2.6052	2.8093	2.8427	2.6052	2.7642	2.8263	2.6052	unable to calculate intermediate correlation	2.8400
$\chi^2_{(4)}$	12.0000	γ_2	12.1789	11.6986	12.5114	12.1789	11.0551	12.0162	12.1789		12.1691
	0.0000	μ	0.0000	0.0002	0.0011	0.0005	0.0055	-0.0010	0.0012		0.0001
	1.0000	σ^2	0.9994	1.0013	1.0022	1.0007	1.0007	0.9996	1.0028		1.0007
	1.4142	γ_1	1.4086	1.4098	1.4175	1.4128	1.4317	1.4090	1.4138		1.4202
$\chi^2_{(1)}$	3.0000	γ_2	2.9624	2.9600	3.0454	3.0083	3.0605	2.9473	3.0107		3.0179
		ρ	0.1024	0.1009	0.0995	0.5134	0.5011	0.4991	0.9230		0.8996
	0.0000	μ	0.0015	-0.0011	-0.0003	0.0015	0.0001	0.0013	0.0015		-0.0008
	1.0000	σ^2	1.0035	0.9968	0.9995	1.0035	1.0121	1.0025	1.0035		0.9988
$\chi^2_{(8)}$	2.8284	γ_1	2.6052	2.8345	2.8348	2.6052	2.9071	2.8433	2.6052	unable to calculate intermediate correlation	2.8384
	12.0000	γ_2	12.1789	12.0233	12.1209	12.1789	13.1897	12.2198	12.1789		12.1572
	0.0000	μ	0.0000	-0.0002	-0.0016	0.0005	0.0003	0.0002	0.0011		-0.0004
	1.0000	σ^2	0.9998	1.0008	0.9988	1.0009	1.0063	1.0011	1.0028		0.9998
$\chi^2_{(1)}$	1.0000	γ_1	0.9960	1.0008	1.0028	0.9974	1.0062	1.0080	0.9979		0.9965
	1.5000	γ_2	1.4809	1.5022	1.5125	1.4998	1.6899	1.5284	1.5013		1.4834
		ρ	0.1022	0.1001	0.0980	0.5122	0.5038	0.4987	0.9209		0.8999
	0.0000	μ	0.0015	-0.0010	-0.0001	0.0015	0.0011	-0.0013	0.0015		-0.0004
$\chi^2_{(16)}$	1.0000	σ^2	1.0035	0.9989	1.0003	1.0035	0.9954	0.9980	1.0035		1.0009
	2.8284	γ_1	2.6052	2.8379	2.8292	2.6052	2.7700	2.8242	2.6052	unable to calculate intermediate correlation	2.8397
	12.0000	γ_2	12.1789	12.3091	11.9023	12.1789	11.3532	12.2732	12.1789		12.1266
	0.0000	μ	0.0000	0.0008	0.0014	0.0004	0.0019	-0.0006	0.0010		-0.0007
$\chi^2_{(16)}$	1.0000	σ^2	1.0001	0.9995	1.0002	1.0010	0.9966	0.9998	1.0028		1.0005
	0.7071	γ_1	0.7040	0.7111	0.7042	0.7040	0.6943	0.7000	0.7041		0.7086
	0.7500	γ_2	0.7388	0.7558	0.7371	0.7484	0.6617	0.7341	0.7483		0.7573
		ρ	0.1020	0.1001	0.1003	0.5114	0.4975	0.5001	0.9194		

$\chi^2_{(1)}$	0.0000	μ	0.0015	-0.0014	0.0001	0.0015	-0.0043	-0.0008	0.0015		
	1.0000	σ^2	1.0035	1.0004	1.0028	1.0035	0.9928	1.0008	1.0035		
	2.8284	γ_1	2.6052	2.8424	2.8514	2.6052	2.8182	2.8435	2.6052	unable to calculate	unable to calculate
	12.0000	γ_2	12.1789	12.1421	12.3585	12.1789	11.8487	12.4566	12.1789	intermediate correlation	intermediate correlation
$\chi^2_{(32)}$	0.0000	μ	0.0000	0.0010	-0.0008	0.0004	-0.0041	-0.0003	0.0010		
	1.0000	σ^2	1.0002	0.9986	1.0007	1.0011	0.9996	1.0010	1.0029		
	0.5000	γ_1	0.4974	0.5019	0.5024	0.4967	0.4757	0.5036	0.4965		
	0.3750	γ_2	0.3675	0.3696	0.3731	0.3739	0.3405	0.3819	0.3722		
$\chi^2_{(1)}$		ρ	0.1016	0.1013	0.1013	0.5088	0.4997	0.5000	0.8623		
	0.0000	μ	0.0015	-0.0010	0.0013	0.0015	0.0025	0.0004	0.0015		
	1.0000	σ^2	1.0035	0.9983	1.0011	1.0035	1.0172	1.0036	1.0035	unable to calculate	unable to calculate
	2.8284	γ_1	2.6052	2.8338	2.8250	2.6052	2.9109	2.8396	2.6052	intermediate correlation	intermediate correlation
Beta ($\alpha=4, \beta=4$)	12.0000	γ_2	12.1789	12.0399	12.1215	12.1789	13.0196	12.1998	12.1789		
	0.0000	μ	0.0000	0.0002	0.0004	0.0004	0.0035	-0.0002	0.0009		
	1.0000	σ^2	1.0006	1.0000	1.0005	1.0012	1.0043	1.0002	1.0026		
	0.0000	γ_1	-0.0008	0.0010	0.0001	-0.0022	0.0199	0.0007	-0.0031		
$\chi^2_{(1)}$	-0.5455	γ_2	-0.5477	-0.5442	-0.5471	-0.5466	-0.5447	-0.5460	-0.5470		
		ρ	0.1013	0.0996	0.1007	0.4963	0.4981	0.4991	0.7976		
	0.0000	μ	0.0015	-0.0001	0.0004	0.0015	-0.0006	-0.0003	0.0015		
	1.0000	σ^2	1.0035	1.0040	1.0003	1.0035	0.9928	1.0041	1.0035	unable to calculate	unable to calculate
Beta ($\alpha=4, \beta=2$)	2.8284	γ_1	2.6052	2.8335	2.8089	2.6052	2.8083	2.8627	2.6052	intermediate correlation	intermediate correlation
	12.0000	γ_2	12.1789	11.9581	11.6572	12.1789	11.7593	12.5889	12.1789		
	0.0000	μ	-0.0001	0.0007	-0.0004	0.0003	0.0015	0.0004	0.0006		
	1.0000	σ^2	1.0006	1.0013	0.9993	1.0014	1.0028	1.0009	1.0027		
$\chi^2_{(1)}$	-0.4677	γ_1	-0.4681	-0.4721	-0.4694	-0.4695	-0.4788	-0.4666	-0.4704		
	-0.3750	γ_2	-0.3767	-0.3687	-0.3721	-0.3743	-0.3692	-0.3777	-0.3733		
		ρ	0.1012	0.1005	0.1009	0.4807		0.5004	0.7604		
	0.0000	μ	0.0015	-0.0011	0.0004	0.0015	unable to calculate	0.0004	0.0015	unable to calculate	unable to calculate
$\chi^2_{(1)}$	1.0000	σ^2	1.0035	0.9966	1.0008	1.0035		0.9983	1.0035		

Beta ($\alpha=4, \beta=3/2$)	2.8284	γ_1	2.6052	2.8132	2.8350	2.6052	intermediate correlation	2.8400	2.6052	intermediate correlation	intermediate correlation
	12.0000	γ_2	12.1789	11.7739	12.1122	12.1789		12.3225	12.1789		
	0.0000	μ	-0.0001	-0.0002	-0.0007	0.0002		0.0010	0.0005		
	1.0000	σ^2	1.0007	1.0014	0.9992	1.0016		0.9994	1.0029		
	-0.6939	γ_1	-0.6940	-0.6940	-0.6913	-0.6957		-0.6930	-0.6968		
	-0.0686	γ_2	-0.0701	-0.0679	-0.0767	-0.0665		-0.0684	-0.0650		
$\chi^2_{(1)}$		ρ	0.1010	0.1002	0.1014	0.4686		0.5005	0.7328		
	0.0000	μ	0.0015	-0.0017	-0.0012	0.0015		0.0014	0.0015		
	1.0000	σ^2	1.0035	0.9950	0.9942	1.0035		1.0019	1.0035		
	2.8284	γ_1	2.6052	2.8175	2.8193	2.6052	unable to calculate intermediate correlation	2.8212	2.6052	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	12.0000	γ_2	12.1789	11.7807	11.9213	12.1789		11.9683	12.1789		
	0.0000	μ	-0.0001	-0.0012	0.0004	0.0002		0.0013	0.0004		
Beta ($\alpha=4, \beta=5/4$)	1.0000	σ^2	1.0008	0.9998	0.9980	1.0017		1.0002	1.0031		
	-0.8482	γ_1	-0.8481	-0.8456	-0.8480	-0.8500		-0.8494	-0.8512		
	0.2210	γ_2	0.2196	0.2126	0.2248	0.2245		0.2244	0.2260		
		ρ	0.1015	0.0991	0.0987	0.5076	0.4974	0.5001	0.8281		
	0.0000	μ	0.0015	0.0009	0.0010	0.0015	-0.0021	0.0000	0.0015		
	1.0000	σ^2	1.0035	1.0036	1.0080	1.0035	0.9925	1.0031	1.0035		
Weibull ($\alpha=6, \beta=10$)	2.8284	γ_1	2.6052	2.8254	2.8445	2.6052	2.8128	2.8391	2.6052	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	12.0000	γ_2	12.1789	11.9385	12.1381	12.1789	11.8417	12.3101	12.1789		
	0.0000	μ	-0.0001	-0.0007	-0.0010	0.0003	0.0022	0.0006	0.0006		
	1.0000	σ^2	1.0006	1.0004	1.0016	1.0016	0.9969	0.9993	1.0032		
	-0.3733	γ_1	-0.3743	-0.3718	-0.3779	-0.3760	-0.3763	-0.3724	-0.3768		
	0.0355	γ_2	0.0335	0.0357	0.0444	0.0390	0.0436	0.0310	0.0383		
$\chi^2_{(1)}$		ρ	0.1022	0.0991	0.0989	0.5127	0.5015	0.5014	0.9214		0.9001
	0.0000	μ	0.0015	-0.0011	0.0000	0.0015	0.0019	0.0003	0.0015	unable to calculate intermediate correlation	0.0011
	1.0000	σ^2	1.0035	1.0001	0.9984	1.0035	1.0054	1.0037	1.0035		1.0023
	2.8284	γ_1	2.6052	2.8443	2.8230	2.6052	2.8396	2.8448	2.6052		2.8365
	12.0000	γ_2	12.1789	12.2139	11.9034	12.1789	11.7431	12.3462	12.1789		12.0381
	0.0000	μ	0.0000	-0.0005	0.0002	0.0005	-0.0026	-0.0012	0.0012		0.0011

$(\alpha=\beta=10)$	1.0000	σ^2	1.0001	1.0005	0.9995	1.0008	0.9999	1.0001	1.0025	0.9999
	0.8222	γ_1	0.8196	0.8223	0.8268	0.8194	0.8103	0.8140	0.8193	0.8306
	0.6000	γ_2	0.5906	0.5994	0.6256	0.5957	0.5982	0.5878	0.5963	0.5988
$\chi^2_{(1)}$		ρ	0.1021	0.1006	0.0997	0.5118	0.5015	0.5015	0.9200	
	0.0000	μ	0.0015	-0.0008	-0.0004	0.0015	0.0015	0.0001	0.0015	
	1.0000	σ^2	1.0035	1.0001	0.9966	1.0035	1.0119	1.0017	1.0035	
	2.8284	γ_1	2.6052	2.8342	2.8290	2.6052	2.8127	2.8496	2.6052	unable to calculate intermediate correlation
	12.0000	γ_2	12.1789	12.0710	12.0114	12.1789	11.5382	12.4986	12.1789	unable to calculate intermediate correlation
Rayleigh $(\alpha=1/2, \mu=\sqrt{\pi/2})$	0.0000	μ	0.0000	0.0003	-0.0005	0.0005	-0.0008	0.0012	0.0011	
	1.0000	σ^2	1.0002	1.0011	0.9987	1.0010	1.0088	1.0012	1.0026	
	0.6311	γ_1	0.6289	0.6324	0.6322	0.6282	0.6388	0.6361	0.6278	
	0.2451	γ_2	0.2383	0.2483	0.2492	0.2416	0.2599	0.2441	0.2410	
		ρ	0.1035	0.1010	0.0997	0.5180	0.4963	0.5017	0.9306	0.9007
$\chi^2_{(1)}$	0.0000	μ	0.0015	0.0004	-0.0009	0.0015	0.0004	0.0006	0.0015	-0.0052
	1.0000	σ^2	1.0032	1.0017	0.9974	1.0032	1.0113	1.0018	1.0032	0.9961
	2.8284	γ_1	2.3379	2.8372	2.8322	2.3379	2.9227	2.8492	2.3379	2.8445
	12.0000	γ_2	9.1130	12.1177	12.1093	9.1130	13.6693	12.4011	9.1130	12.3042
	0.0000	μ	0.0000	0.0000	-0.0011	0.0006	0.0082	0.0011	0.0014	-0.0038
Pareto $(\theta=10, \alpha=1)$	1.0000	σ^2	0.9977	1.0005	0.9953	1.0011	1.0172	0.9985	1.0038	0.9936
	2.8111	γ_1	2.5649	2.8141	2.8008	2.5936	2.8780	2.7656	2.5951	2.8107
	14.8286	γ_2	11.4590	14.7555	14.4641	11.9648	16.0790	13.6252	12.0144	14.7321
		ρ	0.1029	0.0992	0.0989	0.5159	0.4978	0.4993	0.9269	0.8998
	0.0000	μ	0.0015	-0.0012	-0.0001	0.0015	0.0019	-0.0017	0.0015	-0.0008
$\chi^2_{(2)}$	1.0000	σ^2	1.0029	0.9982	0.9976	1.0029	1.0016	0.9975	1.0029	0.9997
	2.0000	γ_1	1.9959	1.9996	1.9882	1.9959	2.0138	2.0051	1.9959	2.0208
	6.0000	γ_2	6.0176	6.0411	5.9318	6.0176	6.2605	6.0609	6.0176	6.2851
	0.0000	μ	0.0000	0.0008	-0.0004	0.0006	0.0056	-0.0009	0.0014	-0.0008
	1.0000	σ^2	0.9988	1.0008	0.9982	1.0006	1.0158	0.9971	1.0030	0.9985
$\chi^2_{(2)}$	2.0000	γ_1	1.9874	1.9954	1.9909	1.9973	2.0310	1.9978	1.9987	2.0040
										1.9854

	6.0000	γ_2	5.9160	5.9579	5.8946	6.0395	6.1128	6.0123	6.0408	6.0317	5.8954
$\chi^2_{(2)}$		ρ	0.1026	0.1005	0.1005	0.5145	0.5028	0.5009	0.9247	0.9004	0.9000
	0.0000	μ	0.0015	0.0006	0.0003	0.0015	-0.0003	0.0008	0.0015	0.0019	-0.0007
	1.0000	σ^2	1.0029	0.9991	0.9989	1.0029	0.9979	1.0033	1.0029	1.0073	1.0014
	2.0000	γ_1	1.9959	1.9862	1.9992	1.9959	2.0039	2.0090	1.9959	2.0374	2.0111
$\chi^2_{(3)}$	6.0000	γ_2	6.0176	5.8673	6.0437	6.0176	6.0052	6.0975	6.0176	6.1701	6.2089
	0.0000	μ	0.0000	0.0010	0.0005	0.0005	0.0022	-0.0009	0.0012	0.0017	-0.0004
	1.0000	σ^2	0.9992	1.0013	0.9979	1.0006	1.0064	1.0014	1.0028	1.0136	1.0015
	1.6330	γ_1	1.6262	1.6342	1.6185	1.6323	1.6202	1.6282	1.6337	1.6768	1.6360
$\chi^2_{(2)}$	4.0000	γ_2	3.9477	4.0144	3.8914	4.0159	3.9974	3.9798	4.0195	4.1904	4.0141
		ρ	0.1024	0.1010	0.1008	0.5137	0.4973	0.4991	0.9234	0.9005	0.9000
	0.0000	μ	0.0015	0.0005	-0.0010	0.0015	-0.0071	-0.0008	0.0015	0.0005	0.0001
	1.0000	σ^2	1.0029	1.0002	0.9957	1.0029	0.9811	0.9971	1.0029	0.9995	1.0025
$\chi^2_{(4)}$	2.0000	γ_1	1.9959	1.9898	2.0003	1.9959	1.9700	2.0002	1.9959	2.0062	2.0138
	6.0000	γ_2	6.0176	5.8493	6.0070	6.0176	5.7791	6.1640	6.0176	6.2975	6.1344
	0.0000	μ	0.0000	0.0008	0.0000	0.0005	-0.0052	0.0005	0.0012	0.0028	-0.0001
	1.0000	σ^2	0.9994	1.0039	0.9981	1.0007	0.9955	0.9992	1.0027	1.0015	1.0008
$\chi^2_{(2)}$	1.4142	γ_1	1.4085	1.4215	1.4117	1.4127	1.3943	1.4078	1.4139	1.3918	1.4206
	3.0000	γ_2	2.9621	3.0629	2.9700	3.0075	2.9790	2.9456	3.0115	2.7686	3.0399
		ρ	0.1021	0.1015	0.1002	0.5121	0.5002	0.4991	0.9209	0.8999	0.8998
	0.0000	μ	0.0015	-0.0001	0.0005	0.0015	-0.0006	0.0019	0.0015	-0.0010	0.0001
$\chi^2_{(8)}$	1.0000	σ^2	1.0029	1.0041	1.0000	1.0029	1.0059	1.0041	1.0029	0.9990	1.0028
	2.0000	γ_1	1.9959	2.0202	1.9843	1.9959	2.0300	1.9974	1.9959	1.9964	2.0179
	6.0000	γ_2	6.0176	6.2314	5.8682	6.0176	6.1190	5.8790	6.0176	5.9256	6.2187
	0.0000	μ	0.0000	0.0013	0.0012	0.0005	0.0004	0.0014	0.0011	-0.0032	0.0004
$\chi^2_{(8)}$	1.0000	σ^2	0.9998	1.0015	1.0005	1.0008	1.0104	0.9997	1.0027	0.9988	1.0013
	1.0000	γ_1	0.9960	0.9950	1.0033	0.9973	1.0322	0.9983	0.9981	0.9877	1.0012
	1.5000	γ_2	1.4807	1.4672	1.5173	1.4993	1.5849	1.4890	1.5021	1.4899	1.5000

