2008

Tests for Correlation on Bivariate Nonnormal Distributions

Louanne Margaret Beversdorf
University of North Florida

TESTS FOR CORRELATION ON BIVARIATE NONNORMAL DISTRIBUTIONS

by

Louanne Margaret Beversdorf

A thesis submitted to the Department of Arts and Sciences
in partial fulfillment of the requirements for the degree of

Master of Science in Mathematical Sciences

UNIVERSITY OF NORTH FLORIDA

COLLEGE OF ARTS AND SCIENCES

June 2008

CERTIFICATE OF APPROVAL

The thesis of Louanne Margaret Beversdorf is approved (Date)

Signature Deleted 6/30/08

Signature Deleted 6-27-08

Signature Deleted 6-27-08

Committee Chairperson

Accepted for the Department:

Signature Deleted 7/3/08

Chairperson

Accepted for the College:

Signature Deleted 7/9/08

Dean

Accepted for the University:

Signature Deleted 11 July 2008

Dean of the Graduate School

ACKNOWLEDGEMENTS

It is a pleasure to thank the many people who made this thesis possible.

I could not overstate my gratitude to my favorite professor, Dr. Ping Sa, you have provided priceless inspiration and guidance. I would have been lost without you during this long process. I would also like to thank the many other people who have taught me statistics and mathematics in high school and as a student at the University of North Florida. Mr. Sentner, Dr. Haywood, Dr. Wilson, Dr. Hochwald, and Dr. Mohr for providing me the knowledge and aid I needed throughout my academic career. More specifically, Dr. Gleaton and Dr. Sen thank you also for serving on my thesis committee.

To my friends who have kept me going and allowed me to vent my frustrations, your humor and caring got me through the hard times. Thank you Albert, Richard, Shelby, Fred, Georgette, and Lori.

Last but not least, my family is most important. Thank you to my mother who passed away during my graduate college years, I wouldn't have even gone to college without your direction. I know that you still see my accomplishments and are proud of your youngest child. My father has been a rock for me to lean on my entire life. Thank you Dad, nothing makes me happier than seeing your proud grin, and I know this accomplishment will provide it for us both. Thank you, Janet, Rob, Amanda and Ian for your endless love and support.

TABLE OF CONTENTS

ABSTRACT

Many samples in the real world are very small in size and often do not follow a normal distribution. Existing tests for correlation have restrictions on the distribution of data and sample sizes, therefore the current tests cannot be used in some real world situations.

In this thesis, two tests are considered to test hypotheses about the population correlation coefficient. The tests are based on statistics transformed by a saddlepoint approximation and by Fisher's Z-transformation. The tests are conducted on small samples of bivariate nonnormal data and found to perform well.

Simulations were run in order to compare the type I error rates and power of the new test with other commonly used tests. The new tests controlled type I error rates well, and have reasonable power performance.

Chapter 1: Introduction

Bivariate data are data in which two variables are measured on an individual. If the variables are quantitative, one may be interested in describing the relationship between them. A scatter plot is often used to demonstrate the relationship in bivariate data. However, interpretation of these plots is subjective, so numerical summaries are preferred or used in conjunction with the graphical information. One measure used to describe the strength of linear relation between two quantitative variables is the linear correlation coefficient.

Sir Francis Galton (1886) published an essay introducing the idea of how two traits varied together (covaried) resulting in use of the term "regression". Karl Pearson (1896) based on suggestions made by Galton on regression, investigated the development of the linear correlation coefficient that would capture the relationship between two variables. Two variables are positively correlated if, whenever the value of one variable increases, the value of the other variable also increases. A negative correlation occurs when the value of one variable increases and the value of the other variable decreases. The parameter used to express this correlation is ρ (Greek letter rho), which has values ranging from -1 to 1, where -1 expresses a perfect negative linear association between the two variables; and 1 indicates a perfect positive linear association. Values of ρ near 0 indicate little or no linear association between the two variables.

The true relationship between the two variables is always unknown. People have proposed different estimators for ρ, and two of them are used frequently. The Spearman Rank Order Correlation is used for ordinal data, whereas the Pearson Product Moment Correlation is applied to interval and ratio data. These two different measures for the relationship between two variables are considered, each having corresponding inferential tests. The maximum likelihood estimator of ρ is the Pearson product-moment correlation coefficient. On the other hand, when the data is not bivariate normal and the sample size exceeds 10 the nonparametric Spearman rank correlation is useful. However, little work has been done when the distribution of the data is unknown and the sample size is relatively small. The methods given hereafter provide insight to useful measures for this situation.

1.1 Pearson Product-Moment Correlation Coefficient Estimator

The most popular estimator of correlation is the Pearson Product-Moment Correlation Coefficient estimator, *r*, which is a biased point estimator for ρ. However, the bias is small when *n* (sample size) is large. This estimator was developed by Pearson in 1896 for use on bivariate normal models.

Pearson's estimator, *r*, provides information about the degree of the linear relationship between the two variables Y_1 and Y_2. The statistic is given by:

$$r = \frac{\sum_{i=1}^{n}\left(Y_{i1} - \overline{Y}_1\right)\left(Y_{i2} - \overline{Y}_2\right)}{\left[\sum_{i=1}^{n}\left(Y_{i1} - \overline{Y}_1\right)^2\left(Y_{i2} - \overline{Y}_2\right)^2\right]^{1/2}}$$

where

(Y_{i1}, Y_{i2}) is the i^{th} observation of the bivariate data $(Y_{11}, Y_{12}),\ldots,(Y_{n1}, Y_{n2})$.