$\chi^2_{(2)}$		ρ	0.1019	0.1019	0.0996	0.5110	0.4976	0.5004	0.9189	0.9004	0.9001
	0.0000	μ	0.0015	0.0000	0.0005	0.0015	0.0033	0.0010	0.0015	-0.0021	0.0004
	1.0000	σ^2	1.0029	1.0004	1.0000	1.0029	1.0156	1.0036	1.0029	1.0018	0.9990
	2.0000	γ_1	1.9959	1.9993	1.9921	1.9959	2.0174	2.0243	1.9959	1.9908	2.0011
$\chi^2_{(16)}$	6.0000	γ_2	6.0176	5.9949	5.9043	6.0176	6.0280	6.4117	6.0176	5.8946	6.0796
	0.0000	μ	0.0000	-0.0017	0.0006	0.0004	-0.0002	0.0011	0.0010	-0.0030	0.0007
	1.0000	σ^2	1.0001	0.9961	1.0006	1.0010	0.9993	1.0019	1.0028	1.0017	0.9997
	0.7071	γ_1	0.7040	0.7049	0.7065	0.7040	0.6990	0.7074	0.7043	0.7224	0.7062
$\chi^2_{(2)}$	0.7500	γ_2	0.7387	0.7431	0.7474	0.7481	0.7169	0.7619	0.7493	0.7926	0.7399
		ρ	0.1017	0.1026	0.1007	0.5102	0.4995	0.5008	0.9175	0.9009	0.9003
	0.0000	μ	0.0015	-0.0001	-0.0002	0.0015	-0.0007	0.0004	0.0015	0.0027	0.0019
	1.0000	σ^2	1.0029	0.9988	1.0011	1.0029	0.9992	1.0036	1.0029	1.0011	1.0030
$\chi^2_{(32)}$	2.0000	γ_1	1.9959	2.0045	1.9981	1.9959	1.9741	2.0131	1.9959	1.9745	1.9972
	6.0000	γ_2	6.0176	6.0981	5.9074	6.0176	5.8560	6.2144	6.0176	5.7063	5.9555
	0.0000	μ	0.0000	0.0014	0.0000	0.0004	0.0001	-0.0011	0.0009	0.0044	0.0021
	1.0000	σ^2	1.0002	1.0014	0.9992	1.0011	1.0055	1.0011	1.0028	1.0020	1.0013
$\chi^2_{(2)}$	0.5000	γ_1	0.4974	0.4991	0.5019	0.4968	0.5010	0.4978	0.4967	0.5175	0.5049
	0.3750	γ_2	0.3674	0.3734	0.3804	0.3736	0.3602	0.3864	0.3737	0.3463	0.3705
		ρ	0.1014	0.1009	0.1003	0.5078	0.4985	0.4987	0.8957		
	0.0000	μ	0.0015	0.0012	0.0011	0.0015	0.0006	0.0005	0.0015		
Beta ($\alpha=4, \beta=4$)	1.0000	σ^2	1.0029	0.9994	1.0046	1.0029	0.9936	1.0008	1.0029		
	2.0000	γ_1	1.9959	1.9873	2.0125	1.9959	1.9538	2.0126	1.9959	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	6.0000	γ_2	6.0176	5.8675	6.1487	6.0176	5.5905	6.1398	6.0176		
	0.0000	μ	0.0000	-0.0009	0.0010	0.0004	0.0029	0.0018	0.0009		
$\chi^2_{(2)}$	1.0000	σ^2	1.0005	1.0027	1.0007	1.0011	0.9993	1.0000	1.0026		
	0.0000	γ_1	-0.0008	0.0022	-0.0015	-0.0021	0.0059	0.0064	-0.0031		
	-0.5455	γ_2	-0.5477	-0.5455	-0.5473	-0.5466	-0.5406	-0.5441	-0.5470		
		ρ	0.1011	0.1006	0.1011	0.5055	0.4992	0.4999	0.8346		
$\chi^2_{(2)}$	0.0000	μ	0.0015	0.0010	0.0015	0.0015	0.0013	-0.0002	0.0015	unable to	unable to

	1.0000	σ^2	1.0029	1.0021	1.0034	1.0029	1.0060	1.0049	1.0029	calculate	calculate
	2.0000	γ_1	1.9959	1.9940	1.9997	1.9959	1.9923	2.0178	1.9959	intermediate	intermediate
	6.0000	γ_2	6.0176	5.8860	6.0506	6.0176	5.7699	6.1813	6.0176	correlation	correlation
Beta	0.0000	μ	-0.0001	-0.0021	-0.0006	0.0003	0.0008	-0.0014	0.0006		
($\alpha=4, \beta=2$)	1.0000	σ^2	1.0006	1.0018	1.0021	1.0014	0.9997	1.0014	1.0027		
	-0.4677	γ_1	-0.4681	-0.4655	-0.4656	-0.4694	-0.4749	-0.4739	-0.4704		
	-0.3750	γ_2	-0.3768	-0.3785	-0.3848	-0.3743	-0.3669	-0.3770	-0.3733		
		ρ	0.1009	0.1001	0.1002	0.5016	0.4995	0.5005	0.7995		
$\chi^2(z)$	0.0000	μ	0.0015	-0.0006	-0.0017	0.0015	-0.0019	0.0010	0.0015		
	1.0000	σ^2	1.0029	0.9991	0.9947	1.0029	1.0035	1.0005	1.0029		
	2.0000	γ_1	1.9959	2.0010	2.0049	1.9959	2.0814	2.0072	1.9959	unable to	unable to
	6.0000	γ_2	6.0176	6.0089	6.0552	6.0176	7.9441	6.1586	6.0176	calculate	calculate
Beta	0.0000	μ	-0.0001	0.0001	0.0002	0.0002	0.0011	0.0002	0.0005	intermediate	intermediate
($\alpha=4, \beta=3/2$)	1.0000	σ^2	1.0007	1.0021	1.0003	1.0016	0.9971	1.0016	1.0030	correlation	correlation
	-0.6939	γ_1	-0.6940	-0.6935	-0.6959	-0.6957	-0.6924	-0.6899	-0.6968		
	-0.0686	γ_2	-0.0702	-0.0704	-0.0641	-0.0665	-0.0636	-0.0789	-0.0650		
		ρ	0.1008	0.0997	0.1004	0.4900	0.5002	0.4996	0.7731		
$\chi^2(z)$	0.0000	μ	0.0015	0.0026	0.0004	0.0015	0.0039	0.0002	0.0015		
	1.0000	σ^2	1.0029	1.0048	1.0005	1.0029	1.0063	1.0021	1.0029		
	2.0000	γ_1	1.9959	1.9861	1.9999	1.9959	2.0120	2.0079	1.9959	unable to	unable to
	6.0000	γ_2	6.0176	5.8290	5.9919	6.0176	5.9518	6.1066	6.0176	calculate	calculate
Beta	0.0000	μ	-0.0001	0.0018	0.0018	0.0002	0.0041	0.0004	0.0004	intermediate	intermediate
($\alpha=4, \beta=5/4$)	1.0000	σ^2	1.0008	0.9990	0.9994	1.0017	0.9963	0.9980	1.0031	correlation	correlation
	-0.8482	γ_1	-0.8481	-0.8490	-0.8515	-0.8500	-0.8453	-0.8474	-0.8512		
	0.2210	γ_2	0.2195	0.2217	0.2286	0.2245	0.2229	0.2220	0.2260		
		ρ	0.1012	0.1000	0.1011	0.5065	0.4980	0.5004	0.8611		
$\chi^2(z)$	0.0000	μ	0.0015	0.0009	0.0014	0.0015	-0.0028	0.0007	0.0015	unable to	unable to
	1.0000	σ^2	1.0029	1.0003	1.0003	1.0029	0.9842	1.0009	1.0029	calculate	calculate
	2.0000	γ_1	1.9959	2.0036	1.9983	1.9959	1.9807	2.0039	1.9959	intermediate	intermediate
	6.0000	γ_2	6.0176	6.1923	6.0442	6.0176	6.1062	6.0984	6.0176	correlation	correlation

Weibull ($\alpha=6, \beta=10$)	0.0000	μ	-0.0001	0.0008	-0.0002	0.0003	-0.0002	0.0001	0.0006		
	1.0000	σ^2	1.0006	1.0010	1.0003	1.0015	0.9913	0.9994	1.0032		
	-0.3733	γ_1	-0.3743	-0.3740	-0.3726	-0.3760	-0.3903	-0.3725	-0.3768		
	0.0355	γ_2	0.0334	0.0338	0.0365	0.0388	0.0420	0.0262	0.0383		
$\chi^2_{(2)}$		ρ	0.1020	0.0995	0.1024	0.5115	0.5001	0.5022	0.9195	0.8997	0.9000
	0.0000	μ	0.0015	-0.0003	-0.0008	0.0015	0.0033	-0.0001	0.0015	0.0009	-0.0005
	1.0000	σ^2	1.0029	0.9996	0.9985	1.0029	1.0009	0.9983	1.0029	1.0143	0.9990
	2.0000	γ_1	1.9959	2.0027	1.9989	1.9959	2.0255	1.9895	1.9959	2.0813	1.9977
	6.0000	γ_2	6.0176	6.0454	5.9858	6.0176	6.3781	5.9266	6.0176	7.2309	6.0811
Gamma ($\alpha=\beta=10$)	0.0000	μ	0.0000	-0.0016	0.0016	0.0005	-0.0018	0.0003	0.0011	0.0000	0.0000
	1.0000	σ^2	1.0001	0.9989	0.9992	1.0008	0.9878	0.9992	1.0024	1.0087	0.9991
	0.8222	γ_1	0.8196	0.8245	0.8196	0.8193	0.8171	0.8274	0.8195	0.8238	0.8133
	0.6000	γ_2	0.5906	0.6086	0.6029	0.5954	0.5940	0.5895	0.5971	0.6532	0.6768
$\chi^2_{(2)}$		ρ	0.1018	0.0977	0.0997	0.5107	0.4997	0.4986	0.9181	0.9004	0.9001
	0.0000	μ	0.0015	0.0001	-0.0004	0.0015	0.0017	-0.0001	0.0015	-0.0004	0.0001
	1.0000	σ^2	1.0029	0.9995	0.9986	1.0029	0.9971	1.0040	1.0029	0.9975	1.0018
	2.0000	γ_1	1.9959	2.0026	2.0001	1.9959	1.9729	2.0180	1.9959	1.9896	2.0036
	6.0000	γ_2	6.0176	5.9756	6.0109	6.0176	5.7846	6.1953	6.0176	5.9530	6.0803
Rayleigh ($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	0.0000	μ	0.0000	0.0014	-0.0012	0.0005	0.0042	-0.0013	0.0010	0.0002	-0.0007
	1.0000	σ^2	1.0002	1.0003	0.9976	1.0009	1.0056	0.9992	1.0025	0.9985	1.0022
	0.6311	γ_1	0.6289	0.6289	0.6309	0.6282	0.6446	0.6235	0.6280	0.6304	0.6263
	0.2451	γ_2	0.2382	0.2363	0.2425	0.2414	0.2486	0.2371	0.2419	0.2488	0.2292
$\chi^2_{(2)}$		ρ	0.1034	0.1006	0.1004	0.5175	0.4997	0.5012	0.9297	0.8999	
	0.0000	μ	0.0015	0.0000	0.0013	0.0015	0.0060	0.0004	0.0015	-0.0003	unable to calculate intermediate correlation
	1.0000	σ^2	1.0029	1.0021	1.0013	1.0029	1.0164	0.9996	1.0029	1.0034	
	2.0000	γ_1	1.9959	2.0128	1.9957	1.9959	2.0165	2.0057	1.9959	2.0269	
	6.0000	γ_2	6.0176	6.1646	5.9261	6.0176	5.9655	6.1093	6.0176	6.3216	
Pareto ($\theta=10, \alpha=1$)	0.0000	μ	0.0000	-0.0003	-0.0008	0.0006	0.0018	-0.0006	0.0014	-0.0002	
	1.0000	σ^2	0.9973	1.0006	0.9942	1.0013	1.0153	0.9975	1.0041	1.0186	

	2.8111	γ_1	2.7817	2.8116	2.7594	2.8214	2.9310	2.7820	2.8241	2.8667	
	14.8286	γ_2	14.1373	14.7442	13.6310	14.9292	16.9216	14.0949	15.0910	15.6473	
$\chi^2_{(3)}$		ρ	0.1022	0.1011	0.1004	0.5130	0.4979	0.5009	0.9224	0.8998	0.9003
	0.0000	μ	0.0014	0.0015	0.0004	0.0014	0.0020	0.0005	0.0014	-0.0007	0.0003
	1.0000	σ^2	1.0029	1.0010	0.9979	1.0029	1.0014	1.0000	1.0029	0.9992	1.0009
	1.6330	γ_1	1.6316	1.6296	1.6216	1.6316	1.6420	1.6425	1.6316	1.6481	1.6411
$\chi^2_{(3)}$	4.0000	γ_2	4.0038	4.0140	3.8944	4.0038	4.1335	4.0383	4.0038	4.1756	4.1183
	0.0000	μ	0.0000	0.0005	0.0014	0.0005	0.0054	-0.0003	0.0012	-0.0007	0.0007
	1.0000	σ^2	0.9992	1.0009	1.0031	1.0006	1.0139	1.0019	1.0028	0.9984	1.0007
	1.6330	γ_1	1.6262	1.6319	1.6337	1.6322	1.6620	1.6398	1.6338	1.6362	1.6324
$\chi^2_{(3)}$	4.0000	γ_2	3.9471	4.0014	3.9988	4.0148	4.0944	4.0369	4.0199	4.0224	3.9468
		ρ	0.1020	0.0998	0.0989	0.5122	0.4981	0.4988	0.9210	0.8991	0.9001
	0.0000	μ	0.0014	0.0002	0.0005	0.0014	0.0020	-0.0004	0.0014	-0.0002	0.0004
	1.0000	σ^2	1.0029	1.0008	1.0024	1.0029	1.0014	0.9993	1.0029	0.9966	1.0012
$\chi^2_{(4)}$	1.6330	γ_1	1.6316	1.6252	1.6430	1.6316	1.6418	1.6422	1.6316	1.6458	1.6412
	4.0000	γ_2	4.0038	3.9037	4.1330	4.0038	4.1310	4.0968	4.0038	4.5037	4.1116
	0.0000	μ	0.0000	-0.0011	-0.0011	0.0005	0.0053	-0.0004	0.0012	-0.0021	0.0001
	1.0000	σ^2	0.9994	0.9960	0.9959	1.0006	1.0128	0.9985	1.0027	0.9959	1.0022
$\chi^2_{(3)}$	1.4142	γ_1	1.4085	1.4113	1.4016	1.4126	1.4418	1.4129	1.4140	1.4139	1.4123
	3.0000	γ_2	2.9617	2.9988	2.9145	3.0067	3.0825	3.0025	3.0119	3.1197	2.9923
		ρ	0.1017	0.1007	0.1002	0.5106	0.4993	0.5005	0.9183	0.9005	0.9000
	0.0000	μ	0.0014	0.0003	-0.0010	0.0014	-0.0051	-0.0005	0.0014	0.0007	0.0004
$\chi^2_{(8)}$	1.0000	σ^2	1.0029	1.0008	0.9969	1.0029	0.9884	1.0001	1.0029	1.0082	1.0022
	1.6330	γ_1	1.6316	1.6341	1.6290	1.6316	1.6219	1.6409	1.6316	1.6665	1.6436
	4.0000	γ_2	4.0038	4.0265	3.9661	4.0038	4.0069	4.0793	4.0038	4.2820	4.1252
	0.0000	μ	0.0000	-0.0007	-0.0004	0.0004	0.0007	-0.0002	0.0010	-0.0023	-0.0005
$\chi^2_{(8)}$	1.0000	σ^2	0.9998	1.0001	1.0008	1.0008	1.0010	0.9998	1.0026	1.0039	1.0013
	1.0000	γ_1	0.9960	1.0026	1.0015	0.9972	0.9982	1.0052	0.9982	1.0104	0.9998
	1.5000	γ_2	1.4806	1.4962	1.4967	1.4988	1.4735	1.5126	1.5027	1.5919	1.5026

		ρ	0.1015	0.1016	0.1011	0.5094	0.5027	0.4995	0.9163	0.9001	0.8996
$\chi^2_{(3)}$	0.0000	μ	0.0014	-0.0010	0.0000	0.0014	0.0004	-0.0005	0.0014	0.0032	-0.0008
	1.0000	σ^2	1.0029	0.9961	1.0021	1.0029	0.9962	0.9986	1.0029	0.9971	0.9962
	1.6330	γ_1	1.6316	1.6269	1.6356	1.6316	1.6538	1.6304	1.6316	1.6280	1.6328
	4.0000	γ_2	4.0038	3.9624	4.0250	4.0038	4.0940	4.0743	4.0038	3.9837	4.1086
$\chi^2_{(16)}$	0.0000	μ	0.0000	-0.0003	0.0009	0.0004	-0.0025	-0.0009	0.0010	0.0011	-0.0001
	1.0000	σ^2	1.0001	0.9972	1.0007	1.0009	0.9943	0.9982	1.0027	0.9992	0.9977
	0.7071	γ_1	0.7040	0.7051	0.7013	0.7040	0.7046	0.7045	0.7044	0.7144	0.7014
	0.7500	γ_2	0.7387	0.7441	0.7222	0.7478	0.7802	0.7430	0.7499	0.7715	0.7323
		ρ	0.1014	0.0986	0.0995	0.5086	0.4992	0.5001	0.9148	0.8994	0.8997
$\chi^2_{(3)}$	0.0000	μ	0.0014	0.0008	-0.0011	0.0014	-0.0013	0.0002	0.0014	0.0057	0.0008
	1.0000	σ^2	1.0029	1.0048	0.9998	1.0029	1.0023	0.9987	1.0029	1.0075	1.0012
	1.6330	γ_1	1.6316	1.6446	1.6364	1.6316	1.6318	1.6391	1.6316	1.6400	1.6338
	4.0000	γ_2	4.0038	4.0679	4.0419	4.0038	4.0097	4.0947	4.0038	3.9517	3.9538
$\chi^2_{(32)}$	0.0000	μ	0.0000	-0.0008	0.0008	0.0004	-0.0052	0.0005	0.0009	0.0043	0.0008
	1.0000	σ^2	1.0002	1.0001	0.9988	1.0010	0.9936	0.9985	1.0027	1.0013	0.9996
	0.5000	γ_1	0.4974	0.4974	0.5007	0.4967	0.5006	0.5000	0.4968	0.5073	0.4997
	0.3750	γ_2	0.3674	0.3747	0.3735	0.3734	0.3764	0.3594	0.3744	0.3705	0.3628
		ρ	0.1010	0.0990	0.1007	0.5062	0.4983	0.5006	0.9103	0.9010	0.8995
$\chi^2_{(3)}$	0.0000	μ	0.0014	0.0003	0.0013	0.0014	-0.0016	-0.0006	0.0014	-0.0044	0.0008
	1.0000	σ^2	1.0029	0.9989	1.0001	1.0029	0.9881	0.9978	1.0029	0.9926	1.0038
	1.6330	γ_1	1.6316	1.6301	1.6267	1.6316	1.6141	1.6338	1.6316	1.5907	1.6506
	4.0000	γ_2	4.0038	3.9719	3.9311	4.0038	3.8603	4.0278	4.0038	3.8502	4.1860
Beta	0.0000	μ	0.0000	0.0003	-0.0005	0.0004	-0.0030	0.0005	0.0009	-0.0040	0.0007
($\alpha=4, \beta=4$)	1.0000	σ^2	1.0005	0.9986	0.9992	1.0011	1.0024	0.9989	1.0025	1.0021	1.0001
	0.0000	γ_1	-0.0008	0.0024	-0.0031	-0.0021	0.0017	0.0013	-0.0031	-0.0148	0.0010
	-0.5455	γ_2	-0.5477	-0.5448	-0.5458	-0.5467	-0.5431	-0.5451	-0.5465	-0.5588	-0.5420
		ρ	0.1007	0.1007	0.1001	0.5040	0.5009	0.5004	0.8746		
$\chi^2_{(3)}$	0.0000	μ	0.0014	-0.0002	-0.0008	0.0014	-0.0019	0.0006	0.0014	unable to	unable to

	1.0000	σ^2	1.0029	0.9978	1.0011	1.0029	0.9982	1.0012	1.0029	calculate intermediate correlation	calculate intermediate correlation
	1.6330	γ_1	1.6316	1.6303	1.6424	1.6316	1.6031	1.6365	1.6316		
	4.0000	γ_2	4.0038	3.9933	4.0637	4.0038	3.6890	4.0340	4.0038		
Beta	0.0000	μ	-0.0001	-0.0003	-0.0005	0.0003	0.0000	0.0011	0.0006		
($\alpha=4, \beta=2$)	1.0000	σ^2	1.0006	0.9989	1.0017	1.0013	0.9994	0.9997	1.0027		
	-0.4677	γ_1	-0.4680	-0.4679	-0.4666	-0.4694	-0.4624	-0.4669	-0.4704		
	-0.3750	γ_2	-0.3768	-0.3716	-0.3767	-0.3744	-0.3667	-0.3719	-0.3733		
		ρ	0.1005	-0.0993	0.0989	0.5028	0.5003	0.4994	0.8436		
$\chi^2_{(3)}$	0.0000	μ	0.0014	-0.0008	-0.0010	0.0014	0.0039	0.0006	0.0014		
	1.0000	σ^2	1.0029	0.9973	1.0021	1.0029	1.0070	0.9983	1.0029		
	1.6330	γ_1	1.6316	1.6310	1.6383	1.6316	1.6556	1.6288	1.6316	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	4.0000	γ_2	4.0038	4.0003	4.0343	4.0038	4.1911	3.9651	4.0038		
Beta	0.0000	μ	-0.0001	0.0003	-0.0011	0.0002	0.0035	0.0012	0.0005		
($\alpha=4, \beta=3/2$)	1.0000	σ^2	1.0007	0.9986	1.0024	1.0015	0.9992	0.9987	1.0030		
	-0.6939	γ_1	-0.6939	-0.6931	-0.6934	-0.6956	-0.6884	-0.6940	-0.6968		
	-0.0686	γ_2	-0.0702	-0.0712	-0.0715	-0.0666	-0.0666	-0.0703	-0.0650		
		ρ	0.1004	0.0986	0.0989	0.5019	0.5002	0.4997	0.8200		
$\chi^2_{(3)}$	0.0000	μ	0.0014	-0.0001	0.0008	0.0014	-0.0006	0.0000	0.0014		
	1.0000	σ^2	1.0029	0.9981	1.0020	1.0029	1.0069	1.0008	1.0029		
	1.6330	γ_1	1.6316	1.6233	1.6286	1.6316	1.6591	1.6400	1.6316	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	4.0000	γ_2	4.0038	3.9471	3.9526	4.0038	4.3829	4.0860	4.0038		
Beta	0.0000	μ	-0.0001	-0.0001	0.0006	0.0002	-0.0003	0.0007	0.0004		
($\alpha=4, \beta=5/4$)	1.0000	σ^2	1.0008	0.9993	0.9976	1.0017	1.0027	0.9982	1.0031		
	-0.8482	γ_1	-0.8480	-0.8463	-0.8482	-0.8500	-0.8609	-0.8508	-0.8512		
	0.2210	γ_2	0.2195	0.2150	0.2240	0.2244	0.2389	0.2300	0.2260		
		ρ	0.1008	0.0998	0.0992	0.5049	0.4970	0.4995	0.8981		
$\chi^2_{(3)}$	0.0000	μ	0.0014	0.0004	0.0003	0.0014	0.0007	0.0009	0.0014	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	1.0000	σ^2	1.0029	0.9991	0.9986	1.0029	0.9973	1.0024	1.0029		
	1.6330	γ_1	1.6316	1.6319	1.6275	1.6316	1.6453	1.6441	1.6316		
	4.0000	γ_2	4.0038	4.0272	3.9343	4.0038	4.1532	4.1008	4.0038		

Weibull ($\alpha=6, \beta=10$)	0.0000	μ	-0.0001	0.0008	-0.0001	0.0002	0.0036	0.0010	0.0006		
	1.0000	σ^2	1.0006	0.9992	1.0010	1.0015	1.0033	0.9992	1.0032		
	-0.3733	γ_1	-0.3743	-0.3728	-0.3753	-0.3759	-0.3825	-0.3734	-0.3768		
	0.0355	γ_2	0.0334	0.0310	0.0322	0.0386	0.0885	0.0409	0.0383		
$\chi^2_{(3)}$		ρ	0.1016	0.1007	0.0985	0.5103	0.5004	0.5010	0.9176	0.8986	0.9001
	0.0000	μ	0.0014	-0.0009	0.0009	0.0014	0.0001	0.0012	0.0014	-0.0027	0.0008
	1.0000	σ^2	1.0029	0.9961	1.0030	1.0029	1.0029	1.0041	1.0029	0.9899	1.0068
	1.6330	γ_1	1.6316	1.6322	1.6429	1.6316	1.6390	1.6367	1.6316	1.6319	1.6502
Gamma ($\alpha=\beta=10$)	4.0000	γ_2	4.0038	4.0018	4.0716	4.0038	4.0318	4.0265	4.0038	3.9765	4.1768
	0.0000	μ	0.0001	0.0002	0.0001	0.0005	0.0009	0.0003	0.0011	-0.0006	-0.0002
	1.0000	σ^2	1.0001	0.9983	1.0012	1.0007	0.9985	1.0022	1.0022	0.9920	1.0035
	0.8222	γ_1	0.8795	0.8221	0.8214	0.8792	0.8081	0.8209	0.8794	0.8231	0.8230
$\chi^2_{(3)}$	0.6000	γ_2	0.5908	0.5976	0.5849	0.5946	0.5731	0.5859	0.5969	0.5836	0.6129
		ρ	0.1015	0.1002	0.1001	0.5091	0.4972	0.5004	0.9156	0.8998	0.8997
	0.0000	μ	0.0014	0.0005	-0.0002	0.0014	0.0004	-0.0006	0.0014	-0.0036	0.0010
	1.0000	σ^2	1.0029	1.0003	1.0013	1.0029	0.9963	0.9974	1.0029	0.9938	0.9998
Rayleigh ($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	1.6330	γ_1	1.6316	1.6210	1.6423	1.6316	1.6410	1.6292	1.6316	1.6236	1.6391
	4.0000	γ_2	4.0038	3.9069	4.0569	4.0038	4.0701	4.0108	4.0038	3.9562	4.0960
	0.0000	μ	0.0000	0.0013	-0.0002	0.0004	0.0028	0.0004	0.0010	-0.0028	0.0017
	1.0000	σ^2	1.0002	0.9999	0.9992	1.0009	0.9953	1.0002	1.0024	1.0025	0.9998
$\chi^2_{(3)}$	0.6311	γ_1	0.6289	0.6328	0.6342	0.6282	0.6181	0.6268	0.6281	0.6341	0.6349
	0.2451	γ_2	0.2382	0.2535	0.2613	0.2412	0.1933	0.2251	0.2425	0.2635	0.2469
		ρ	0.1036	0.0994	0.0995	0.5152	0.4947	0.5025	0.9276	0.9013	
	0.0000	μ	-0.0001	0.0005	-0.0003	-0.0006	0.0053	0.0011	0.0015	0.0014	unable to calculate intermediate correlation
Pareto ($\theta=10, \alpha=1$)	1.0000	σ^2	1.0004	0.9999	0.9983	0.9989	1.0101	1.0027	1.0017	1.0159	
	1.6330	γ_1	1.6394	1.6274	1.6286	1.6316	1.6595	1.6511	1.6274	1.6907	
	4.0000	γ_2	4.0617	3.9378	3.9772	3.9900	4.1164	4.1956	3.9580	4.2902	
	0.0000	μ	0.0010	0.0006	0.0012	-0.0006	0.0020	0.0008	0.0015	0.0022	
	σ^2	1.0014	1.0007	1.0032	1.0014	1.0015	1.0018	1.0006	1.0133		

	2.8111	γ_1	2.8275	2.8095	2.8685	2.8130	2.8615	2.8390	2.7788	2.8177	
	14.8286	γ_2	15.5209	14.6991	16.7207	14.5412	15.5158	15.2349	14.1296	13.8462	
$\chi^2_{(4)}$	0.0000	ρ	0.1004	0.1007	0.1002	0.5104	0.4980	0.5015	0.9195	0.8998	0.9001
	1.0000	μ	0.0005	-0.0002	-0.0008	0.0003	0.0020	0.0008	-0.0004	-0.0002	-0.0006
	1.4142	σ^2	1.0004	1.0007	0.9996	1.0002	1.0013	1.0008	1.0013	0.9995	1.0036
	3.0000	γ_1	1.4108	1.4151	1.4149	1.4115	1.4211	1.4163	1.4137	1.4484	1.4216
$\chi^2_{(4)}$	0.0000	γ_2	2.9915	2.9657	3.0010	2.9804	3.0853	3.0330	3.0066	6.5176	3.0761
	1.0000	μ	-0.0003	-0.0002	-0.0008	0.0009	0.0053	0.0008	-0.0009	-0.0003	-0.0006
	1.4142	σ^2	1.0024	0.9997	0.9977	1.0011	1.0128	1.0046	1.0000	0.9976	1.0021
	3.0000	γ_1	1.4224	1.4164	1.4237	1.4176	1.4418	1.4251	1.4111	1.4220	1.4097
$\chi^2_{(4)}$	0.0000	γ_2	3.0601	2.9945	3.0683	3.0333	3.0820	3.0350	2.9845	6.0227	2.9914
	1.0000	ρ	0.1006	0.0999	0.1002	0.5082	0.5005	0.5008	0.9163	0.8983	0.9001
	1.4142	μ	-0.0008	0.0010	0.0008	0.0003	-0.0012	0.0018	-0.0003	0.0058	0.0005
	3.0000	σ^2	0.9985	1.0036	1.0013	0.9995	1.0034	1.0012	0.9996	0.9966	0.9998
$\chi^2_{(8)}$	1.4142	γ_1	1.4128	1.4222	1.4105	1.4095	1.4048	1.4137	1.4062	1.4030	1.4202
	0.0000	γ_2	2.9899	3.0745	2.9821	2.9685	3.0178	2.9993	2.9466	2.8132	3.0883
	1.0000	μ	-0.0009	0.0011	0.0000	-0.0008	0.0004	0.0005	0.0000	0.0057	0.0003
	1.4142	σ^2	0.9980	1.0009	0.9999	0.9975	1.0005	1.0013	1.0020	0.9938	0.9999
$\chi^2_{(4)}$	0.0000	γ_1	0.9965	0.9972	1.0023	0.9976	0.9880	1.0032	0.9994	0.9950	0.9986
	1.0000	γ_2	1.4864	1.4861	1.5093	1.4841	1.4558	1.5158	1.4976	1.4299	1.4891
	1.4142	ρ	0.1008	0.1012	0.0988	0.5092	0.4999	0.5014	0.9145	0.9010	0.8998
	3.0000	μ	0.0005	0.0013	-0.0020	-0.0019	0.0040	-0.0011	0.0010	0.0014	0.0002
$\chi^2_{(16)}$	0.0000	σ^2	0.9997	1.0033	0.9951	0.9957	1.0160	0.9986	0.9995	1.0093	0.9999
	1.4142	γ_1	1.4091	1.4128	1.4094	1.4131	1.4690	1.4100	1.4103	1.4239	1.4198
	0.0000	γ_2	2.9812	3.0165	2.9864	2.9998	3.3391	3.0303	2.9792	3.2470	3.0658
	1.0000	μ	-0.0010	-0.0001	0.0021	-0.0015	0.0011	0.0008	0.0013	0.0018	0.0001
$\chi^2_{(16)}$	0.7071	σ^2	1.0016	1.0014	1.0003	1.0001	0.9999	0.9985	1.0010	1.0088	0.9979
	0.7500	γ_1	0.7036	0.7033	0.7084	0.7064	0.6938	0.7082	0.7065	0.7008	0.7077
	0.7500	γ_2	0.7393	0.7244	0.7554	0.7482	0.6788	0.7577	0.7481	0.8175	0.7565

$\chi^2_{(4)}$	0.0000	ρ	0.1009	0.0997	0.1012	0.5071	0.4990	0.4997	0.9133	0.9004	0.9001
	1.0000	μ	-0.0005	-0.0014	-0.0005	-0.0001	0.0011	-0.0012	-0.0004	0.0031	0.0010
	1.4142	σ^2	0.9996	0.9953	0.9990	1.0022	1.0008	0.9990	0.9993	1.0016	1.0004
	3.0000	γ_1	1.4161	1.4162	1.4168	1.4183	1.4120	1.4187	1.4099	1.3981	1.4169
	0.0000	γ_2	3.0145	3.0269	3.0121	3.0289	3.0426	3.0397	2.9724	2.8539	3.0565
$\chi^2_{(32)}$	0.0000	μ	-0.0012	0.0000	-0.0003	-0.0004	-0.0007	-0.0019	-0.0002	0.0030	0.0007
	1.0000	σ^2	1.0020	0.9991	1.0010	1.0017	1.0043	0.9975	1.0014	0.9985	1.0000
	0.5000	γ_1	0.5042	0.4957	0.5053	0.4995	0.4871	0.4920	0.4984	0.4958	0.5003
	0.3750	γ_2	0.3850	0.3627	0.3909	0.3783	0.3255	0.3730	0.3674	0.3271	0.3757
	$\chi^2_{(4)}$	0.0000	ρ	0.1002	0.1003	0.0998	0.5047	0.4965	0.4989	0.9089	0.8998
1.0000		μ	0.0013	0.0004	0.0003	0.0007	-0.0035	0.0011	0.0006	-0.0029	0.0009
1.4142		σ^2	1.0049	1.0003	1.0007	1.0023	0.9894	1.0004	1.0022	0.9944	1.0011
3.0000		γ_1	1.4196	1.4141	1.4171	1.4205	1.3813	1.4235	1.4124	1.3940	1.4105
0.0000		γ_2	3.0318	2.9954	3.0041	3.0408	2.8751	3.0390	2.9658	2.9730	2.9801
Beta ($\alpha=4, \beta=4$)	0.0000	μ	0.0000	0.0018	-0.0013	0.0006	-0.0002	-0.0002	0.0005	-0.0022	0.0008
	1.0000	σ^2	0.9996	0.9998	1.0018	1.0008	0.9987	0.9987	1.0021	1.0005	1.0006
	0.0000	γ_1	-0.0005	-0.0025	0.0020	0.0002	0.0030	0.0048	0.0013	-0.0164	-0.0001
	-0.5455	γ_2	-0.5450	-0.5447	-0.5491	-0.5417	-0.5510	-0.5474	-0.5482	-0.5558	-0.5441
	$\chi^2_{(4)}$	0.0000	ρ	0.0997	0.1010	0.1014	0.5025	0.4985	0.5009	0.8958	
1.0000		μ	0.0000	0.0015	-0.0007	-0.0002	0.0015	0.0010	0.0001		
1.4142		σ^2	0.9994	1.0003	0.9992	1.0008	0.9989	0.9997	1.0018		
3.0000		γ_1	1.4111	1.4113	1.4130	1.4167	1.4054	1.4067	1.4097	unable to calculate intermediate correlation	unable to calculate intermediate correlation
0.0000		γ_2	2.9710	3.0124	2.9802	3.0254	2.8187	2.9723	2.9530		
Beta ($\alpha=4, \beta=2$)	0.0000	μ	-0.0009	-0.0003	-0.0009	-0.0012	0.0002	0.0013	-0.0008		
	1.0000	σ^2	0.9991	1.0002	1.0015	1.0008	0.9992	0.9996	1.0025		
	-0.4677	γ_1	-0.4663	-0.4672	-0.4674	-0.4675	-0.4613	-0.4615	-0.4671		
	-0.3750	γ_2	-0.3779	-0.3754	-0.3755	-0.3777	-0.4017	-0.3826	-0.3803		
	$\chi^2_{(4)}$	0.0000	ρ	0.1006	0.1021	0.0977	0.5010	0.4995	0.4989	0.8668	
		μ	0.0007	-0.0008	0.0023	-0.0007	-0.0017	0.0003	-0.0016	unable to	unable to