$\overline{Y}_1$ is the sample mean of Y_1 and $\overline{Y}_2$ is the sample mean of Y_2.

The range for r is from -1 to 1, with properties and interpretation corresponding to what it estimates, ρ.

The correlation coefficient r is a random variable, thus having a distribution function which depends on the population value of the correlation coefficient, ρ, and the sample size n. Researchers have done intensive work on the distribution of r (Fisher 1915; Stuart 1994). Tbey found that when $n = 2$ the distribution of r can be regarded as an extreme case of a U-shaped distribution. For $n = 3$ the density is still U-shaped, but if $n = 4$ the distribution is uniform when $\rho = 0$ and J-shaped otherwise. For $n > 4$ the density function is unimodal and increasingly skew as $|\rho|$ increases, as follows from the fact that the mode moves with ρ and r. For any ρ, the distribution of r slowly tends to normality as $n \rightarrow \infty$. (Stuart 1994)

When the population is bivariate normal and has equal variance parameters a test statistic can be derived to test H_o: $\rho = 0$. The three possible alternative hypotheses are:

1.) $H_a: \rho \neq 0$ for two tail test
2.) $H_a: \rho > 0$ for right tail test
3.) $H_a: \rho < 0$ for left tail test

The test statistic is $t^* = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

Under H_o, t* follows the Student's t-distribution with $(n-2)$ degrees of freedom, denoted $t_{(n-2)}$. The decision rule is to reject the null hypothesis if $|t^*| > t_{\alpha/2}$, or $t^* > t_\alpha$, or $t^* < -t_\alpha$, respectively, for tests 1.), 2.), and 3.), where t_p is the p^{th} upper percentile of $t_{(n-2)}$.

Disadvantages of this test include the need of a large sample of bivariate normal data and the ability to test only for $\rho = 0$.

1.2 Spearman Rank Correlation Coefficient

When the population is not bivariate normal and the sample size exceeds 10, a non-parametric statistic, Spearman Rank Correlation Coefficient (Spearman 1904), is usually used to measure the association between two variables when no transformation for the data can be found to approximate a bivariate normal distribution. The range for Spearman's statistic, r_s, is between -1 and 1, inclusive. The coefficient r_s equals 1 when the ranks for Y_1 are identical to those for Y_2, that is, when the case with rank 1 for Y_1 also has rank 1 for Y_2, etc. There is a perfect inverse association when r_s equals -1, meaning Y_1 has rank 1 and Y_2 has rank n. When there is little or no association between the ranks of Y_1 and Y_2, the Spearman rank correlation coefficient has a value near zero.

The Spearman rank correlation coefficient, denoted by r_s, is then defined as the ordinary Pearson product-moment correlation coefficient based on the ranks of the data:

$$r_s = \frac{\sum\left(R_{i1} - \overline{R}_1\right)\left(R_{i2} - \overline{R}_2\right)}{\left[\sum\left(R_{i1} - \overline{R}_1\right)^2\left(R_{i2} - \overline{R}_2\right)^2\right]^{1/2}},$$

where (R_{i1}, R_{i2}) are the ranks of (Y_{i1}, Y_{i2}) respectively, and $\overline{R}_1$ is the mean of the ranks of R_{i1} ($i = 1,2,\ldots n$) and $\overline{R}_2$ is the mean of the ranks of R_{i2} ($i = 1,2,\ldots n$).

The Spearman Rank Correlation Coefficient can also be used to test about the association between the two variables with the following hypotheses:

Ho: There is no association between Y_1 and Y_2

versus

1) Ha: There is an association between Y_1 and Y_2 (for a two-tail test)

2) Ha: There is a positive association between Y_1 and Y_2 (one-tail, upper)

3) Ha: There is a negative association between Y_1 and Y_2 (one-tail, lower)

When sample size *n*, exceeds 10, we use the following test statistic:

$$t^* = \frac{r_s\sqrt{n-2}}{\sqrt{1-r_s^2}}$$

t^* is approximately a *t*-distribution with *n-2* degrees of freedom under Ho. The decision rule is the same as for Pearson's statistic. This is a nonparametric test and thus may result in a lower power performance, and this test can also be used only for testing whether an association exists.

1.3 Summary

The motivation for this study is to test H_o: $\rho = \rho_0$, where ρ_0 can be values other than zero, for bivariate nonnormal data. Fisher's Z-transformation and a saddlepoint transformation are investigated and tested.

A detailed explanation of the methods is given in Chapter 2. A simulation study is introduced in Chapter 3 to examine type I error rates and the power performance. Simulation results are discussed in Chapter 4 and conclusions are stated in Chapter 5.

Chapter 2: Methods

In this research we investigate two statistics for testing the correlation coefficient of bivariate nonnormal populations. The two statistics are Fisher's z-transformation, denoted r_F, and the saddlepoint approximation, denoted r_L. These methods are used on bivariate nonnormal data sets with a small sample size. Bivariate data is represented as pairs of observations, namely of the form $(Y_{11}, Y_{12}), (Y_{21}, Y_{22}), \ldots (Y_{n1}, Y_{n2})$, where n is the sample size and where (Y_{i1}, Y_{i2}) is the ith observation of the bivariate data. The goal is to test if either of the two methods is appropriate for hypothesis testing about the population correlation coefficient, specifically for bivariate nonnormal data sets with a small sample size.

2.1 Fisher's Z-Transformation

The sampling distribution of r is complicated when $\rho \neq 0$, so Fisher (1915) derived an approximation procedure based on a transformation. Fisher's Z-transformation has limitations, it must be used on bivariate normal data for interval estimation of ρ when n is greater than 25. Also, the variance in the first variable's values must be independent of the other variable's values and the relationship between them must be linear. Fisher's Z transform can be regarded as the hyperbolic slope of the standardized least-squares regression line or more simply;

$$z' = \frac{1}{2}\log\frac{1+r}{1-r} = \text{arctanh}(r)$$

With large sample sizes, the distribution of the transformation is approximately normal with mean $\frac{1}{2}\log\frac{1+\rho}{1-\rho}$ and standard deviation $\frac{1}{\sqrt{n-3}}$. After standardizing, the statistic for Fisher's classical transformation is given by:

$$r_F = \left(\frac{1}{2}\log\frac{1+r}{1-r} - \frac{1}{2}\log\frac{1+\rho}{1-\rho} - \frac{\rho}{2(n-1)}\right)\sqrt{n-3}$$

and can be compared to a standard normal distribution. This transformation tends to normality much faster than r, with a variance almost independent of ρ.

2.2 Saddlepoint Approximation

Saddlepoint approximations were introduced to statistics by Daniels (1954). However, computations of these approximations only recently became feasible with the availability of inexpensive computing. In practice, statistical inference often involves test statistics with normal distributions, which are valid as sample sizes get large. For small sample size problems, these distributions give inaccurate results. Saddlepoint methods give approximations that are accurate to a higher order than these first-order approximations, and the accuracy holds for extremely small sample sizes (Huzurbazar 1999). Also, saddlepoint approximations provide good approximations to very small tail probabilities or to the density in the tails of the distributions.

The main requirement for calculating a saddlepoint approximation is that it must be possible to calculate a Laplace transform. Not necessarily a Laplace transform of the statistic of interest itself, but, rather, the Laplace transform of a low-dimensional variable that becomes transformed into the statistic of interest. Although the theory of saddlepoint approximations is very complex and outside the scope of this research, the application of the resulting approximations is straightforward.

Jensen (1995) transforms the Pearson correlation coefficient using the method of Laplace transformations to derive a function of r that can be normalized. Assuming a bivariate normal data set with correlation ρ, an approximation to the distribution of r is needed. The maximum likelihood estimate, Pearson's r, does not depend on the variances of Y_1 and Y_2, so these are set equal to one. We then obtain the joint density of r and $\sum_{i=1}^{n}\left(Y_{i1}-\overline{Y}_1\right)^2/(n-1)$, and $\sum_{i=1}^{n}\left(Y_{i2}-\overline{Y}_2\right)^2/(n-1)$ from Anderson (1984). Then the saddlepoint approximation, denoted r_L, can be calculated as follows:

$r_L = v + \frac{1}{v}\log\frac{u}{v}$, where

$$v = \operatorname{sgn}(r-\rho)\left\{2m\log\left(\frac{1-\rho r}{\sqrt{1-\rho^2}\sqrt{1-r^2}}\right)\right\}^{\frac{1}{2}},$$

$$u = \sqrt{m}\left(\frac{1-\rho r}{1-\rho^2}\right)^{\frac{3}{2}}\frac{r-\rho}{1-r^2},$$

$m = n - 4$, ρ is the correlation coefficient and r is the sample Pearson correlation coefficient

Jensen (1995) claims r_L is normally distributed to a high accuracy and that in most situations of practical interest after numerical analysis, Fisher's classical transformation, r_F, is very close to r_L.

2.3 Proposed Test

Both Fisher's and the saddlepoint transformations are derived for bivariate normal data. This research will investigate if they can be used for hypothesis testing on nonnormal bivariate data. The outcome of tests using the saddlepoint approximation is compared side-by-side with results using Fisher's statistic on small samples of bivariate nonnormal data. Also, since both of the statistics involve ρ, the tests are additionally conducted for nonzero values of ρ_0. The following hypotheses are tested:

$H_o : \rho = \rho_0$ versus
1.) $H_a : \rho \neq \rho_0$ for two tail test
2.) $H_a : \rho > \rho_0$ for right tail test
3.) $H_a : \rho < \rho_0$ for left tail test

The decision rule is to reject the null hypothesis respectively for tests 1, 2, and 3 when:

1.) $|r_F| > z_{\alpha/2}$ or $|r_L| > z_{\alpha/2}$, or

2.) $r_F, r_L > z_\alpha$

3.) $r_F, r_L < -z_\alpha$,

Chapter 3: Simulation Study

A comparative study via simulation is provided in this chapter. To carry out the simulation, bivariate nonnormal data with a specified correlation is needed. After the data is generated, all needed statistics are calculated and comparisons given as decision rules are performed. Finally, the process is repeated a large number of times to obtain simulation results. Section 3.1 illustrates how bivariate nonnormal data are generated, and Section 3.2 gives a description of how the simulation study is executed.

3.1 Generating Bivariate Nonnormal Data

In order to generate data to test on the r_F and r_L statistics, bivariate nonnormal data with a specified correlation structure is needed. Fleishman (1978) derived a method of generating univariate nonnormal random variables. Vale and Maurelli (1983) proposed generating multivariate nonnormal random variables with a specified correlation structure by combining the matrix decomposition procedure and Fleishman's method.