Beta ($\alpha=4, \beta=3/2$)	1.0000	σ^2	1.0010	0.9988	1.0038	0.9989	0.9898	1.0020	1.0017	calculate intermediate correlation	calculate intermediate correlation
	1.4142	γ_1	1.4147	1.4134	1.4201	1.4247	1.3840	1.4248	1.4256		
	3.0000	γ_2	3.0018	2.9925	3.0400	3.0801	2.8720	3.1781	3.0819		
	0.0000	μ	-0.0001	0.0023	0.0002	0.0004	-0.0013	-0.0004	-0.0022		
	1.0000	σ^2	1.0001	0.9980	1.0012	0.9977	0.9957	1.0001	1.0021		
	-0.6939	γ_1	-0.6944	-0.6973	-0.6955	-0.6932	-0.6992	-0.7003	-0.6933		
	-0.0686	γ_2	-0.0680	-0.0584	-0.0638	-0.0661	-0.0483	-0.0609	-0.0664		
$\chi^2_{(4)}$		ρ	0.0998	0.0992	0.1001	0.5007	0.4996	0.4999	0.8451	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	0.0000	μ	0.0009	-0.0003	0.0004	0.0009	-0.0014	-0.0013	0.0004		
	1.0000	σ^2	1.0025	0.9989	0.9994	1.0013	1.0013	0.9976	1.0018		
	1.4142	γ_1	1.4111	1.4135	1.4059	1.4124	1.4280	1.4167	1.4172		
	3.0000	γ_2	2.9677	3.0033	2.9462	2.9906	3.1205	3.0834	3.0091		
	0.0000	μ	0.0007	-0.0006	0.0014	0.0004	-0.0025	-0.0009	0.0000		
	1.0000	σ^2	0.9997	0.9989	0.9982	1.0022	1.0017	1.0001	1.0005		
Beta ($\alpha=4, \beta=5/4$)	-0.8482	γ_1	-0.8508	-0.8475	-0.8510	-0.8499	-0.8417	-0.8481	-0.8473	0.8991 0.0024 1.0095 1.4585 3.1980 0.0027 0.9992 -0.3528	0.8997 -0.0014 0.9987 1.4109 3.0544 -0.0019 1.0018 -0.3844
	0.2210	γ_2	0.2300	0.2228	0.2307	0.2211	0.1969	0.2229	0.2154		
		ρ	0.0995	0.0982	0.1009	0.5030	0.5020	0.4999	0.9067		
	0.0000	μ	0.0017	0.0016	-0.0007	-0.0003	-0.0012	-0.0001	-0.0006		
	1.0000	σ^2	1.0042	1.0003	0.9988	0.9989	0.9964	1.0008	0.9973		
	1.4142	γ_1	1.4181	1.4105	1.4144	1.4148	1.4026	1.4151	1.4065		
	3.0000	γ_2	3.0231	2.9545	3.0069	2.9998	2.9213	2.9961	2.9696		
Weibull ($\alpha=6, \beta=10$)	0.0000	μ	-0.0017	-0.0001	0.0009	-0.0004	-0.0027	0.0005	-0.0007	0.0027	-0.0019
	1.0000	σ^2	0.9989	0.9992	0.9992	1.0012	1.0024	1.0003	1.0011	0.9992	1.0018
	-0.3733	γ_1	-0.3777	-0.3724	-0.3766	-0.3736	-0.3690	-0.3710	-0.3786	-0.3528	-0.3844
	0.0355	γ_2	0.0377	0.0323	0.0449	0.0381	0.0041	0.0338	0.0382	0.0259	0.0421
		ρ	0.1008	0.1003	0.0985	0.5086	0.5050	0.5009	0.9157	0.9000	0.9001
	0.0000	μ	0.0006	0.0004	0.0002	-0.0009	-0.0035	-0.0007	0.0011	0.0003	0.0005
	1.0000	σ^2	1.0023	0.9982	1.0006	0.9986	0.9951	1.0031	1.0033	0.9958	1.0001
$\chi^2_{(4)}$	1.4142	γ_1	1.4154	1.4043	1.4159	1.4189	1.4339	1.4208	1.4159	1.3972	1.4206
	3.0000	γ_2	3.0005	2.9186	3.0244	3.0401	3.2428	3.0696	2.9962	2.9232	3.0310

Gamma ($\alpha=\beta=10$)	0.0000	μ	0.0004	0.0013	-0.0007	-0.0014	-0.0048	0.0003	-0.0002	0.0021	0.0005
	1.0000	σ^2	1.0010	1.0019	0.9998	0.9998	0.9960	1.0010	1.0020	1.0016	1.0002
	0.8222	γ_1	0.8256	0.8234	0.8242	0.8207	0.8287	0.8245	0.8262	0.8153	0.8320
	0.6000	γ_2	0.6141	0.5996	0.6113	0.6068	0.6430	0.5979	0.6081	0.5903	0.5913
$\chi^2_{(4)}$		ρ	0.1011	0.0988	0.1007	0.5075	0.4952	0.5008	0.9139	0.8990	0.9000
	0.0000	μ	0.0006	0.0005	-0.0014	-0.0011	0.0008	0.0010	-0.0012	-0.0002	-0.0009
	1.0000	σ^2	1.0008	1.0022	0.9987	0.9963	1.0031	0.9996	0.9997	0.9929	1.0007
	1.4142	γ_1	1.4148	1.4226	1.4160	1.4149	1.4542	1.4160	1.4178	1.3989	1.4147
Rayleigh ($\alpha=1/2,$ $\mu=\sqrt{\pi/2}$)	3.0000	γ_2	3.0076	3.1003	2.9800	3.0185	3.3024	2.9987	3.0310	2.9049	3.0293
	0.0000	μ	0.0006	-0.0009	-0.0018	0.0008	-0.0002	-0.0006	-0.0007	0.0016	-0.0013
	1.0000	σ^2	0.9991	0.9982	0.9985	1.0004	0.9965	0.9986	0.9997	0.9953	0.9997
	0.6311	γ_1	0.6337	0.6261	0.6309	0.6368	0.6517	0.6279	0.6299	0.6221	0.6255
$\chi^2_{(4)}$	0.2451	γ_2	0.2528	0.2266	0.2439	0.2593	0.3125	0.2470	0.2449	0.2196	0.2392
		ρ	0.1030	0.1006	0.0984	0.5147	0.4991	0.5014	0.9262	0.9008	
	0.0000	μ	0.0003	-0.0010	0.0002	0.0005	0.0054	-0.0001	-0.0010	-0.0003	
	1.0000	σ^2	0.9999	0.9972	1.0010	0.9993	1.0052	1.0001	0.9995	1.0033	
Pareto ($\theta=10, \alpha=1$)	1.4142	γ_1	1.4147	1.4108	1.4168	1.4091	1.4148	1.4176	1.4156	1.4207	unable to calculate intermediate correlation
	3.0000	γ_2	3.0246	2.9754	3.0064	2.9599	2.9162	3.0696	3.0234	2.9992	
	0.0000	μ	-0.0007	-0.0001	0.0013	0.0004	0.0002	-0.0007	-0.0006	-0.0015	
	1.0000	σ^2	1.0025	1.0014	1.0053	0.9984	1.0000	0.9973	0.9989	1.0046	
$\chi^2_{(8)}$	2.8111	γ_1	2.8686	2.8007	2.8498	2.8054	2.8070	2.7793	2.8147	2.7634	
	14.8286	γ_2	15.9940	14.6149	16.3166	14.5866	14.6172	14.0576	14.7923	13.7767	
		ρ	0.1005	0.1011	0.0999	0.5076	0.4984	0.5008	0.9139	0.8995	0.9003
	0.0000	μ	0.0002	-0.0002	0.0010	0.0005	0.0020	0.0004	0.0005	0.0020	-0.0001
$\chi^2_{(8)}$	1.0000	σ^2	0.9994	1.0022	0.9998	1.0000	1.0012	0.9997	1.0036	1.0011	1.0019
	1.0000	γ_1	0.9991	1.0009	1.0023	1.0042	1.0043	0.9954	1.0034	1.0108	1.0103
	1.5000	γ_2	1.4974	1.5087	1.5270	1.5221	1.5319	1.4678	1.5026	1.5934	1.5500
	0.0000	μ	0.0005	0.0009	0.0008	0.0006	0.0050	-0.0013	0.0008	0.0034	0.0001
$\chi^2_{(8)}$	1.0000	σ^2	1.0006	1.0004	1.0003	1.0023	1.0107	1.0001	1.0047	1.0061	1.0022

	1.0000	γ_1	0.9964	0.9926	0.9943	1.0033	1.0243	0.9929	1.0041	1.0193	1.0106
	1.5000	γ_2	1.4784	1.4532	1.4912	1.5055	1.5571	1.4781	1.4989	1.5666	1.5353
$\chi^2_{(8)}$		ρ	0.1012	0.0996	0.0997	0.5074	0.4974	0.4983	0.9117	0.9004	0.8999
	0.0000	μ	-0.0001	-0.0013	-0.0003	0.0006	-0.0021	-0.0013	-0.0006	-0.0007	-0.0008
	1.0000	σ^2	1.0014	1.0003	1.0007	1.0011	0.9964	1.0001	1.0015	1.0115	0.9997
	1.0000	γ_1	1.0004	1.0009	0.9991	0.9958	0.9850	1.0055	0.9990	1.0109	0.9970
$\chi^2_{(16)}$	1.5000	γ_2	1.5034	1.5131	1.4853	1.4770	1.4857	1.5579	1.4818	1.5142	1.5207
	0.0000	μ	-0.0017	0.0007	-0.0003	0.0005	-0.0025	-0.0006	-0.0006	-0.0004	-0.0011
	1.0000	σ^2	0.9979	0.9990	1.0001	1.0011	0.9942	1.0006	1.0024	1.0111	0.9989
	0.7071	γ_1	0.7050	0.7073	0.7110	0.7114	0.6967	0.7060	0.7103	0.7172	0.7021
$\chi^2_{(8)}$	0.7500	γ_2	0.7481	0.7473	0.7800	0.7568	0.7608	0.7419	0.7510	0.7780	0.7381
		ρ	0.1012	0.1009	0.1000	0.5038	0.5011	0.4992	0.9102	0.9001	0.9000
	0.0000	μ	-0.0005	0.0015	-0.0009	0.0011	-0.0015	0.0006	0.0003	0.0045	0.0000
	1.0000	σ^2	1.0005	0.9995	0.9983	0.9977	1.0005	1.0019	0.9998	1.0077	0.9989
$\chi^2_{(32)}$	1.0000	γ_1	0.9978	0.9979	1.0008	0.9931	1.0045	1.0057	0.9978	1.0281	1.0055
	1.5000	γ_2	1.4838	1.4757	1.5006	1.4875	1.4561	1.5146	1.4964	1.6247	1.5437
	0.0000	μ	-0.0010	-0.0007	-0.0002	0.0009	-0.0074	-0.0001	0.0008	0.0079	0.0001
	1.0000	σ^2	0.9977	0.9981	0.9999	0.9993	0.9953	0.9977	0.9997	1.0074	0.9984
$\chi^2_{(8)}$	0.5000	γ_1	0.4982	0.4957	0.4941	0.4990	0.4751	0.4998	0.4963	0.5215	0.5038
	0.3750	γ_2	0.3675	0.3722	0.3537	0.3672	0.3419	0.3684	0.3713	0.3825	0.3771
		ρ	0.0997	0.1002	0.0997	0.5028	0.4980	0.4994	0.9060	0.9003	0.9001
	0.0000	μ	0.0001	-0.0010	0.0014	-0.0016	0.0006	0.0000	-0.0003	0.0008	0.0001
Beta ($\alpha=4, \beta=4$)	1.0000	σ^2	0.9985	1.0008	1.0035	0.9966	1.0010	0.9990	0.9969	1.0014	0.9995
	1.0000	γ_1	0.9996	1.0057	1.0014	0.9984	1.0060	0.9999	0.9965	0.9869	1.0024
	1.5000	γ_2	1.4936	1.5296	1.5002	1.5037	1.5135	1.4974	1.4929	1.5037	1.5113
	0.0000	μ	-0.0003	-0.0003	-0.0002	0.0008	0.0003	-0.0003	0.0003	0.0011	0.0005
Beta ($\alpha=4, \beta=4$)	1.0000	σ^2	0.9988	1.0019	1.0005	0.9993	1.0018	1.0004	0.9986	1.0030	0.9995
	0.0000	γ_1	0.0019	0.0011	0.0006	-0.0002	0.0220	-0.0013	-0.0018	-0.0236	0.0020
	-0.5455	γ_2	-0.5445	-0.5478	-0.5440	-0.5451	-0.5603	-0.5461	-0.5418	-0.5504	-0.5451

		ρ	0.0985	0.0987	0.0991	0.5014	0.5008	0.4980	0.9018	0.9013	0.8999
$\chi^2_{(8)}$	0.0000	μ	0.0018	-0.0013	-0.0002	0.0014	-0.0040	0.0002	0.0011	-0.0062	0.0005
	1.0000	σ^2	1.0012	1.0020	0.9994	1.0006	0.9906	1.0015	1.0023	0.9837	1.0020
	1.0000	γ_1	0.9966	1.0042	0.9949	0.9993	0.9725	1.0088	1.0050	0.9677	1.0047
	1.5000	γ_2	1.4716	1.5144	1.4777	1.4972	1.3615	1.5565	1.5284	1.4204	1.5312
Beta ($\alpha=4, \beta=2$)	0.0000	μ	-0.0006	-0.0010	-0.0006	0.0003	-0.0062	-0.0006	0.0009	-0.0040	0.0003
	1.0000	σ^2	0.9988	1.0022	1.0025	0.9996	1.0094	1.0006	1.0001	0.9957	1.0008
	-0.4677	γ_1	-0.4656	-0.4676	-0.4681	-0.4675	-0.4892	-0.4692	-0.4669	-0.4789	-0.4685
	-0.3750	γ_2	-0.3761	-0.3779	-0.3793	-0.3730	-0.3712	-0.3731	-0.3760	-0.3689	-0.3725
		ρ	0.0988	0.0991	0.1012	0.4990	0.4998	0.4992	0.8998		0.9001
$\chi^2_{(8)}$	0.0000	μ	0.0017	0.0007	-0.0004	-0.0001	0.0020	-0.0003	0.0013		0.0009
	1.0000	σ^2	1.0014	1.0018	0.9992	0.9998	1.0011	0.9989	1.0007		0.9988
	1.0000	γ_1	1.0056	0.9980	1.0043	1.0065	1.0048	1.0044	0.9967	unable to calculate intermediate correlation	1.0009
	1.5000	γ_2	1.5260	1.4804	1.5352	1.5238	1.5384	1.5411	1.4847		1.4976
Beta ($\alpha=4, \beta=3/2$)	0.0000	μ	0.0002	0.0005	-0.0007	0.0012	0.0029	-0.0001	0.0013		0.0015
	1.0000	σ^2	1.0001	0.9978	0.9989	0.9993	1.0020	0.9997	0.9998		0.9970
	-0.6939	γ_1	-0.6919	-0.6936	-0.6939	-0.6965	-0.6904	-0.6946	-0.6953		-0.6900
	-0.0686	γ_2	-0.0733	-0.0658	-0.0704	-0.0619	-0.0760	-0.0691	-0.0669		-0.0716
		ρ	0.1001	0.0988	0.0986	0.4995	0.4996	0.5005	0.8883		
$\chi^2_{(8)}$	0.0000	μ	-0.0002	0.0004	-0.0017	-0.0002	-0.0022	0.0000	-0.0008		
	1.0000	σ^2	0.9999	0.9987	0.9999	1.0004	0.9926	0.9991	0.9974		
	1.0000	γ_1	1.0004	0.9974	0.9970	0.9975	0.9859	0.9998	1.0009	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	1.5000	γ_2	1.5007	1.4769	1.4714	1.4801	1.4734	1.5209	1.5037		
Beta ($\alpha=4, \beta=5/4$)	0.0000	μ	0.0004	0.0000	0.0006	0.0000	-0.0038	0.0007	-0.0003		
	1.0000	σ^2	0.9974	1.0016	0.9978	1.0006	1.0024	1.0016	0.9986		
	-0.8482	γ_1	-0.8470	-0.8481	-0.8493	-0.8477	-0.8428	-0.8477	-0.8460		
	0.2210	γ_2	0.2202	0.2166	0.2253	0.2184	0.2153	0.2218	0.2135		
		ρ	0.0979	0.0981	0.1015	0.5012	0.5040	0.5005	0.9038	0.8996	0.8999
$\chi^2_{(8)}$	0.0000	μ	0.0000	0.0008	0.0007	0.0004	0.0014	-0.0002	0.0007	0.0004	0.0004

Weibull ($\alpha=6, \beta=10$)	1.0000	σ^2	0.9990	1.0020	0.9997	0.9993	1.0069	0.9987	1.0011	0.9988	0.9998
	1.0000	γ_1	0.9981	1.0055	1.0026	0.9963	1.0117	0.9967	1.0010	1.0104	1.0023
	1.5000	γ_2	1.4687	1.5340	1.5233	1.4915	1.4823	1.4982	1.5038	1.4727	1.5311
	0.0000	μ	-0.0021	-0.0006	0.0000	0.0005	-0.0007	0.0004	0.0003	0.0003	0.0007
	1.0000	σ^2	0.9999	1.0031	1.0012	1.0002	1.0022	0.9986	1.0008	1.0036	0.9987
	-0.3733	γ_1	-0.3728	-0.3756	-0.3704	-0.3739	-0.3670	-0.3759	-0.3713	-0.3730	-0.3752
	0.0355	γ_2	0.0349	0.0306	0.0342	0.0382	0.0390	0.0423	0.0310	0.0616	0.0372
$\chi^2_{(8)}$		ρ	0.1012	0.0999	0.1002	0.5070	0.4958	0.5000	0.9129	0.8991	0.8998
	0.0000	μ	0.0011	-0.0003	0.0014	-0.0014	-0.0066	0.0008	-0.0001	0.0017	0.0004
	1.0000	σ^2	0.9985	1.0006	0.9995	0.9967	0.9982	1.0019	1.0009	1.0038	0.9996
	1.0000	γ_1	0.9991	1.0059	0.9938	0.9983	0.9832	1.0056	1.0006	1.0175	1.0040
	1.5000	γ_2	1.4883	1.5301	1.4538	1.5009	1.5099	1.5408	1.4907	1.5937	1.5383
	0.0000	μ	0.0002	-0.0002	0.0007	-0.0006	-0.0043	0.0002	0.0003	0.0010	0.0003
	1.0000	σ^2	1.0004	0.9992	0.9981	0.9995	0.9961	1.0002	1.0012	0.9978	0.9986
Gamma ($\alpha=\beta=10$)	0.8222	γ_1	0.8255	0.8262	0.8187	0.8182	0.8129	0.8244	0.8206	0.8232	0.8247
	0.6000	γ_2	0.6066	0.6202	0.5791	0.5905	0.5816	0.5883	0.5886	0.6042	0.5966
		ρ	0.0996	0.1000	0.1003	0.5063	0.4987	0.5002	0.9114	0.8999	0.9001
	0.0000	μ	-0.0009	-0.0004	0.0014	0.0009	0.0039	-0.0004	-0.0006	0.0008	0.0007
	1.0000	σ^2	0.9980	0.9991	1.0030	1.0020	1.0065	0.9987	1.0007	1.0026	1.0026
	1.0000	γ_1	0.9979	0.9963	1.0017	1.0051	1.0382	0.9990	0.9964	0.9992	1.0021
	1.5000	γ_2	1.4991	1.4738	1.5157	1.5134	1.5835	1.5460	1.4824	1.5263	1.5055
Rayleigh ($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	0.0000	μ	-0.0004	0.0010	0.0021	0.0009	-0.0046	0.0026	-0.0007	0.0014	0.0003
	1.0000	σ^2	1.0004	1.0004	1.0018	1.0005	0.9897	1.0016	1.0020	1.0058	1.0023
	0.6311	γ_1	0.6298	0.6305	0.6339	0.6342	0.6260	0.6406	0.6319	0.6336	0.6319
	0.2451	γ_2	0.2445	0.2526	0.2581	0.2600	0.2369	0.2632	0.2458	0.2623	0.2464
		ρ	0.1027	0.1002	0.0979	0.5117	0.4984	0.5015	0.9237	0.8980	
	0.0000	μ	-0.0002	-0.0008	-0.0002	0.0004	0.0024	0.0012	-0.0006	0.0003	unable to calculate intermediate
	1.0000	σ^2	0.9971	0.9997	0.9987	0.9998	0.9957	1.0009	0.9990	0.9933	
1.0000	γ_1	1.0021	1.0005	0.9954	0.9989	0.9648	1.0021	1.0025	0.9747		

											correlation
Pareto ($\theta=10, \alpha=1$)	1.5000	γ_2	1.5009	1.5003	1.4676	1.5005	1.3074	1.4836	1.5135	1.3780	
	0.0000	μ	0.0003	-0.0009	0.0017	-0.0011	0.0020	0.0008	-0.0004	0.0002	
	1.0000	σ^2	1.0005	0.9995	1.0013	0.9958	0.9916	1.0020	1.0008	0.9897	
	2.8111	γ_1	2.8842	2.8415	2.7931	2.7671	2.7593	2.7827	2.8041	2.8073	
	14.8286	γ_2	17.6680	15.0984	14.4651	13.7451	13.6564	14.1655	14.2744	14.3969	
$\chi^2_{(16)}$		ρ	0.1014	0.1003	0.1017	0.5050	0.4987	0.5015	0.9097	0.8995	0.9003
	0.0000	μ	-0.0007	0.0007	0.0011	0.0001	0.0019	0.0001	0.0002	0.0020	0.0005
	1.0000	σ^2	1.0005	1.0013	1.0009	0.9981	1.0012	1.0015	1.0009	1.0008	1.0007
	0.7071	γ_1	0.7113	0.7049	0.7068	0.7034	0.7103	0.7027	0.7098	0.7148	0.7058
	0.7500	γ_2	0.7616	0.7405	0.7605	0.7391	0.7638	0.7326	0.7639	0.7974	0.7522
$\chi^2_{(16)}$	0.0000	μ	-0.0007	0.0000	0.0001	0.0006	0.0048	-0.0005	-0.0002	0.0033	0.0008
	1.0000	σ^2	1.0010	0.9997	0.9982	1.0020	1.0093	1.0007	1.0001	1.0050	1.0010
	0.7071	γ_1	0.7065	0.7093	0.7028	0.7072	0.7289	0.7081	0.7134	0.7239	0.7040
	0.7500	γ_2	0.7491	0.7566	0.7374	0.7522	0.7897	0.7458	0.7752	0.7930	0.7282
	$\chi^2_{(16)}$		ρ	0.1010	0.0989	0.0988	0.5043	0.4974	0.4995	0.9083	0.9000
0.0000		μ	0.0006	-0.0018	-0.0002	-0.0001	-0.0008	0.0001	-0.0024	0.0009	0.0011
1.0000		σ^2	1.0030	0.9966	0.9996	1.0004	0.9985	1.0001	0.9994	1.0027	1.0014
0.7071		γ_1	0.7068	0.7037	0.7070	0.7095	0.7091	0.7070	0.7112	0.7235	0.7127
0.7500		γ_2	0.7425	0.7397	0.7551	0.7617	0.8392	0.7632	0.7646	0.7932	0.7832
$\chi^2_{(32)}$	0.0000	μ	-0.0005	0.0021	-0.0011	-0.0008	-0.0040	-0.0009	-0.0028	0.0007	0.0015
	1.0000	σ^2	0.9988	0.9999	0.9984	1.0021	0.9972	1.0010	0.9993	1.0017	1.0015
	0.5000	γ_1	0.4955	0.5012	0.5028	0.4952	0.4764	0.4954	0.5005	0.5107	0.5043
	0.3750	γ_2	0.3630	0.3788	0.3792	0.3644	0.3613	0.3651	0.3773	0.3936	0.3826
	$\chi^2_{(16)}$		ρ	0.0993	0.1014	0.1005	0.5009	0.5018	0.5013	0.9042	0.8999
0.0000		μ	0.0020	-0.0003	0.0004	-0.0006	-0.0020	-0.0009	-0.0015	-0.0025	0.0017
1.0000		σ^2	1.0024	0.9987	1.0000	0.9970	1.0035	1.0001	1.0003	1.0013	1.0009
0.7071		γ_1	0.7103	0.7097	0.7058	0.7009	0.6986	0.7020	0.7111	0.7207	0.7100
0.7500		γ_2	0.7541	0.7566	0.7445	0.7305	0.7459	0.7611	0.7694	0.9403	0.7446
Beta	0.0000	μ	0.0013	-0.0005	0.0006	0.0011	0.0008	-0.0011	-0.0016	-0.0010	0.0011

$(\alpha=4, \beta=4)$	1.0000	σ^2	1.0016	1.0002	0.9978	0.9992	1.0008	1.0013	0.9995	0.9968	0.9995
	0.0000	γ_1	0.0018	0.0019	0.0009	-0.0020	0.0136	0.0000	0.0008	0.0000	0.0017
	-0.5455	γ_2	-0.5472	-0.5451	-0.5439	-0.5470	-0.5516	-0.5429	-0.5411	-0.5403	-0.5472
$\chi^2_{(16)}$		ρ	0.1001	0.0995	0.0992	0.4994	0.5050	0.4981	0.8999	0.8997	0.9000
	0.0000	μ	0.0003	0.0003	0.0002	0.0017	0.0043	0.0019	-0.0002	0.0047	-0.0011
	1.0000	σ^2	1.0009	1.0001	1.0022	1.0017	1.0016	1.0020	0.9997	1.0028	0.9985
	0.7071	γ_1	0.7110	0.7096	0.7127	0.7062	0.7042	0.7205	0.7072	0.7280	0.7011
	0.7500	γ_2	0.7610	0.7561	0.7577	0.7470	0.6761	0.7696	0.7479	0.7698	0.7597
Beta $(\alpha=4, \beta=2)$	0.0000	μ	0.0015	-0.0015	0.0006	0.0015	-0.0012	0.0005	0.0001	0.0047	-0.0001
	1.0000	σ^2	1.0002	1.0018	0.9991	0.9986	0.9990	0.9994	1.0000	0.9991	0.9986
	-0.4677	γ_1	-0.4706	-0.4650	-0.4700	-0.4701	-0.4730	-0.4641	-0.4688	-0.4587	-0.4720
$\chi^2_{(16)}$	-0.3750	γ_2	-0.3730	-0.3813	-0.3710	-0.3703	-0.3631	-0.3761	-0.3731	-0.3744	-0.3690
		ρ	0.0985	0.1017	0.1011	0.4981	0.5026	0.5004	0.8979	0.8997	0.8999
	0.0000	μ	0.0010	-0.0008	-0.0008	0.0004	0.0033	0.0003	0.0001	-0.0004	-0.0019
	1.0000	σ^2	1.0002	0.9997	0.9992	0.9994	1.0110	1.0017	1.0000	0.9919	0.9988
	0.7071	γ_1	0.7087	0.7063	0.7081	0.7090	0.6989	0.7019	0.7050	0.6959	0.7068
Beta $(\alpha=4, \beta=3/2)$	0.7500	γ_2	0.7498	0.7454	0.7612	0.7561	0.7124	0.7493	0.7442	0.7930	0.7608
	0.0000	μ	0.0003	-0.0010	0.0020	-0.0003	-0.0033	0.0009	0.0001	0.0012	-0.0018
	1.0000	σ^2	0.9982	1.0017	0.9978	0.9997	1.0072	0.9993	0.9997	0.9944	1.0004
$\chi^2_{(16)}$	-0.6939	γ_1	-0.6936	-0.6935	-0.6972	-0.6943	-0.6956	-0.6947	-0.6968	-0.7059	-0.6991
	-0.0686	γ_2	-0.0692	-0.0689	-0.0598	-0.0651	-0.0781	-0.0667	-0.0591	-0.0500	-0.0614
		ρ	0.0989	0.0976	0.0999	0.4976	0.4970	0.5007	0.8963		0.9001
	0.0000	μ	0.0011	0.0023	-0.0013	-0.0001	0.0040	0.0004	0.0011		0.0000
	1.0000	σ^2	1.0015	1.0022	0.9983	1.0008	1.0052	1.0014	1.0029		0.9979
Beta $(\alpha=4, \beta=5/4)$	0.7071	γ_1	0.7077	0.7101	0.7068	0.7082	0.7120	0.7110	0.7037	unable to calculate intermediate correlation	0.7119
	0.7500	γ_2	0.7499	0.7672	0.7403	0.7621	0.7316	0.7685	0.7457		0.7512
	0.0000	μ	0.0019	0.0001	0.0008	-0.0007	-0.0013	0.0013	0.0003		0.0003
	1.0000	σ^2	0.9978	1.0015	0.9984	1.0007	1.0030	0.9994	1.0031		0.9979
	-0.8482	γ_1	-0.8515	-0.8503	-0.8478	-0.8436	-0.8451	-0.8510	-0.8513		-0.8407
	γ_2	0.2301	0.2278	0.2226	0.2049	0.2101	0.2308	0.2262		0.2124	

$\chi^2_{(16)}$	0.0000	ρ	0.0987	0.1002	0.1001	0.5012	0.4967	0.4995	0.9015	0.8998	0.9002
	1.0000	μ	-0.0015	0.0003	-0.0007	-0.0006	-0.0053	0.0008	0.0011	0.0034	0.0012
	0.7071	σ^2	0.9960	1.0000	0.9993	0.9988	0.9978	1.0011	1.0020	0.9960	1.0014
	0.7500	γ_1	0.7049	0.7022	0.7118	0.7066	0.6926	0.7116	0.7062	0.7232	0.7094
Weibull ($\alpha=6, \beta=10$)	0.0000	γ_2	0.7430	0.7280	0.7769	0.7487	0.7897	0.7694	0.7501	0.8160	0.7602
	1.0000	μ	0.0008	0.0012	0.0002	-0.0004	-0.0046	0.0003	0.0009	0.0030	0.0012
	-0.3733	σ^2	0.9998	0.9983	0.9992	0.9989	1.0028	0.9994	1.0014	0.9886	1.0007
	0.0355	γ_1	-0.3740	-0.3765	-0.3741	-0.3711	-0.3790	-0.3740	-0.3741	-0.3727	-0.3700
$\chi^2_{(16)}$	0.0000	γ_2	0.0304	0.0458	0.0355	0.0331	0.0356	0.0335	0.0355	0.0821	0.0331
	1.0000	ρ	0.0997	0.1010	0.1014	0.5054	0.4996	0.5008	0.9107	0.9002	0.8999
	0.7071	μ	0.0009	-0.0007	0.0006	0.0000	-0.0106	-0.0014	-0.0004	0.0043	0.0000
	0.7500	σ^2	1.0017	0.9992	1.0010	0.9992	0.9992	0.9995	0.9982	1.0007	1.0002
Gamma ($\alpha=\beta=10$)	0.0000	γ_1	0.7085	0.7070	0.7060	0.7081	0.7053	0.7044	0.7037	0.7017	0.7079
	1.0000	γ_2	0.7489	0.7505	0.7514	0.7526	0.7914	0.7489	0.7286	0.7307	0.7757
	0.8222	μ	0.0014	0.0007	-0.0010	0.0004	-0.0027	-0.0017	-0.0004	0.0053	-0.0005
	0.6000	σ^2	1.0025	0.9989	1.0018	0.9992	0.9956	0.9990	0.9993	1.0068	0.9986
$\chi^2_{(16)}$	0.0000	γ_1	0.8194	0.8215	0.8267	0.8230	0.8149	0.8157	0.8226	0.8204	0.8167
	1.0000	γ_2	0.5874	0.5952	0.6036	0.6068	0.5570	0.5961	0.5917	0.5809	0.5806
	0.7071	ρ	0.1012	0.0986	0.1001	0.5040	0.4948	0.4986	0.9094	0.8998	0.9000
	0.7500	μ	-0.0011	0.0001	0.0005	0.0007	-0.0008	0.0000	0.0004	0.0028	-0.0012
Rayleigh ($\alpha=\frac{1}{2}, \mu=\sqrt{\pi/2}$)	1.0000	σ^2	0.9973	1.0006	0.9999	1.0004	0.9964	0.9987	1.0015	1.0065	0.9987
	0.6311	γ_1	0.7090	0.7072	0.7070	0.7083	0.7457	0.7119	0.7080	0.7109	0.7021
	0.2451	γ_2	0.7660	0.7554	0.7396	0.7486	0.8673	0.7643	0.7490	0.7697	0.7555
	0.0000	μ	-0.0004	-0.0007	-0.0009	-0.0003	-0.0023	0.0003	0.0000	0.0012	-0.0010
	σ^2	0.9964	0.9999	1.0004	0.9969	1.0019	1.0014	1.0005	1.0014	0.9994	
	γ_1	0.6314	0.6336	0.6330	0.6308	0.6459	0.6402	0.6294	0.6261	0.6289	
	γ_2	0.2525	0.2533	0.2472	0.2403	0.2761	0.2624	0.2384	0.2751	0.2364	
	ρ	0.1023	0.1005	0.1002	0.5120	0.4989	0.4989	0.9219	0.9016		

$\chi^2_{(16)}$	0.0000	μ	0.0003	-0.0013	-0.0007	0.0000	0.0006	-0.0008	-0.0006	0.0016	
	1.0000	σ^2	0.9981	0.9986	0.9997	1.0011	0.9987	0.9984	0.9989	1.0040	
	0.7071	γ_1	0.7026	0.7102	0.7059	0.7081	0.7247	0.7037	0.7007	0.7149	unable to calculate intermediate correlation
	0.7500	γ_2	0.7325	0.7501	0.7422	0.7516	0.8191	0.7522	0.7420	0.8072	
Pareto ($\theta=10, \alpha=1$)	0.0000	μ	0.0015	-0.0017	0.0017	-0.0006	0.0029	-0.0005	-0.0006	0.0010	
	1.0000	σ^2	1.0095	0.9955	1.0062	0.9934	1.0175	0.9956	0.9946	1.0002	
	2.8111	γ_1	2.8259	2.8075	2.8056	2.7605	2.9288	2.7910	2.7701	2.7944	
	14.8286	γ_2	14.6680	14.8680	14.4399	13.9140	16.7874	14.3857	13.7884	14.5889	
$\chi^2_{(32)}$		ρ	0.0992	0.0994	0.0985	0.5025	0.4990	0.4997	0.9067	0.8997	0.8999
	0.0000	μ	-0.0011	0.0003	-0.0014	0.0006	0.0019	0.0003	0.0006	-0.0003	-0.0002
	1.0000	σ^2	0.9980	1.0014	1.0012	0.9992	1.0012	0.9999	1.0026	0.9980	1.0001
	0.5000	γ_1	0.5008	0.5034	0.5010	0.5009	0.5026	0.4983	0.5013	0.5047	0.5019
$\chi^2_{(32)}$	0.3750	γ_2	0.3780	0.3812	0.3807	0.3776	0.3821	0.3748	0.3880	0.3992	0.3687
	0.0000	μ	0.0001	0.0005	-0.0016	0.0000	0.0046	0.0000	0.0010	-0.0003	0.0002
	1.0000	σ^2	0.9991	1.0027	0.9999	0.9986	1.0083	0.9980	1.0026	0.9980	0.9991
	0.5000	γ_1	0.4987	0.5015	0.5012	0.4983	0.5201	0.4964	0.4991	0.5020	0.5011
$\chi^2_{(32)}$	0.3750	γ_2	0.3808	0.3773	0.3739	0.3645	0.4029	0.3652	0.3825	0.3808	0.3658
		ρ	0.0975	0.0998	0.0986	0.5002	0.5009	0.4997	0.9024	0.8999	0.9002
	0.0000	μ	0.0004	0.0011	0.0006	0.0006	0.0016	-0.0001	-0.0005	-0.0019	-0.0007
	1.0000	σ^2	1.0004	0.9995	0.9987	1.0008	1.0043	1.0004	0.9991	0.9947	1.0038
Beta ($\alpha=4, \beta=4$)	0.5000	γ_1	0.4983	0.5003	0.4976	0.4982	0.5090	0.5043	0.5019	0.4901	0.5007
	0.3750	γ_2	0.3720	0.3862	0.3703	0.3751	0.4215	0.3865	0.3893	0.3921	0.3805
	0.0000	μ	-0.0007	0.0007	-0.0003	0.0004	-0.0018	-0.0001	-0.0008	0.0002	-0.0003
	1.0000	σ^2	0.9989	1.0016	0.9997	0.9981	0.9990	0.9994	1.0000	0.9991	1.0018
$\chi^2_{(32)}$	0.0000	γ_1	0.0003	-0.0047	-0.0004	0.0003	-0.0148	0.0015	-0.0018	-0.0024	0.0016
	-0.5455	γ_2	-0.5422	-0.5479	-0.5416	-0.5473	-0.5410	-0.5416	-0.5431	-0.5420	-0.5450
		ρ	0.0996	0.1000	0.0996	0.4989	0.5000	0.4999	0.8987	0.8985	0.9001
	0.0000	μ	0.0003	-0.0004	0.0005	0.0013	0.0024	-0.0007	-0.0015	0.0017	0.0004
$\chi^2_{(32)}$	1.0000	σ^2	1.0004	1.0008	0.9990	0.9996	0.9979	0.9998	0.9976	0.9979	0.9988