3.1.1 Fleishman's Method

Fleishman's method of generating univariate nonnormal random variables is based on the variable Y defined as $Y = a + bZ + cZ^2 + dZ^3$. Where Z is a standard normal random variable, and *a, b, c,* and *d* are constants chosen in such a way that Y has the desired coefficients of skewness and kurtosis (γ_1 and γ_2). For a standard distribution (with mean 0 and variance 1), after using the first fourteen moments of the standard normal variable and doing considerable algebraic manipulation, Fleishman showed that $a = -c$ and the constants *b, c,* and *d* need to be determined by simultaneously solving the following three nonlinear equations:

$$b^2 + 6bd + 2c^2 + 15d^2 - 1 = 0$$
$$2c\left(b^2 + 24bd + 105d^2 + 2\right) - \gamma_1 = 0$$
$$24\left\{bd + c^2\left(1 + b^2 + 28bd\right) + d^2\left(12 + 48bd + 141c^2 + 225d^2\right)\right\} - \gamma_2 = 0$$

Generate a standard normal variable Z and the constants *a, b, c,* and *d* are used to transform it, yielding a univariate nonnormal variable Y.

3.1.2 Vale and Maurelli's Expansion of Fleishman's Method

Vale and Maurelli (1983) suggested a method to generate bivariate nonnormal random data, (Y_1, Y_2). First choosing desired coefficients of skewness and kurtosis for the two populations (γ_{11} and γ_{21} and γ_{12} and γ_{22}) one must find solutions to the system of equations given in Fleishman's method. Using the set of skewness and kurtosis for the first population (γ_{11} and γ_{21}) the solutions to the system are $a_1 = - c_1$ and the constants b_1, c_1, and d_1. Again solving the system using the other set of skewness and kurtosis (γ_{12} and γ_{22}), for the second population the solutions are a_2, b_2, c_2, and d_2. Let Z_1, Z_2 be two standard normal correlated variables then Y_1 and Y_2 can be calculated with the following equations:

$$
\begin{aligned}
Y_1 &= a_1 + b_1 Z_1 + c_1 Z_1^{\,2} + d_1 Z_1^{\,3}, \\
Y_2 &= a_2 + b_2 Z_2 + c_2 Z_2^{\,2} + d_2 Z_2^{\,3}
\end{aligned}
\qquad (1)
$$

The correlation coefficient between Y_1 and Y_2 is then determined as follows:

$$\rho_{Y_1,Y_2} = \rho_{Z_1,Z_2}\left(b_1 b_2 + 3b_1 d_2 + 3b_2 d_1 + 9d_1 d_2\right) + \rho_{Z_1,Z_2}^2 2c_1 c_2 + \rho_{Z_1,Z_2}^3 6d_1 d_2$$

Given the desired correlation, ρ_{y_1,y_2}, the intermediate correlation, ρ_{Z_1,Z_2}, can be found by solving the above cubic equation. In general, there are three roots to a cubic polynomial. The root within the range of -1 and +1 is chosen. Next, apply the Cholesky factorization to the variance-covariance matrix, Σ, to find an upper triangular matrix, R, such that $\Sigma = R'R$. Since $\sigma_1^2 = \sigma_2^2 = 1$, the covariance matrix is

$$\Sigma = \begin{bmatrix} \operatorname{cov}(z_1, z_2) & 1 \\ 1 & \operatorname{cov}(z_1, z_2) \end{bmatrix} = \begin{bmatrix} \rho_{z_1,z_2} & 1 \\ 1 & \rho_{z_1,z_2} \end{bmatrix} = R'R .$$

Bivariate normal random variates, Z_1 and Z_2, with intermediate correlation ρ_{Z_1,Z_2}, can be obtained by $Z^* \times R$ where Z^* is a vector of independent standard normal variates, $Z^* = \begin{pmatrix} z_1^* \\ z_2^* \end{pmatrix} \sim N\left(\hat{0}, I_{2x2}\right)$. These Z_1 and Z_2 are input to Fleishman's transformation procedure in (1). This transforms the correlated standard normal variables, Z_1 and Z_2 into correlated nonnormal variables, Y_1 and Y_2.

Steyn (1993) has used this method in his construction of multivariate distributions with coefficient of kurtosis greater than one. Other limitations on Fleishman's method as well as some alternatives are further explained in Tadikamalla's paper (1980).

3.2 Simulation Description

Simulations are run with Fortran 77 for Windows on a Toshiba Satellite-A105 Laptop Computer. All the type I error rates and power comparisons for the test procedures use a simulation size of 100,000 in order to reduce experimental noise. Three programs for type I error rates are used. One program is constructed and used for each of left-tailed, right-tailed, and two-tailed tests given in Appendix B. The program is slightly modified for critical values for the three different tests. Another program is used to evaluate left-tailed power, given in Appendix C. Fortran 77 IMSL library was used for many important elements of the analysis. The DNEQNF function is called to solve the system of nonlinear equations to generate the data. The DZREAL function is called to solve for the correlation coefficient. The DCHFAC function is called to compute a Cholesky factorization on the covariance matrix needed to generate the data. The DRNMVN function is used to generate random bivariate standard normal variables with the covariance matrix from the previous function.

Population parameters of skewness and kurtosis are needed to generate the nonnormal data. Skewness is a measure of asymmetry of the distribution of a population. Skewness of zero indicates a symmetric distribution such as a normal distribution. Negative skewness indicates a longer left tail, meaning more data is in the left tail than would be for a normal distribution. Positive skewness indicates the same but in the right tail. Skewness is defined as $\gamma_1 = \frac{E(X-\mu)^3}{\sigma^3}$.

Kurtosis is a measure of tail behavior of a distribution (Johnson 1980). Higher kurtosis indicates more of the variance is due to infrequent extreme deviations, as opposed to frequent modestly-sized deviations. Kurtosis is defined as

$\gamma_2 = \frac{E(X-\mu)^4}{\sigma^4} - 3$. A normal distribution has kurtosis of zero.

Different values of skewness and kurtosis are chosen for this analysis in order to reflect different population distributions. Skewness values are -3, -1, 1, 3, chosen to represent some negatively skewed and some positively skewed distributions. Kurtosis values must be greater than one, so 3, 7, and 25 are chosen to represent a range of lighter to heavier-tailed distributions. Kurtosis of 3 is referred to as "small", 7 is "medium" and 25 is "large". All permutations of these pairs are used where both populations have either positive skewness or both have negative skewness, yielding $3 \times 2 \times 1 + 3 \times 2 \times 1 = 12$ different sets of population parameters for the two populations.

A relatively small sample size of 10 is used in the study and the test statistics r_L and r_F were investigated for type I error rates of left-tail, right-tail, and two-tail tests with the nominal levels of 0.01 and 0.05 for each sample. Comparisons in the simulation study use r_L and r_F and three critical values to evaluate the decision rule. Each test used z_α, $t_{(n-2,\alpha)}$, and $(z_\alpha + t_{(n-2,\alpha)})/2$ as critical values.

Algorithm

1. Input population parameters of skewness and kurtosis for the two populations
    a. Choosing two of these six pairs, (3,25) (-3,25) (1,7) (-1,7) (1,3) (-1,3), where the sign of skewness is the same for both populations.
2. Input population correlation for data generation (ρ = 0, 0.5, 0.7, 0.9)
3. Solve the system of equations to calculate coefficients *a, b, c,* and *d* for the two populations

4. Calculate the $\rho_{z_1 z_2}$ value needed for the standard normal variables* in order to produce desired correlation for data
5. Generate 10 independent random bivariate standard normal variables, Z_1^*, Z_2^*
6. Use Cholesky factorization to transform the independent standard normal variables, Z_1^*, Z_2^*, to correlated bivariate normal data, Z_1 and Z_2, with correlation $\rho_{z_1 z_2}$
7. Apply the transformation in (1) to obtain nonnormal sample data Y_1 and Y_2
8. Calculate the r_L and r_F values and compare to critical values for $t_{(n-2)}$ and z distribution and also the value of the average of the t and z critical values
    a. If $\rho = 0$, also calculate the Pearson and Spearman test statistics and compare to $t_{(n-2)}$ critical values.
9. Repeat steps 5 -8 for 99,999 more samples
10. Calculate the proportion (out of 100,000) that each test statistic falls in the rejection region

* When the value of the desired correlation is zero, the value for the correlation of the standard normal variables, $\rho_{z_1 z_2}$, must also be zero. This follows from the equations on page 12.

Since the type I error rates are calculated by simulation, there is an error involved in their computation. The type I error estimate is accurate with 95% confidence within the limits of $\pm 1.96\sqrt{\frac{p(1-p)}{100000}}$, where p is the nominal alpha level of .05 and .01. The result of adding will give a higher value for acceptance of the type I error rate. The consequential confidence limits are .051351, and .010617. Any type I error rates within these limits is considered controlled.

Chapter 4: Simulation Results

In the following discussion, the population parameters of skewness and kurtosis are referred to as pairs with skewness first and kurtosis second. For example the population of (1,3) has small skewness of one and small kurtosis of three. This analysis uses bivariate data, requiring two independent populations, so two pairs are presented and called a set of parameters. For example, (1,3) and (3,25) is a set of parameters where the first population has skewness equal to one and kurtosis equal to three and the second population has skewness equal to three and a larger kurtosis equal to 25.

4.1 Type I Error Rate Comparison

The type I error rates are the probability the null hypothesis is rejected when it is actually true, so this number should be at least as small as the nominal level of significance. Appendix tables A1 to A6 show complete type I error rate results for all distributions with a sample size of 10 for population correlations of 0, 0.5, 0.7, and 0.9 and levels of significance 0.05 and 0.01. The set of population parameters for skewness and kurtosis are in the first column with the first population's parameters in the first row and the second in the second row. Pearson and Spearman are evaluated with a t test for $\rho = 0$ only, and the type I error rates are reported in the first column with Pearson first and then Spearman underneath. Comparisons were made between the tests for saddlepoint and Fisher's transformation, given in the table as the two adjacent numbers within a given correlation column, r_L and r_F respectively.

Type I error rates are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row. Type I error rates falling above the bounds mentioned in Chapter 3 are in bold print (these limits are 0.051351 for $\alpha = 0.05$ and 0.010617 for $\alpha = 0.01$). The most important column is that where $\rho = 0$, since this can be used to test whether or not some correlation exists or whether a correlation exists that is positive or negative.

4.1.1 Left-Tail Type I Error Rate Comparisons

Tables 1 through 5 reference type I error rate results for the left-tail tests of five diverse populations. The sets of skewness and kurtosis are (3,25)(3,25); (1,7),(1,7); (-1,3),(-1,3); (-1,3),(-1,7); and (3,25),(1,7). The results for each of the r_L and r_F statistics are given when using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as the first, second, and third number.

First looking at the very important case when $\rho = 0$, Table 1 shows both the r_L and r_F statistics have controlled type I error rates using any of the three critical values with the 0.05 significance level. When the significance level is lowered to 0.01, Table 2 reveals some of the type I error rates using the z critical value are slightly inflated (in bold print). Spearman has inflated type I error rates for almost all populations and at both significance levels. Pearson has some controlled error rates for the 0.05 significance level, but those involving a population with large kurtosis are slightly inflated when the significance level is lowered to 0.01.