	0.5000	γ_1	0.4994	0.5001	0.4980	0.4950	0.5039	0.5030	0.4983	0.5313	0.5038
	0.3750	γ_2	0.3750	0.3692	0.3792	0.3618	0.4087	0.3938	0.3724	0.4704	0.3832
Beta	0.0000	μ	0.0000	-0.0003	0.0006	0.0000	-0.0019	-0.0005	-0.0008	0.0007	-0.0001
($\alpha=4, \beta=2$)	1.0000	σ^2	1.0021	0.9999	1.0010	0.9986	1.0020	1.0017	0.9991	0.9940	0.9987
	-0.4677	γ_1	-0.4697	-0.4699	-0.4662	-0.4671	-0.4734	-0.4690	-0.4695	-0.4643	-0.4656
	-0.3750	γ_2	-0.3768	-0.3679	-0.3785	-0.3754	-0.3669	-0.3747	-0.3719	-0.3770	-0.3795
		ρ	0.0984	0.0991	0.1007	0.4981	0.5015	0.4998	0.8965	0.8989	0.8998
$\chi^2_{(32)}$	0.0000	μ	0.0003	-0.0008	-0.0006	0.0017	0.0024	0.0011	-0.0025	-0.0034	0.0001
	1.0000	σ^2	1.0019	0.9984	0.9991	1.0023	0.9978	1.0026	0.9996	0.9968	0.9995
	0.5000	γ_1	0.4975	0.5011	0.5005	0.4981	0.4901	0.5056	0.4991	0.5027	0.5009
	0.3750	γ_2	0.3691	0.3761	0.3822	0.3715	0.3574	0.3826	0.3756	0.4281	0.3867
Beta	0.0000	μ	-0.0016	-0.0002	-0.0001	0.0005	0.0089	-0.0001	-0.0028	-0.0034	0.0006
($\alpha=4, \beta=3/2$)	1.0000	σ^2	1.0002	1.0018	1.0005	0.9972	1.0004	0.9998	1.0026	0.9931	0.9985
	-0.6939	γ_1	-0.6924	-0.6957	-0.6950	-0.6905	-0.6785	-0.6915	-0.6952	-0.6948	-0.6927
	-0.0686	γ_2	-0.0712	-0.0671	-0.0678	-0.0709	-0.0628	-0.0670	-0.0661	-0.0727	-0.0672
		ρ	0.0997	0.0996	0.0988	0.4975	0.4989	0.4992	0.8949	0.8996	0.8997
$\chi^2_{(32)}$	0.0000	μ	-0.0003	0.0013	-0.0004	-0.0006	0.0012	-0.0007	-0.0011	-0.0010	-0.0005
	1.0000	σ^2	0.9988	1.0029	1.0004	1.0003	0.9917	0.9990	1.0002	0.9938	0.9988
	0.5000	γ_1	0.4987	0.5011	0.4984	0.4997	0.5011	0.4919	0.5036	0.5048	0.5005
	0.3750	γ_2	0.3651	0.3763	0.3688	0.3741	0.4395	0.3609	0.3758	0.4114	0.3918
Beta	0.0000	μ	0.0003	-0.0004	0.0020	0.0002	-0.0008	-0.0010	-0.0013	0.0000	-0.0002
($\alpha=4, \beta=5/4$)	1.0000	σ^2	0.9986	1.0005	0.9974	0.9998	0.9986	1.0029	0.9996	1.0008	1.0001
	-0.8482	γ_1	-0.8512	-0.8492	-0.8464	-0.8477	-0.8777	-0.8509	-0.8463	-0.8490	-0.8508
	0.2210	γ_2	0.2293	0.2256	0.2119	0.2166	0.2488	0.2168	0.2191	0.2367	0.2223
		ρ	0.0996	0.0989	0.1018	0.5004	0.5024	0.5009	0.9002	0.9003	0.9000
$\chi^2_{(32)}$	0.0000	μ	0.0012	-0.0002	-0.0010	0.0003	-0.0013	-0.0013	-0.0018	-0.0008	0.0002
	1.0000	σ^2	1.0038	0.9999	0.9978	1.0001	0.9999	0.9997	1.0005	1.0028	1.0004
	0.5000	γ_1	0.5039	0.4971	0.4966	0.5004	0.4865	0.4974	0.5028	0.4971	0.5003
	0.3750	γ_2	0.3826	0.3615	0.3636	0.3732	0.3460	0.3796	0.3877	0.3474	0.3707
Weibull	0.0000	μ	0.0024	-0.0006	0.0015	-0.0002	-0.0024	-0.0017	-0.0011	0.0000	0.0001

$(\alpha=6, \beta=10)$	1.0000	σ^2	1.0012	1.0023	0.9997	0.9982	1.0040	0.9992	1.0009	1.0030	0.9991
	-0.3733	γ_1	-0.3745	-0.3737	-0.3751	-0.3704	-0.3740	-0.3793	-0.3706	-0.3793	-0.3728
	0.0355	γ_2	0.0456	0.0268	0.0367	0.0301	0.0254	0.0427	0.0291	0.0752	0.0317
$\chi^2_{(32)}$		ρ	0.1004	0.1006	0.1007	0.5051	0.4970	0.4982	0.9094	0.9000	0.8998
	0.0000	μ	-0.0006	-0.0004	0.0003	0.0007	-0.0026	-0.0021	-0.0003	0.0036	-0.0003
	1.0000	σ^2	1.0002	0.9990	1.0026	0.9996	0.9931	0.9982	0.9992	1.0043	0.9992
	0.5000	γ_1	0.5008	0.5021	0.5039	0.5023	0.4881	0.4952	0.4998	0.5095	0.4984
	0.3750	γ_2	0.3699	0.3890	0.3929	0.3824	0.3671	0.3618	0.3723	0.3719	0.3752
Gamma $(\alpha=\beta=10)$	0.0000	μ	-0.0004	-0.0013	-0.0001	-0.0006	-0.0021	-0.0010	-0.0002	0.0037	-0.0004
	1.0000	σ^2	0.9962	0.9968	0.9991	0.9983	0.9927	0.9952	1.0006	1.0012	1.0000
	0.8222	γ_1	0.8147	0.8195	0.8199	0.8250	0.8116	0.8192	0.8234	0.8300	0.8182
$\chi^2_{(32)}$	0.6000	γ_2	0.5758	0.5879	0.5954	0.6115	0.5503	0.6556	0.6005	0.6092	0.6125
		ρ	0.1003	0.0987	0.1015	0.5039	0.5012	0.5010	0.9076	0.9014	0.8995
	0.0000	μ	-0.0020	-0.0019	0.0009	0.0013	0.0037	0.0006	0.0014	-0.0006	-0.0003
	1.0000	σ^2	0.9979	1.0006	1.0009	1.0007	0.9986	1.0001	0.9987	1.0098	0.9999
	0.5000	γ_1	0.4981	0.4989	0.4986	0.4982	0.5060	0.5086	0.4993	0.5075	0.5035
Rayleigh $(\alpha=1/2,$ $\mu=\sqrt{(\pi/2)})$	0.3750	γ_2	0.3760	0.3621	0.3765	0.3686	0.3580	0.3859	0.3731	0.4298	0.3926
	0.0000	μ	-0.0018	-0.0001	0.0003	0.0000	0.0041	0.0003	0.0010	0.0007	-0.0007
	1.0000	σ^2	0.9992	0.9976	0.9984	0.9999	0.9987	1.0029	0.9978	1.0105	0.9997
	0.6311	γ_1	0.6341	0.6298	0.6315	0.6294	0.6474	0.6386	0.6279	0.6365	0.6358
	0.2451	γ_2	0.2563	0.2397	0.2571	0.2438	0.2725	0.2515	0.2403	0.2624	0.2668
$\chi^2_{(32)}$		ρ	0.1027	0.1005	0.1001	0.5116	0.5008	0.4991	0.9155	0.9013	
	0.0000	μ	0.0008	0.0008	-0.0004	0.0000	-0.0021	-0.0002	0.0017	0.0032	
	1.0000	σ^2	1.0019	0.9993	0.9970	0.9996	0.9994	0.9987	0.9979	1.0006	
	0.5000	γ_1	0.4993	0.5016	0.5001	0.5026	0.4790	0.5020	0.5022	0.5010	unable to
	0.3750	γ_2	0.3764	0.3838	0.3680	0.3882	0.3673	0.3931	0.3771	0.3452	calculate
Pareto $(\theta=10, \alpha=1)$	0.0000	μ	0.0004	0.0010	-0.0014	-0.0004	-0.0012	-0.0005	0.0009	0.0030	intermediate
	1.0000	σ^2	0.9993	1.0068	0.9936	0.9978	1.0077	0.9979	1.0000	0.9966	correlation
	2.8111	γ_1	2.8130	2.8237	2.7945	2.7929	2.7881	2.8078	2.8098	2.7159	

	14.8286	γ_2	15.4936	14.9484	14.6523	14.4564	14.5417	14.6887	14.7429	13.0857	
		ρ	0.1013	0.1003	0.0999	0.5004	0.4992	0.5002	0.8994	0.8997	0.9001
Beta	0.0000	μ	-0.0007	0.0016	0.0014	-0.0005	-0.0001	0.0006	-0.0001	-0.0002	0.0001
($\alpha=4$, $\beta=4$)	1.0000	σ^2	1.0015	0.9994	0.9987	1.0009	0.9986	0.9991	1.0003	0.9976	1.0006
delta = 0.025	0.0000	γ_1	0.0000	0.0001	-0.0032	0.0011	-0.0005	0.0025	0.0016	0.0013	0.0019
	-0.5455	γ_2	-0.5498	-0.5475	-0.5423	-0.5443	-0.5442	-0.5422	-0.5453	-0.5431	-0.5459
Beta	0.0000	μ	-0.0009	0.0010	0.0011	-0.0003	-0.0002	0.0015	0.0007	-0.0002	0.0001
($\alpha=4$, $\beta=4$)	1.0000	σ^2	0.9990	0.9987	1.0013	0.9988	0.9994	0.9987	1.0007	0.9981	1.0003
time =	0.0000	γ_1	-0.0016	-0.0018	0.0007	-0.0010	-0.0010	0.0058	0.0017	0.0012	0.0038
1208 seconds	-0.5455	γ_2	-0.5464	-0.5477	-0.5472	-0.5433	-0.5450	-0.5461	-0.5445	-0.5436	-0.5483
		ρ	0.0975	0.0999	0.1010	0.4972	0.5008	0.5007	0.8957	0.8999	0.9001
Beta	0.0000	μ	-0.0007	0.0011	-0.0004	-0.0003	0.0018	0.0014	0.0014	0.0001	0.0004
($\alpha=4$, $\beta=4$)	1.0000	σ^2	1.0006	1.0020	1.0015	1.0010	1.0041	1.0005	1.0001	1.0008	1.0004
	0.0000	γ_1	0.0008	-0.0031	0.0040	-0.0001	0.0047	0.0022	-0.0040	-0.0109	-0.0010
	-0.5455	γ_2	-0.5477	-0.5439	-0.5457	-0.5467	-0.5420	-0.5482	-0.5477	-0.5334	-0.5442
Beta	0.0000	μ	0.0021	0.0020	0.0008	0.0008	-0.0045	0.0011	0.0014	0.0009	0.0006
($\alpha=4$, $\beta=2$)	1.0000	σ^2	0.9963	0.9990	1.0025	0.9989	1.0033	0.9996	0.9994	1.0018	1.0015
	-0.4677	γ_1	-0.4675	-0.4678	-0.4693	-0.4633	-0.4763	-0.4673	-0.4694	-0.4828	-0.4683
	-0.3750	γ_2	-0.3704	-0.3762	-0.3788	-0.3819	-0.3642	-0.3749	-0.3741	-0.3529	-0.3735
		ρ	0.0980	0.1009	0.0998	0.4966	0.4988	0.5002	0.8937	0.9008	0.9002
Beta	0.0000	μ	-0.0010	0.0002	0.0012	0.0006	-0.0022	0.0020	-0.0005	0.0035	0.0007
($\alpha=4$, $\beta=4$)	1.0000	σ^2	1.0001	0.9999	0.9996	1.0016	1.0046	1.0023	0.9997	0.9971	0.9997
	0.0000	γ_1	0.0021	0.0009	-0.0024	-0.0004	-0.0130	-0.0005	0.0035	-0.0048	-0.0018
	-0.5455	γ_2	-0.5464	-0.5457	-0.5436	-0.5421	-0.5466	-0.5471	-0.5461	-0.5453	-0.5438
Beta	0.0000	μ	-0.0013	0.0004	0.0004	-0.0007	0.0010	0.0026	-0.0004	0.0050	0.0010
($\alpha=4$, $\beta=3/2$)	1.0000	σ^2	1.0013	1.0019	1.0004	1.0007	1.0010	0.9991	0.9994	1.0006	1.0002
	-0.6939	γ_1	-0.6925	-0.6948	-0.6951	-0.6909	-0.6829	-0.6889	-0.6901	-0.6944	-0.6969
	-0.0686	γ_2	-0.0748	-0.0706	-0.0686	-0.0766	-0.0700	-0.0803	-0.0745	-0.0595	-0.0601
		ρ	0.0996	0.1015	0.0999	0.4953	0.4998	0.5001	0.8926	0.9000	0.9001

Beta ($\alpha=4, \beta=4$)	0.0000	μ	0.0023	0.0011	-0.0007	0.0009	-0.0020	-0.0004	0.0001	0.0055	0.0004
	1.0000	σ^2	0.9986	1.0004	1.0007	1.0000	1.0053	0.9991	1.0006	0.9994	1.0005
	0.0000	γ_1	-0.0031	-0.0027	0.0003	-0.0015	0.0037	-0.0085	0.0000	0.0013	-0.0018
	-0.5455	γ_2	-0.5502	-0.5439	-0.5450	-0.5474	-0.5625	-0.5447	-0.5491	-0.5451	-0.5435
Beta ($\alpha=4, \beta=5/4$)	0.0000	μ	-0.0005	0.0006	-0.0016	-0.0008	0.0003	-0.0002	-0.0003	0.0056	0.0002
	1.0000	σ^2	1.0012	0.9999	1.0018	1.0008	0.9980	1.0020	1.0004	0.9949	1.0001
	-0.8482	γ_1	-0.8461	-0.8512	-0.8477	-0.8505	-0.8490	-0.8578	-0.8465	-0.8491	-0.8505
	0.2210	γ_2	0.2114	0.2303	0.2214	0.2254	0.2107	0.2313	0.2132	0.2170	0.2238
Beta ($\alpha=4, \beta=4$)		ρ	0.1007*	0.0984	0.1004	0.4973	0.5018	0.5009	0.8967	0.9009	0.9000
	0.0000	μ	0.0009	-0.0002	-0.0004	0.0008	0.0033	0.0011	-0.0001	-0.0013	0.0012
	1.0000	σ^2	1.0038	0.9998	1.0003	0.9996	0.9981	1.0002	1.0016	1.0068	0.9999
	0.0000	γ_1	0.0009	-0.0007	-0.0021	-0.0049	0.0103	0.0037	-0.0001	-0.0222	0.0035
Weibull ($\alpha=6, \beta=10$)	-0.5455	γ_2	-0.5521	-0.5447	-0.5430	-0.5429	-0.5421	-0.5438	-0.5461	-0.5573	-0.5455
	0.0000	μ	0.0014	-0.0009	-0.0015	0.0011	0.0039	0.0005	-0.0008	-0.0006	0.0024
	1.0000	σ^2	0.9995	0.9981	0.9969	0.9981	1.0024	1.0001	1.0017	1.0052	0.9997
	-0.3733	γ_1	-0.3767	-0.3699	-0.3722	-0.3766	-0.3785	-0.3733	-0.3745	-0.3853	-0.3655
Beta ($\alpha=4, \beta=4$)	0.0355	γ_2	0.0377	0.0330	0.0289	0.0426	0.0740	0.0397	0.0308	0.0134	0.0360
		ρ	0.1024	0.1002	0.1010	0.5027	0.4962	0.4992	0.9055	0.9007	0.9002
	0.0000	μ	-0.0006	0.0004	0.0014	0.0006	0.0020	-0.0005	-0.0005	0.0034	0.0012
	1.0000	σ^2	1.0000	1.0001	0.9990	0.9991	1.0007	0.9996	1.0012	1.0040	1.0003
Gamma ($\alpha=\beta=10$)	0.0000	γ_1	0.0022	-0.0008	-0.0019	-0.0034	-0.0044	-0.0047	0.0017	0.0194	0.0001
	-0.5455	γ_2	-0.5478	-0.5459	-0.5477	-0.5446	-0.5463	-0.5446	-0.5480	-0.5458	-0.5480
	0.0000	μ	0.0009	0.0002	-0.0012	0.0001	-0.0017	-0.0013	-0.0007	0.0034	0.0013
	1.0000	σ^2	1.0003	0.9984	1.0010	0.9999	1.0009	0.9994	0.9997	1.0067	1.0020
Beta ($\alpha=4, \beta=4$)	0.8222	γ_1	0.8221	0.8227	0.8195	0.8281	0.8124	0.8214	0.8235	0.8279	0.8204
	0.6000	γ_2	0.6037	0.6012	0.6161	0.6221	0.6077	0.6212	0.6103	0.5599	0.5872
		ρ	0.0982	0.0994	0.0992	0.5001	0.5018	0.5009	0.9040	0.9000	0.8998
	0.0000	μ	0.0002	-0.0001	-0.0002	-0.0006	0.0014	0.0010	-0.0018	-0.0004	-0.0001
Beta ($\alpha=4, \beta=4$)	1.0000	σ^2	0.9990	1.0007	1.0009	0.9984	0.9957	1.0015	1.0016	0.9990	1.0002
	0.0000	γ_1	-0.0006	0.0038	0.0004	0.0019	0.0034	0.0051	-0.0013	-0.0103	-0.0044

Rayleigh ($\alpha=1/2$, $\mu=\sqrt{(\pi/2)}$)	-0.5455	γ_2	-0.5444	-0.5438	-0.5463	-0.5449	-0.5442	-0.5479	-0.5449	-0.5446	-0.5430
	0.0000	μ	0.0007	0.0002	0.0020	0.0000	0.0022	0.0011	-0.0016	-0.0012	-0.0005
	1.0000	σ^2	1.0007	0.9977	1.0036	0.9984	1.0016	0.9999	0.9997	1.0014	1.0007
	0.6311	γ_1	0.6337	0.6259	0.6362	0.6323	0.6510	0.6381	0.6288	0.6175	0.6273
	0.2451	γ_2	0.2482	0.2276	0.2547	0.2513	0.2629	0.2497	0.2412	0.2228	0.2429
Beta ($\alpha=4$, $\beta=4$)		ρ	0.1018	0.1010	0.1014	0.5075	0.4973		0.8522		
	0.0000	μ	0.0000	-0.0007	0.0017	0.0006	-0.0034		-0.0007		
	1.0000	σ^2	1.0014	0.9994	0.9967	0.9974	0.9991		0.9984		
	0.0000	γ_1	0.0007	-0.0002	-0.0019	-0.0009	-0.0157	unable to calculate intermediate correlation	-0.0010	unable to calculate intermediate correlation	unable to calculate intermediate correlation
Pareto ($\theta=10$, $\alpha=1$)	-0.5455	γ_2	-0.5441	-0.5432	-0.5469	-0.5444	-0.5545		-0.5441		
	0.0000	μ	0.0009	0.0007	0.0015	-0.0020	0.0028		-0.0013		
	1.0000	σ^2	1.0019	0.9991	1.0017	0.9941	1.0147		0.9968		
	2.8111	γ_1	2.8352	2.8252	2.8152	2.8743	2.8740		2.8487		
	14.8286	γ_2	15.8458	15.0666	15.0088	16.1822	15.3198		15.9355		
Beta ($\alpha=4$, $\beta=2$)		ρ	0.0991	0.0995	0.0998	0.4940	0.4994	0.5013	0.90404*	0.8997	0.9004
	0.0000	μ	-0.0008	-0.0010	0.0021	-0.0012	0.0001	0.0002	0.0003	0.0001	-0.0011
	1.0000	σ^2	0.9996	0.9991	1.0002	1.0000	0.9987	1.0004	1.0020	0.9973	1.0030
	-0.4677	γ_1	-0.4670	-0.4647	-0.4696	-0.4659	-0.4684	-0.4693	-0.4717	-0.4666	-0.4740
Beta ($\alpha=4$, $\beta=2$)	-0.3750	γ_2	-0.3745	-0.3812	-0.3720	-0.3764	-0.3748	-0.3700	-0.3751	-0.3748	-0.3591
	0.0000	μ	-0.0005	0.0006	-0.0001	-0.0006	-0.0001	-0.0010	0.0005	0.0000	-0.0013
	1.0000	σ^2	1.0009	0.9986	1.0008	0.9998	0.9997	1.0030	1.0016	0.9979	1.0020
	-0.4677	γ_1	-0.4685	-0.4675	-0.4704	-0.4675	-0.4684	-0.4745	-0.4721	-0.4665	-0.4702
	-0.3750	γ_2	-0.3738	-0.3742	-0.3697	-0.3720	-0.3751	-0.3720	-0.3746	-0.3749	-0.3685
Beta ($\alpha=4$, $\beta=2$)		ρ	0.0984	0.0997	0.0999	0.4928	0.5029	0.4992	0.9046*	0.8996	0.8997
	0.0000	μ	0.0009	-0.0005	-0.0017	-0.0003	-0.0011	-0.0005	0.0005	-0.0028	0.0011
	1.0000	σ^2	0.9986	0.9987	1.0018	0.9994	1.0098	0.9996	1.0001	1.0001	0.9989
	-0.4677	γ_1	-0.4672	-0.4669	-0.4678	-0.4669	-0.4734	-0.4701	-0.4683	-0.4720	-0.4582
	-0.3750	γ_2	-0.3753	-0.3711	-0.3768	-0.3775	-0.3658	-0.3762	-0.3743	-0.3732	-0.3839
Beta	0.0000	μ	-0.0023	0.0003	0.0005	-0.0003	0.0033	0.0002	0.0002	-0.0015	0.0006

(α=4, β=3/2)	1.0000	σ^2	0.9999	0.9994	1.0004	0.9986	1.0094	0.9985	0.9990	0.9985	0.9989
	-0.6939	γ_1	-0.6933	-0.6946	-0.6960	-0.6932	-0.7081	-0.6984	-0.6945	-0.6858	-0.6865
	-0.0686	γ_2	-0.0726	-0.0674	-0.0634	-0.0688	-0.0535	-0.0609	-0.0669	-0.0863	-0.0793
Beta (α=4, β=2)	0.0000	ρ	0.0954	0.1008	0.0991	0.4944	0.5011	0.5005	0.9055*	0.8996	0.9000
	0.0000	μ	0.0000	0.0004	0.0021	0.0001	-0.0057	-0.0004	0.0007	0.0011	-0.0002
	1.0000	σ^2	0.9995	1.0010	0.9987	0.9983	0.9997	1.0001	0.9990	0.9975	1.0009
Beta (α=4, β=5/4)	-0.4677	γ_1	-0.4679	-0.4694	-0.4659	-0.4668	-0.4788	-0.4669	-0.4683	-0.4566	-0.4706
	-0.3750	γ_2	-0.3735	-0.3687	-0.3786	-0.3706	-0.3644	-0.3769	-0.3745	-0.3831	-0.3737
	0.0000	μ	0.0010	-0.0017	0.0006	-0.0019	-0.0035	0.0015	0.0007	0.0008	0.0002
Weibull (α=6, β=10)	1.0000	σ^2	0.9995	1.0007	0.9998	1.0014	0.9948	0.9978	0.9988	0.9953	0.9997
	-0.8482	γ_1	-0.8510	-0.8452	-0.8484	-0.8430	-0.8582	-0.8463	-0.8487	-0.8354	-0.8503
	0.2210	γ_2	0.2335	0.2180	0.2324	0.2051	0.2193	0.2264	0.2257	0.1873	0.2257
Beta (α=4, β=2)	0.0000	ρ	0.0997	0.1002	0.1002	0.4965	0.4995	0.4996	0.9035*	0.9000	0.9003
	0.0000	μ	0.0001	0.0006	-0.0010	0.0011	-0.0007	-0.0011	-0.0005	-0.0021	0.0009
	1.0000	σ^2	1.0004	0.9990	0.9997	0.9998	0.9984	1.0000	0.9980	1.0040	1.0009
Weibull (α=6, β=10)	-0.4677	γ_1	-0.4685	-0.4660	-0.4672	-0.4674	-0.4457	-0.4671	-0.4663	-0.4896	-0.4649
	-0.3750	γ_2	-0.3752	-0.3767	-0.3733	-0.3755	-0.3862	-0.3770	-0.3756	-0.3348	-0.3722
	0.0000	μ	-0.0003	-0.0011	-0.0008	0.0020	-0.0007	0.0004	-0.0010	-0.0030	0.0015
Beta (α=4, β=2)	1.0000	σ^2	1.0020	1.0009	1.0019	0.9991	0.9929	0.9981	0.9975	1.0012	1.0015
	-0.3733	γ_1	-0.3742	-0.3732	-0.3714	-0.3742	-0.3655	-0.3748	-0.3708	-0.3913	-0.3698
	0.0355	γ_2	0.0348	0.0399	0.0294	0.0341	0.0442	0.0364	0.0324	0.0774	0.0413
Beta (α=4, β=2)	0.0000	ρ	0.0995	0.1004	0.1002	0.5005	0.4999	0.5001	0.9014	0.8988	0.9001
	0.0000	μ	0.0013	-0.0007	0.0005	-0.0008	0.0075	-0.0011	0.0002	-0.0012	0.0013
	1.0000	σ^2	0.9982	1.0001	0.9997	0.9995	1.0036	0.9992	0.9985	1.0009	0.9995
Gamma (α=β=10)	-0.4677	γ_1	-0.4685	-0.4663	-0.4689	-0.4645	-0.4511	-0.4650	-0.4702	-0.4691	-0.4640
	-0.3750	γ_2	-0.3729	-0.3740	-0.3744	-0.3783	-0.3846	-0.3752	-0.3664	-0.3726	-0.3787
	0.0000	μ	-0.0010	-0.0004	-0.0003	-0.0011	0.0058	-0.0008	0.0003	0.0000	0.0014
Gamma (α=β=10)	1.0000	σ^2	0.9993	1.0022	0.9972	0.9986	1.0101	0.9984	0.9993	1.0043	1.0012
	0.8222	γ_1	0.8226	0.8237	0.8185	0.8232	0.8558	0.8197	0.8224	0.8429	0.8216
	0.6000	γ_2	0.6031	0.6043	0.5741	0.5982	0.6764	0.5981	0.6040	0.6648	0.5894

Beta ($\alpha=4, \beta=2$)	0.0000	ρ	0.1002	0.0986	0.0993	0.4989	0.4970	0.5009	0.9001	0.9005	0.9001
		μ	-0.0002	-0.0017	0.0008	-0.0002	0.0014	-0.0007	-0.0002	-0.0020	-0.0014
	1.0000	σ^2	1.0004	1.0019	1.0009	0.9982	0.9991	1.0028	1.0002	1.0023	1.0006
	-0.4677	γ_1	-0.4675	-0.4660	-0.4673	-0.4670	-0.4833	-0.4693	-0.4688	-0.4635	-0.4740
	-0.3750	γ_2	-0.3742	-0.3783	-0.3753	-0.3723	-0.3686	-0.3705	-0.3737	-0.3840	-0.3687
Rayleigh ($\alpha=1/2, \mu=\sqrt{(\pi/2)}$)	0.0000	μ	0.0007	-0.0016	-0.0009	-0.0006	0.0001	-0.0016	0.0002	-0.0020	-0.0013
	1.0000	σ^2	1.0012	0.9999	0.9997	0.9988	1.0021	1.0018	0.9999	1.0032	1.0006
	0.6311	γ_1	0.6320	0.6324	0.6317	0.6291	0.6160	0.6321	0.6285	0.6260	0.6260
	0.2451	γ_2	0.2442	0.2451	0.2451	0.2398	0.2266	0.2480	0.2356	0.2150	0.2403
		ρ	0.1016	0.0986	0.1006	0.4893	0.5033		0.7882		
Beta ($\alpha=4, \beta=2$)	0.0000	μ	0.0008	0.0009	0.0003	-0.0001	0.0020		-0.0005		
	1.0000	σ^2	0.9990	0.9991	0.9993	0.9979	1.0056		1.0013		
	-0.4677	γ_1	-0.4686	-0.4692	-0.4687	-0.4651	-0.4709	unable to calculate intermediate correlation	-0.4694	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	-0.3750	γ_2	-0.3700	-0.3690	-0.3734	-0.3756	-0.3811		-0.3775		
	0.0000	μ	0.0009	-0.0006	0.0005	-0.0005	-0.0015		-0.0006		
Pareto ($\theta=10, \alpha=1$)	1.0000	σ^2	1.0060	0.9980	0.9978	0.9970	0.9972		0.9935		
	2.8111	γ_1	2.8164	2.7930	2.7916	2.8076	2.7515		2.7670		
	14.8286	γ_2	14.4994	14.6783	14.4444	14.7280	13.6716		13.9189		
		ρ	0.0976	0.0997	0.1005	0.4915	0.4994	0.4993	0.8957*	0.8997	0.8997
	0.0000	μ	0.0023	0.0008	-0.0013	0.0004	0.0001	0.0014	0.0001	0.0002	0.0005
Beta ($\alpha=4, \beta=3/2$)	1.0000	σ^2	0.9971	0.9998	1.0018	1.0007	0.9988	0.9993	1.0010	0.9973	0.9983
	-0.6939	γ_1	-0.6970	-0.6968	-0.6924	-0.6945	-0.6943	-0.6925	-0.6911	-0.6929	-0.6876
	-0.0686	γ_2	-0.0560	-0.0651	-0.0738	-0.0672	-0.0683	-0.0624	-0.0732	-0.0685	-0.0618
	0.0000	μ	0.0012	0.0005	-0.0007	-0.0002	-0.0001	-0.0003	0.0000	0.0001	0.0003
	1.0000	σ^2	0.9992	1.0007	1.0012	0.9992	0.9998	0.9999	1.0016	0.9978	0.9994
Beta ($\alpha=4, \beta=3/2$)	-0.6939	γ_1	-0.6931	-0.6922	-0.6945	-0.6953	-0.6942	-0.6922	-0.6924	-0.6926	-0.6909
	-0.0686	γ_2	-0.0674	-0.0724	-0.0698	-0.0649	-0.0697	-0.0693	-0.0720	-0.0697	-0.0693
		ρ	0.0982	0.0999	0.0998	0.4919	0.4973	0.5004	0.9059*	0.8995	0.8999
	0.0000	μ	0.0000	-0.0004	0.0000	-0.0003	0.0020	0.0006	0.0009	-0.0039	-0.0002

$(\alpha=4, \beta=3/2)$	1.0000	σ^2	1.0003	0.9996	0.9992	1.0026	0.9999	0.9985	0.9983	0.9970	0.9995
	-0.6939	γ_1	-0.6960	-0.6950	-0.6921	-0.6944	-0.6974	-0.6886	-0.6946	-0.6921	-0.6963
	-0.0686	γ_2	-0.0640	-0.0633	-0.0697	-0.0723	-0.0730	-0.0732	-0.0690	-0.0834	-0.0665
Beta	0.0000	μ	-0.0003	-0.0001	-0.0005	0.0007	0.0002	-0.0008	0.0008	-0.0043	-0.0005
$(\alpha=4, \beta=5/4)$	1.0000	σ^2	0.9987	1.0005	1.0003	0.9979	0.9989	1.0018	0.9974	0.9946	1.0009
	-0.8482	γ_1	-0.8443	-0.8491	-0.8468	-0.8473	-0.8385	-0.8466	-0.8477	-0.8442	-0.8514
	0.2210	γ_2	0.2102	0.2222	0.2158	0.2195	0.2071	0.2184	0.2197	0.1956	0.2201
Beta		ρ	0.1001	0.0992	0.1021	0.4952	0.5011	0.5003	0.9047*	0.8984	0.8999
	0.0000	μ	-0.0004	-0.0005	0.0014	0.0006	0.0004	-0.0009	0.0000	0.0006	0.0002
	$(\alpha=4, \beta=3/2)$	1.0000	σ^2	1.0002	1.0011	0.9981	0.9980	1.0005	1.0012	1.0019	0.9970
Weibull	-0.6939	γ_1	-0.6942	-0.6939	-0.6967	-0.6927	-0.7011	-0.6980	-0.6951	-0.6912	-0.6889
	-0.0686	γ_2	-0.0725	-0.0677	-0.0623	-0.0689	-0.0581	-0.0634	-0.0683	-0.0386	-0.0689
	0.0000	μ	-0.0016	0.0021	0.0012	-0.0002	-0.0017	-0.0012	-0.0003	0.0002	0.0003
$(\alpha=6, \beta=10)$	1.0000	σ^2	1.0008	0.9981	1.0001	0.9986	1.0060	1.0034	1.0023	0.9941	0.9988
	-0.3733	γ_1	-0.3679	-0.3693	-0.3749	-0.3718	-0.3871	-0.3748	-0.3718	-0.3590	-0.3676
	0.0355	γ_2	0.0289	0.0263	0.0345	0.0328	0.0462	0.0348	0.0282	0.0381	0.0297
Beta		ρ	0.0991	0.1013	0.1015	0.4995	0.4998	0.4991	0.8993	0.8992	0.9000
	0.0000	μ	0.0004	-0.0005	-0.0002	0.0004	0.0042	-0.0012	0.0006	0.0026	-0.0004
	$(\alpha=4, \beta=3/2)$	1.0000	σ^2	0.9999	0.9998	0.9982	1.0000	1.0034	1.0012	0.9997	1.0025
Gamma	-0.6939	γ_1	-0.6939	-0.6942	-0.6933	-0.6944	-0.6898	-0.6932	-0.6938	-0.6922	-0.6951
	-0.0686	γ_2	-0.0720	-0.0678	-0.0671	-0.0701	-0.0667	-0.0688	-0.0714	-0.0617	-0.0658
	0.0000	μ	-0.0005	0.0015	-0.0002	0.0003	-0.0024	-0.0011	0.0010	0.0045	-0.0002
$(\alpha=\beta=10)$	1.0000	σ^2	1.0007	1.0002	1.0013	1.0003	0.9942	0.9996	0.9999	1.0097	1.0006
	0.8222	γ_1	0.8243	0.8171	0.8212	0.8200	0.8167	0.8240	0.8210	0.8379	0.8227
	0.6000	γ_2	0.6043	0.5739	0.5947	0.5994	0.5852	0.6186	0.5964	0.6032	0.5983
Beta		ρ	0.0997	0.1007	0.0991	0.4989	0.4996	0.4992	0.8980	0.8997	0.8997
	0.0000	μ	-0.0003	-0.0003	-0.0010	-0.0002	0.0042	0.0004	-0.0013	-0.0008	0.0004
	$(\alpha=4, \beta=3/2)$	1.0000	σ^2	1.0033	1.0013	1.0005	0.9989	0.9963	1.0009	1.0030	0.9965
	-0.6939	γ_1	-0.6932	-0.6946	-0.6920	-0.6946	-0.6878	-0.6697	-0.6943	-0.7015	-0.6921
	-0.0686	γ_2	-0.0729	-0.0639	-0.0749	-0.0687	-0.0977	0.5110	-0.0693	-0.0623	-0.0659

Rayleigh ($\alpha=1/2$, $\mu=\sqrt{(\pi/2)}$)	0.0000	μ	-0.0004	-0.0001	0.0002	0.0000	0.0076	-0.0006	-0.0008	-0.0017	0.0002
	1.0000	σ^2	1.0001	0.9990	1.0012	0.9999	1.0094	0.9995	1.0025	0.9963	1.0006
	0.6311	γ_1	0.6318	0.6293	0.6310	0.6314	0.6679	0.6332	0.6309	0.6161	0.6343
	0.2451	γ_2	0.2415	0.2417	0.2489	0.2483	0.2928	0.2416	0.2522	0.2505	0.2461
Beta ($\alpha=4$, $\beta=3/2$)		ρ	0.1018	0.1020	0.1013	0.4734	0.4992		0.7496		
	0.0000	μ	-0.0007	0.0008	0.0012	-0.0004	-0.0044		-0.0004		
	1.0000	σ^2	1.0004	1.0009	1.0006	0.9993	1.0073		1.0027		
	-0.6939	γ_1	-0.6922	-0.6964	-0.6978	-0.6927	-0.7167	unable to calculate intermediate correlation	-0.6948	unable to calculate intermediate correlation	unable to calculate intermediate correlation
Pareto ($\theta=10$, $\alpha=1$)	-0.0686	γ_2	-0.0743	-0.0651	-0.0598	-0.0687	-0.0737		-0.0721		
	0.0000	μ	0.0007	-0.0001	0.0014	-0.0012	-0.0046		0.0007		
	1.0000	σ^2	1.0015	1.0008	1.0016	0.9999	0.9912		1.0021		
	2.8111	γ_1	2.7840	2.8368	2.8021	2.7988	2.7527		2.7875		
Beta ($\alpha=4$, $\beta=5/4$)	14.8286	γ_2	14.2145	15.1600	15.0286	14.1990	14.1522		14.1772		
		ρ	0.0972	0.1002	0.1004	0.4932	0.4994	0.4996	0.9071*	0.8997	0.8998
	0.0000	μ	0.0014	0.0007	0.0010	-0.0012	0.0002	0.0005	0.0016	0.0003	-0.0010
	1.0000	σ^2	0.9993	0.9964	0.9998	1.0012	0.9989	0.9977	1.0013	0.9973	1.0011
Beta ($\alpha=4$, $\beta=5/4$)	-0.8482	γ_1	-0.8499	-0.8455	-0.8484	-0.8464	-0.8484	-0.8503	-0.8501	-0.8476	-0.8501
	0.2210	γ_2	0.2227	0.2161	0.2183	0.2176	0.2216	0.2281	0.2223	0.2236	0.2256
	0.0000	μ	-0.0005	0.0002	-0.0002	0.0007	0.0000	0.0005	0.0017	0.0002	-0.0011
	1.0000	σ^2	1.0015	0.9997	0.9987	1.0022	0.9999	1.0003	1.0003	0.9977	1.0022
Weibull ($\alpha=6$, $\beta=10$)	-0.8482	γ_1	-0.8478	-0.8474	-0.8451	-0.8515	-0.8482	-0.8461	-0.8482	-0.8468	-0.8515
	0.2210	γ_2	0.2182	0.2161	0.2142	0.2308	0.2187	0.2202	0.2173	0.2188	0.2212
		ρ	0.0969	0.0999	0.0984	0.4935	0.5018	0.4990	0.8893	0.8998	0.9005
	0.0000	μ	-0.0001	0.0017	-0.0023	-0.0003	0.0050	0.0015	0.0010	-0.0035	-0.0021
Weibull ($\alpha=6$, $\beta=10$)	1.0000	σ^2	0.9986	0.9989	1.0026	0.9998	1.0015	0.9982	1.0007	0.9967	1.0037
	-0.8482	γ_1	-0.8495	-0.8525	-0.8454	-0.8464	-0.8308	-0.8469	-0.8490	-0.8575	-0.8521
	0.2210	γ_2	0.2301	0.2301	0.2095	0.2134	0.2126	0.2223	0.2219	0.2472	0.2334
	0.0000	μ	-0.0003	0.0010	-0.0005	0.0006	0.0028	0.0014	0.0013	-0.0061	-0.0023
Weibull ($\alpha=6$, $\beta=10$)	1.0000	σ^2	0.9999	1.0009	0.9985	0.9975	1.0007	0.9995	1.0011	0.9991	1.0015