Table 1. Type I Error Rates for Left-Tail Test, $\alpha = 0.05$, $\rho = 0$

(3,25),(3,25)			(1,7), (1,7)			(-1,3),(-1,3)			(-1,3), (-1,7)			(3,25), (1,7)		
Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F
0.042	0.028	0.028	0.048	0.032	0.031	0.046	0.029	0.028	0.048	0.031	0.03	0.046	0.031	0.03
0.052	0.035	0.034	**0.052**	0.04	0.039	**0.052**	0.037	0.036	**0.052**	0.039	0.038	**0.052**	0.038	0.037
	0.042	0.041		0.049	0.047		0.047	0.045		0.049	0.047		0.047	0.045

Table 2. Type I Error Rates for Left-Tail Test, $\alpha = 0.01$, $\rho = 0$

(3,25),(3,25)			(1,7), (1,7)			(-1,3),(-1,3)			(-1,3), (-1,7)			(3,25), (1,7)		
Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F
0.011	0.004	0.005	0.011	0.002	0.003	0.008	0.002	0.002	0.01	0.002	0.003	**0.011**	0.003	0.004
0.012	0.007	0.008	**0.011**	0.005	0.006	**0.012**	0.004	0.005	**0.012**	0.005	0.006	**0.011**	0.006	0.007
	0.012	**0.012**		**0.011**	**0.012**		0.009	0.01		0.01	**0.011**		**0.012**	**0.013**

The results of the next case, $\rho = 0.5$, are in Table 3. Type I error rates are controlled for both r_L and r_F statistics using all critical values.

Table 3. Type I Error Rates for Left-Tail Test, $\alpha = 0.05$, $\rho = 0.5$

(3,25),(3,25)		(1,7), (1,7)		(-1,3),(-1,3)		(-1,3), (-1,7)		(3,25), (1,7)	
r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
0.02843	0.0281	0.02825	0.02518	0.0313	0.02778	0.02929	0.02627	0.02209	0.01944
0.03479	0.03394	0.03598	0.03182	0.03994	0.03552	0.03738	0.03312	0.02897	0.02528
0.04235	0.04077	0.04532	0.03994	0.05047	0.04441	0.04712	0.04147	0.0375	0.0325

When $\rho = 0.7$, the t and the averaged critical values have controlled rates for both significance levels. The z critical value is also satisfactory when $\alpha = 0.01$, and has some slightly inflated rates when $\alpha = 0.05$ (Table 4).

Table 4. Type I Error Rates for Left-Tail Test, $\alpha = 0.05$, $\rho = 0.7$

(3,25),(3,25)		(1,7), (1,7)		(-1,3),(-1,3)		(-1,3), (-1,7)		(3,25), (1,7)	
r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
0.03231	0.02626	0.02919	0.0245	0.03264	0.02713	0.02959	0.02437	0.02174	0.01746
0.04213	0.03443	0.03741	0.0312	0.04109	0.03455	0.03734	0.03128	0.02879	0.02304
0.05372	0.04436	0.04753	0.03922	**0.05167**	0.04326	0.04691	0.03916	0.03774	0.03057

For the largest population correlation, $\rho = 0.9$, Table 5 shows the result that both statistics are controlled when using the *t* critical value. However, some inflated type I error rates occur, at both significance levels, for the z and averaged critical values when both populations have the same population parameters for skewness and kurtosis.

Table 5. Type I Error Rates for Left-Tail Test, $\alpha = 0.05$, $\rho = 0.9$

(3,25),(3,25)		(1,7), (1,7)		(-1,3),(-1,3)		(-1,3), (-1,7)		(3,25), (1,7)	
r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
0.04582	0.03555	0.03156	0.02352	0.03488	0.02641	0.02544	0.019	0.01123	0.00787
0.05638	0.04469	0.04029	0.03069	0.04378	0.03384	0.03332	0.02469	0.01511	0.01085
0.0688	**0.055**	**0.05153**	0.03916	**0.05497**	0.04257	0.04221	0.03206	0.02072	0.01456

Only slight differences in type I error rates are present between the results for the saddlepoint and Fisher's transformation. Results using the *t* critical value achieves very good type I error rates for all of the distributions. The *z* critical value only results in a few slightly inflated type I error rates and more often for the saddlepoint approximation than for the Fisher's transformation. The average of the *t* and *z* has similar results, only inflated twice out of 19 times when the *z* is inflated for the saddlepoint approximation. The averaged critical value is not inflated at all using Fisher's transformation.

Overall, the most cases of inflation occur when the population correlation is higher, 0.7 or 0.9 or when the *z*-test is used. One more result worth mentioning is that the only place the error rate is at all inflated, for the averaged critical value tests, is when both populations have the same population parameters for skewness and kurtosis and kurtosis is large. The mixed distributions have controlled rates for r_L and r_F and both levels of significance using the t critical value and only slightly inflated in the $\rho = 0$ case, for the z values in the 0.01 significance level. The most unusual instance when comparing the two results from the two different significance levels is that when $\alpha = 0.05$ none of the three tests fail for $\rho = 0$, but the z is slightly inflated for $\alpha = 0.01$.

More elaborate left-tail type I error rate results are given in Appendix Tables A1 and A2. Table A1 uses a significance level of 0.05, while Table A2 uses a significance level of 0.01.

4.1.2 Right-Tail Type I Error Rate Comparison

Right-tail type I error rates for the above-referenced distributions are given in Tables 6 and 7 for the $\rho = 0$ case, and significance levels $\alpha = 0.05$ and $\alpha = 0.01$, respectively. With the right-tail test, most type I error rates are very inflated, the only values that really stand out are the tests where the t critical value are used.