	-0.3733	γ_1	-0.3728	-0.3722	-0.3722	-0.3719	-0.3600	-0.3658	-0.3736	-0.3861	-0.3816
	0.0355	γ_2	0.0333	0.0332	0.0358	0.0402	0.0453	0.0292	0.0304	0.0551	0.0435
		ρ	0.0981	0.1012	0.0998	0.4992	0.4958	0.5004	0.8979		
Beta	0.0000	μ	0.0013	-0.0009	-0.0001	-0.0013	0.0022	-0.0007	-0.0015		
($\alpha=4, \beta=5/4$)	1.0000	σ^2	0.9986	1.0001	0.9993	1.0003	1.0034	0.9986	1.0020		
	-0.8482	γ_1	-0.8494	-0.8452	-0.8487	-0.8455	-0.8639	-0.8526	-0.8494	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	0.2210	γ_2	0.2249	0.2145	0.2277	0.2163	0.2251	0.2259	0.2277		
Gamma	0.0000	μ	-0.0008	0.0015	-0.0001	0.0007	-0.0014	-0.0006	-0.0015		
($\alpha=\beta=10$)	1.0000	σ^2	0.9986	1.0032	1.0015	1.0015	0.9982	0.9988	0.9996		
	0.8222	γ_1	0.8229	0.8200	0.8206	0.8227	0.8067	0.8142	0.8213		
	0.6000	γ_2	0.6069	0.5919	0.5882	0.5948	0.5777	0.6136	0.6029		
		ρ	0.1005	0.0996	0.1017	0.4974	0.5030	0.4994	0.8964		0.9001
Beta	0.0000	μ	0.0007	0.0013	-0.0014	-0.0017	0.0017	-0.0003	-0.0014		-0.0008
($\alpha=4, \beta=5/4$)	1.0000	σ^2	0.9980	0.9998	1.0021	0.9995	1.0008	1.0002	0.9997		0.9981
	-0.8482	γ_1	-0.8476	-0.8516	-0.8460	-0.8444	-0.8495	-0.8454	-0.8465	unable to calculate intermediate correlation	-0.8437
	0.2210	γ_2	0.2175	0.2290	0.2119	0.2170	0.2113	0.2174	0.2176		0.2175
Rayleigh	0.0000	μ	0.0016	0.0004	0.0000	-0.0018	0.0017	0.0013	-0.0017		-0.0014
($\alpha=1/2, \mu=\sqrt{\pi/2}$)	1.0000	σ^2	1.0031	1.0000	1.0018	0.9980	0.9967	1.0001	0.9990		0.9960
	0.6311	γ_1	0.6356	0.6299	0.6335	0.6322	0.6299	0.6356	0.6333		0.6331
	0.2451	γ_2	0.2595	0.2480	0.2470	0.2515	0.2321	0.2461	0.2517		0.2458
		ρ	0.1010	0.1007	0.1002	0.4611	0.5008		0.7213		
Beta	0.0000	μ	0.0016	-0.0009	0.0001	-0.0008	0.0004		0.0000		
($\alpha=4, \beta=5/4$)	1.0000	σ^2	0.9991	1.0008	0.9999	1.0017	0.9960		1.0001		
	-0.8482	γ_1	-0.8502	-0.8473	-0.8491	-0.8478	-0.8556	unable to calculate intermediate correlation	-0.8481	unable to calculate intermediate correlation	unable to calculate intermediate correlation
	0.2210	γ_2	0.2237	0.2149	0.2274	0.2192	0.2360		0.2193		
Pareto	0.0000	μ	-0.0004	-0.0023	-0.0013	0.0016	-0.0010		0.0003		
($\theta=10, \alpha=1$)	1.0000	σ^2	1.0018	0.9924	0.9933	1.0129	0.9890		1.0026		
	2.8111	γ_1	2.8202	2.8070	2.7565	2.8603	2.7582		2.8311		
	14.8286	γ_2	14.6813	15.1172	13.6898	15.1862	14.0719		15.1393		

		ρ	0.1000*	0.1005	0.0989	0.5019*	0.4993	0.4986	0.9036*	0.8997	0.9004
Weibull	0.0000	μ	-0.0003	-0.0005	0.0003	0.0014	0.0000	-0.0005	0.0000	0.0001	0.0002
($\alpha=6, \beta=10$)	1.0000	σ^2	1.0010	1.0010	1.0010	1.0017	0.9987	1.0002	1.0008	0.9972	1.0016
	-0.3733	γ_1	-0.3739	-0.3764	-0.3702	-0.3758	-0.3739	-0.3745	-0.3736	-0.3714	-0.3724
	0.0355	γ_2	0.0261	0.0398	0.0325	0.0376	0.0393	0.0398	0.0420	0.0370	0.0343
Weibull	0.0000	μ	-0.0002	-0.0015	0.0008	0.0009	-0.0002	0.0009	0.0000	0.0000	-0.0002
($\alpha=6, \beta=10$)	1.0000	σ^2	0.9991	0.9969	0.9998	1.0000	0.9994	0.9982	1.0007	0.9976	1.0022
	-0.3733	γ_1	-0.3753	-0.3715	-0.3737	-0.3764	-0.3738	-0.3718	-0.3727	-0.3711	-0.3690
	0.0355	γ_2	0.0363	0.0265	0.0294	0.0409	0.0339	0.0378	0.0361	0.0327	0.0279
		ρ	0.0997	0.1010	0.1003	0.5015	0.5012	0.5013	0.9030	0.8994	0.9002
Weibull	0.0000	μ	0.0006	0.0015	-0.0008	-0.0012	0.0016	-0.0003	0.0007	0.0027	0.0008
($\alpha=6, \beta=10$)	1.0000	σ^2	1.0011	0.9984	0.9986	0.9997	1.0017	1.0002	0.9981	1.0002	1.0000
	-0.3733	γ_1	-0.3750	-0.3736	-0.3733	-0.3697	-0.3790	-0.3737	-0.3690	-0.3636	-0.3718
	0.0355	γ_2	0.0352	0.0297	0.0345	0.0344	0.0239	0.0357	0.0327	0.0453	0.0367
Gamma	0.0000	μ	0.0018	-0.0013	-0.0011	-0.0005	-0.0031	0.0015	-0.0001	0.0028	0.0005
($\alpha=\beta=10$)	1.0000	σ^2	1.0016	1.0009	0.9995	0.9992	0.9965	1.0015	0.9998	1.0019	0.9993
	0.8222	γ_1	0.8211	0.8217	0.8230	0.8189	0.8136	0.8182	0.8239	0.8396	0.8242
	0.6000	γ_2	0.5963	0.5919	0.5972	0.5860	0.5604	0.6093	0.6012	0.5849	0.6041
		ρ	0.0997	0.0992	0.0996	0.5003	0.4963	0.5004	0.9015	0.9000	0.9001
Weibull	0.0000	μ	0.0016	-0.0002	0.0004	0.0001	0.0008	0.0003	0.0006	-0.0032	0.0024
($\alpha=6, \beta=10$)	1.0000	σ^2	1.0004	1.0011	1.0009	1.0017	0.9941	1.0011	1.0013	0.9991	1.0006
	-0.3733	γ_1	-0.3731	-0.3733	-0.3723	-0.3743	-0.3646	-0.3702	-0.3717	-0.3921	-0.3693
	0.0355	γ_2	0.0450	0.0359	0.0321	0.0445	0.0289	0.0330	0.0362	0.0544	0.0303
Rayleigh	0.0000	μ	0.0010	0.0020	0.0026	-0.0011	0.0009	0.0002	0.0004	-0.0039	0.0029
($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	1.0000	σ^2	1.0010	1.0036	1.0004	1.0004	0.9988	0.9994	1.0029	1.0008	1.0013
	0.6311	γ_1	0.6305	0.6363	0.6309	0.6325	0.6487	0.6248	0.6315	0.6114	0.6328
	0.2451	γ_2	0.2385	0.2552	0.2470	0.2471	0.2687	0.2327	0.2445	0.2410	0.2302
		ρ	0.1007	0.1015	0.0998	0.5016	0.5018		0.8189		
Weibull	0.0000	μ	-0.0022	0.0016	0.0019	0.0006	0.0022	unable to	-0.0004	unable to	unable to

$(\alpha=6, \beta=10)$	1.0000	σ^2	1.0011	0.9960	0.9996	1.0014	0.9952	calculate intermediate correlation	0.9987	calculate intermediate correlation	calculate intermediate correlation
	-0.3733	γ_1	-0.3720	-0.3741	-0.3721	-0.3758	-0.3605		-0.3738		
	0.0355	γ_2	0.0346	0.0301	0.0313	0.0363	0.0294		0.0361		
Pareto $(\theta=10, \alpha=1)$	0.0000	μ	-0.0003	0.0010	0.0020	0.0011	-0.0001		-0.0010		
	1.0000	σ^2	0.9951	1.0012	1.0055	1.0019	0.9846		0.9960		
	2.8111	γ_1	2.7732	2.8248	2.8005	2.8358	2.6974		2.8201		
	14.8286	γ_2	13.9209	15.1452	14.2546	15.2307	12.9086		15.4302		
Gamma $(\alpha=\beta=10)$	0.0000	ρ	0.0985	0.0993	0.1011	0.5053	0.4990	0.4995	0.9088	0.8997	0.8999
		μ	0.0001	0.0023	-0.0010	0.0001	-0.0003	0.0013	-0.0015	-0.0005	-0.0010
	1.0000	σ^2	0.9985	1.0023	1.0012	1.0014	0.9987	1.0020	0.9992	0.9982	0.9993
Gamma $(\alpha=\beta=10)$	0.8222	γ_1	0.6267	0.8225	0.8278	0.6310	0.8245	0.8263	0.6287	0.8262	0.8050
	0.6000	γ_2	0.2348	0.6051	0.6004	0.2406	0.6189	0.6681	0.2360	0.6238	0.7146
	0.0000	μ	0.0012	0.0001	0.0002	-0.0007	-0.0004	0.0008	-0.0019	-0.0005	-0.0006
Gamma $(\alpha=\beta=10)$	1.0000	σ^2	0.9979	1.0004	0.9994	1.0002	0.9988	0.9994	0.9994	0.9983	0.9999
	0.8222	γ_1	0.6327	0.8242	0.8247	0.6305	0.8215	0.8288	0.6290	0.8238	0.8084
	0.6000	γ_2	0.2488	0.6097	0.5934	0.2392	0.6015	0.6013	0.2329	0.6071	0.6568
Gamma $(\alpha=\beta=10)$	0.0000	ρ	0.1009	0.1005	0.1003	0.5069	0.4966	0.5006	0.9103	0.8999	0.9004
		μ	0.0007	-0.0013	-0.0017	0.0014	-0.0019	0.0015	-0.0013	-0.0004	0.0010
	1.0000	σ^2	0.9998	1.0005	0.9974	1.0021	0.9980	1.0013	0.9982	1.0041	1.0018
Rayleigh $(\alpha=1/2,$ $\mu=\sqrt{(\pi/2)})$	0.8222	γ_1	0.8230	0.8249	0.8255	0.8210	0.7998	0.8199	0.8220	0.8467	0.8276
	0.6000	γ_2	0.5966	0.6123	0.5996	0.5987	0.5333	0.6506	0.6029	0.6762	0.6227
	0.0000	μ	0.0015	0.0005	0.0014	0.0003	0.0003	0.0010	-0.0013	-0.0022	0.0008
Gamma $(\alpha=\beta=10)$	1.0000	σ^2	1.0025	0.9999	1.0012	1.0011	0.9995	0.9992	0.9984	1.0068	1.0017
	0.6311	γ_1	0.6313	0.6306	0.6322	0.6298	0.6338	0.6339	0.6293	0.6539	0.6372
	0.2451	γ_2	0.2390	0.2511	0.2468	0.2410	0.2416	0.2551	0.2438	0.2872	0.2554
Gamma $(\alpha=\beta=10)$	0.0000	ρ	0.1027	0.0999	0.1011	0.5119	0.5014	0.5006	0.9216	0.9000	unable to calculate intermediate
		μ	-0.0014	0.0021	-0.0013	0.0010	0.0048	-0.0014	-0.0022	-0.0017	
	1.0000	σ^2	0.9991	1.0025	0.9999	1.0019	1.0013	0.9986	0.9991	1.0015	
	0.8222	γ_1	0.8244	0.8244	0.8222	0.8233	0.8297	0.8116	0.8242	0.8331	

	0.6000	γ_2	0.6016	0.6089	0.6127	0.5987	0.5711	0.6099	0.6088	0.6344	correlation
Pareto	0.0000	μ	-0.0005	0.0007	-0.0007	0.0015	0.0011	-0.0018	-0.0015	0.0001	
($\theta=10, \alpha=1$)	1.0000	σ^2	1.0027	1.0031	0.9999	1.0008	0.9980	0.9936	1.0004	1.0126	
	2.8111	γ_1	2.8346	2.8385	2.8327	2.7716	2.7245	2.7570	2.8348	2.8650	
	14.8286	γ_2	14.9970	15.8420	15.2727	13.9350	14.5061	13.7268	15.0958	15.1684	
		ρ	0.1006	0.0998	0.0998	0.5053	0.4990	0.5001	0.9092	0.8997	0.8998
Rayleigh	0.0000	μ	0.0011	-0.0010	0.0012	0.0011	-0.0002	-0.0004	0.0011	-0.0004	-0.0007
($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	1.0000	σ^2	1.0026	0.9995	0.9995	1.0026	0.9987	0.9997	1.0026	0.9981	0.9992
	0.6311	γ_1	0.6276	0.6339	0.6298	0.6276	0.6328	0.6307	0.6276	0.6345	0.6248
	0.2451	γ_2	0.2397	0.2547	0.2439	0.2397	0.2586	0.2581	0.2397	0.2623	0.2382
Rayleigh	0.0000	μ	0.0000	-0.0004	-0.0006	0.0004	-0.0003	-0.0007	0.0010	-0.0005	-0.0010
($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	1.0000	σ^2	1.0002	1.0020	1.0000	1.0008	0.9990	0.9967	1.0023	0.9982	0.9993
	0.6311	γ_1	0.6290	0.6323	0.6354	0.6282	0.6302	0.6259	0.6282	0.6327	0.6259
	0.2451	γ_2	0.2382	0.2438	0.2560	0.2410	0.2461	0.2469	0.2430	0.2509	0.2378
		ρ	0.1020	0.1005	0.1007	0.5117	0.4985	0.5022	0.9204	0.9015	
Rayleigh	0.0000	μ	0.0011	0.0003	0.0001	0.0011	-0.0037	0.0012	0.0011	0.0024	
($\alpha=1/2,$ $\mu=\sqrt{(\pi/2)}$)	1.0000	σ^2	1.0026	0.9991	0.9993	1.0026	0.9989	1.0006	1.0026	1.0060	unable to calculate intermediate correlation
	0.6311	γ_1	0.6276	0.6285	0.6312	0.6276	0.6078	0.6415	0.6276	0.6453	
	0.2451	γ_2	0.2397	0.2374	0.2464	0.2397	0.1990	0.2637	0.2397	0.2431	
Pareto	0.0000	μ	0.0000	-0.0010	-0.0002	0.0006	-0.0045	0.0004	0.0015	0.0025	
($\theta=10, \alpha=1$)	1.0000	σ^2	0.9977	0.9913	1.0002	1.0010	0.9978	1.0008	1.0040	1.0061	
	2.8111	γ_1	2.5642	2.7868	2.8130	2.5932	2.8157	2.7885	2.8227	2.7495	
	14.8286	γ_2	11.4506	14.5827	14.6175	11.9531	14.6560	14.1250	15.1463	13.5282	
		ρ	0.1036	0.1001	0.1010	0.5185	0.4989	0.5003	0.9316	0.8998	0.9008
Pareto	0.0000	μ	0.0015	0.0017	-0.0002	0.0015	-0.0004	0.0016	0.0015	-0.0005	0.0016
($\theta=10, \alpha=1$)	1.0000	σ^2	1.0035	1.0044	0.9982	1.0035	1.0015	1.0095	1.0035	1.0016	1.0030
	2.8111	γ_1	2.5910	2.7737	2.8072	2.5910	2.8521	2.9622	2.5910	2.8778	2.8103
	14.8286	γ_2	12.0005	14.0691	14.5515	12.0005	15.5181	19.4885	12.0005	16.3055	14.7608

Pareto	0.0000	μ	0.0000	-0.0007	0.0002	0.0006	-0.0005	0.0018	0.0014	-0.0006	0.0012
($\theta=10, \alpha=1$)	1.0000	σ^2	0.9977	0.9991	1.0011	1.0011	0.9978	1.0078	1.0038	0.9981	1.0018
	2.8111	γ_1	2.5649	2.8002	2.8132	2.5938	2.8132	2.8440	2.5951	2.8216	2.7976
	14.8286	γ_2	11.4601	14.5060	15.0909	11.9691	14.8300	15.0600	12.0141	14.8922	14.2911

GLD = Generalized Lambda Distribution Method, **FPM** = Flieshman Power Method, and **Fifth-Order** = Fifth-Order Polynomial Transformation Method

ρ = correlation

μ = mean

σ^2 = variance

γ_1 = skewness

γ_2 = kurtosis

** attempts were made to calculate an accurate correlation using delta values of 0.025, 0.02, and 0.01

* a delta value of 0.025 was needed to calculate an accurate correlation