A great result is for the t test when $\rho = 0$, type I error rates for both significance levels are controlled for the saddlepoint approximation, r_L. When $\alpha = 0.05$, r_F is controlled as well, but with $\alpha = 0.01$, r_F is least inflated when the kurtosis of at least one population is small or medium. The Pearson and Spearman t- test all have inflated type I errors, except two for Spearman (opposite of the left-tail test) when $\alpha = 0.05$.

Table 6. Type I Error Rates for Right-Tail Test, $\alpha = 0.05$, $\rho = 0$

(3,25),(3,25)			(1,7), (1,7)			(-1,3),(-1,3)			(-1,3), (-1,7)			(3,25), (1,7)		
Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F
0.064	0.048	0.047	**0.053**	0.036	0.035	**0.054**	0.036	0.035	**0.052**	0.034	0.034	**0.058**	0.04	0.04
0.051	**0.055**	**0.054**	0.051	0.044	0.043	**0.053**	0.044	0.043	**0.052**	0.043	0.042	**0.053**	0.049	0.047
	0.064	**0.063**		**0.054**	**0.052**		**0.054**	**0.052**		**0.053**	0.051		**0.059**	**0.057**

Table 7. Type I Error Rates for Right-Tail Test, $\alpha = 0.01$, $\rho = 0$

(3,25),(3,25)			(1,7), (1,7)			(-1,3),(-1,3)			(-1,3), (-1,7)			(3,25), (1,7)		
Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F	Pearson, Spearman	r_L	r_F
0.024	0.01	**0.012**	**0.013**	0.003	0.005	**0.013**	0.003	0.004	**0.013**	0.003	0.004	**0.017**	0.005	0.006
0.011	**0.016**	**0.018**	**0.013**	0.007	0.008	**0.012**	0.007	0.008	**0.012**	0.007	0.008	**0.012**	0.009	**0.011**
	0.025	**0.026**		**0.014**	**0.015**		**0.013**	**0.014**		**0.013**	**0.014**		**0.017**	**0.018**

Not all results are consistent in the right-tail tests, so the rest of the type I error rates can be examined as a comprehensive result in Appendix Tables A3 and A4. For example, when $\alpha = 0.01$ some slightly inflated type I errors occur for the saddlepoint statistic with the medium correlation $\rho = 0.7$ and controlled values for $\rho = 0$, $\rho = 0.5$, and $\rho = 0.9$, such as for the distribution of (3,25) (1,3) and (-3,25)(-1,3). So, it seems the saddlepoint is a little bit more controlled than Fisher's and both more controlled when $\alpha =$ 0.01.

Overall, the *t* tests perform the best for right-tail tests with the least amount of inflated type I error rates. When $\rho = 0$, the *t* tests are controlled for both levels of significance for the saddlepoint statistic and this is also true for Fisher's when the kurtosis of both populations is not large.

4.1.3 Two-Tailed Type I Error Rate Comparison

The results of the two-tailed tests are similar to that of the right-tail test, but more controlled. These results are given in Appendix Tables A5 and A6 with significance levels of 0.05 and 0.01, respectively.

The outcomes of the *z* tests are inflated for all situations so this will not be discussed further for two-tail tests. The outcome of the *t* test is controlled for all combinations of population correlation and significance levels when both populations have small kurtosis or one is small and the other medium or large. However, when one population has medium kurtosis and the other has large, or both are large, the type I error rates are inflated.

The important case when $\rho = 0$ has controlled type I error rates when $\alpha = 0.05$ for all distributions when the *t* test or the average of *t* and *z* is used. However, using the averaged critical value with $\alpha = 0.01$ the type I errors are slightly inflated for the two sets of populations with both populations having large kurtosis.

The rest of the correlations are inconsistent across the two significance levels. The best way to sum up these results of the *t* test is to say that it is controlled as long as both populations do not have large kurtosis, or when one is large and the other medium. The averaged critical value results in inflated type I error rates when either population has large kurtosis and also for larger population correlation values in the (-1,7) (-1,7) and (1,7) (1,7) populations. When $\alpha = 0.01$, the averaged critical value works better for the saddlepoint statistic, even for some situations where one population has large kurtosis.

Type I error rates for the tests using the r_L statistic are smaller than those using the r_F in the tests that result in controlled error rates except for some when $\rho = 0.9$ for $\alpha = 0.05$.

4.2 Power Results

The power tables give the power of left-tail tests which can be explained as the probability of rejecting the null hypothesis that $\rho = 0.7$ given that the true population correlation is actually less than 0.7. Ideally the power should be equal to one. Right-tail power is not evaluated as the power would not be realistic due to the high type I error rates. Power could not be evaluated for $\rho = 0$ since negatively correlated data could not be generated using Vale and Maurelli's methods. Also, power was examined for correlation of 0.5 but due to the very slow convergence, the results were inconclusive without using negatively correlated data, and therefore not included. The other correlation, 0.9, is not considered since the type I error rates for those instances were not consistent.

4.2.1 Left-Tail Test Power Results

The tables with the results of power analysis are given in Appendix Tables A7 and A8. Power results for all three tests show a relatively slow convergence. Again, the results are inconsistent across the two significance levels. Within the *t*- tests, the r_F statistic has higher power than r_L for $\alpha = 0.01$. Opposite, for $\alpha = 0.05$, the r_L statistic has higher power than r_F for tests that had controlled type I error rates. Looking at the *t* tests when $\alpha = 0.05$ about 10% power is added with each 0.10 step away from the hypothesized value. However, with $\alpha = 0.01$ the power doubles each step away and never gets larger than .2 for the t tests.

As expected, the z tests have higher power than the other two tests, but sometimes exceeded the type 1 error rate limits. The averaged critical value has higher power for saddlepoint than for Fisher's transformations on both significance levels.

Chapter 5: Conclusions

The proposed tests for saddlepoint transformation, r_L, and Fisher's transformation, r_F , perform similarly. Both r_L and r_F control type I error rates in the left-tailed test very well. The z critical point can be used for almost all left-tailed tests except for when $\rho =$ 0.9. This corresponds to a population with large kurtosis (heavy tails). However, even in this case of a larger population correlation the type I error rates for r_L and r_F using the z critical point are only slightly inflated. We are not able to furnish an explanation for this at this time. The results for the t test using r_L and r_F is definitely acceptable for a left-tail test which means that you can use this to test when the population correlation is zero and nonzero. The distinction of which of the two statistics has better power for these left-tailed tests is not clear regarding all three critical values.

Right- tailed and two-tailed tests did not achieve type I error rates as controlled as that of the left-tailed test. For a right-tail test, the saddlepoint and Fisher's transformation only perform well when both populations have small kurtosis, or when the kurtosis are small and medium and the population correlation is 0.7 or less. In these cases, the t test performs best. For two-tailed tests, the proposed tests only work for small to medium kurtosis. When the population correlation is 0.7 or less the saddlepoint statistic is slightly more controlled than Fisher's with the t test.

The most consistent and least inflated results come from a population with $\rho = 0$. The lowest type I error rates are achieved when using the t critical value with n-2 degrees of freedom. Pearson and Spearman also can be used for this test, but their results are less stable than that of r_L and r_F. The power performance is not as good as one would hope, but still reasonable.

When generating small samples of nonnormal data using Vale and Maurelli's method, the correlation is often not what is expected which can cause the spurious results of the power test. Further analysis should be conducted using different methods of generating the data, this would require further research in the area of generating small samples of nonnormal bivariate data. Once negatively correlated data can be generated, the power of the Pearson, Spearman, saddlepoint, and Fisher's statistics could be compared when $\rho = 0$. Increased sample sizes are expected to increase power performance as well. Overall, the new statistics can be useful for testing hypotheses on bivariate nonnormal populations.

APPENDIX

Appendix Table A1. Type I Error Rates for Left Tail Test, 0.05 level of significance

			RHO = 0		RHO = .5		RHO = .7		RHO = .9	
Skewness	Kurtosis	Pearson, Spearman	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	0.0416	0.0284	0.0281	0.021	0.0179	0.0323	0.0263	0.0458	0.0356
3	25	**0.0522**	0.0348	0.0339	0.029	0.0248	0.0421	0.0344	**0.0564**	0.0447
			0.0424	0.0408	0.0395	0.0334	**0.0537**	0.0444	**0.0688**	**0.055**
-3	25	0.0428	0.0292	0.0289	0.0218	0.0188	0.0325	0.0261	0.0445	0.0342
-3	25	**0.0538**	0.0357	0.0348	0.0298	0.0255	0.0429	0.0348	**0.0553**	0.0433
			0.0436	0.042	0.0397	0.0337	**0.0549**	0.0451	**0.0674**	**0.054**
-1	7	0.0475	0.0302	0.0298	0.0285	0.0256	0.0289	0.0241	0.0311	0.0235
-1	7	**0.0516**	0.0381	0.037	0.0368	0.0324	0.0369	0.0307	0.0395	0.0302
			0.0483	0.0463	0.0462	0.0409	0.0469	0.0387	0.0502	0.0383
1	7	0.0483	0.0316	0.0312	0.0283	0.0252	0.0292	0.0245	0.0316	0.0235
1	7	**0.0521**	0.0396	0.0385	0.036	0.0318	0.0374	0.0312	0.0403	0.0307
			0.0491	0.0473	0.0453	0.0399	0.0475	0.0392	**0.0515**	0.0392
1	3	0.0463	0.0286	0.0281	0.0309	0.0277	0.0321	0.0268	0.0354	0.0276
1	3	**0.0514**	0.0374	0.0362	0.0398	0.0351	0.0411	0.034	0.0448	0.0345
			0.0473	0.0454	0.0501	0.0444	**0.0527**	0.0434	**0.0555**	0.0434
-1	3	0.0461	0.0286	0.028	0.0313	0.0278	0.0326	0.0271	0.0349	0.0264
-1	3	**0.0517**	0.0371	0.0358	0.0399	0.0355	0.0411	0.0346	0.0438	0.0338
			0.047	0.0449	0.0505	0.0444	**0.0517**	0.0433	**0.055**	0.0426
-3	25	0.0441	0.0278	0.0272	0.0228	0.0198	0.0189	0.0153	0.002	0.0013
-1	3	**0.0519**	0.0357	0.0346	0.0304	0.0264	0.025	0.0203	0.0029	0.0019
			0.045	0.0431	0.0392	0.0343	0.0328	0.0266	0.0043	0.0028
3	25	0.0446	0.0284	0.0279	0.0226	0.0201	0.0199	0.0163	0.0019	0.0012
1	3	0.0512	0.0359	0.0349	0.0293	0.0259	0.026	0.0212	0.0027	0.0018
			0.0455	0.0436	0.0382	0.0329	0.0341	0.0274	0.0043	0.0026
3	25	0.0464	0.0307	0.0302	0.0221	0.0194	0.0217	0.0175	0.0112	0.0079
1	7	**0.0521**	0.0383	0.0371	0.029	0.0253	0.0288	0.023	0.0151	0.0109
			0.0472	0.0454	0.0375	0.0325	0.0377	0.0306	0.0207	0.0146
-3	25	0.0474	0.0317	0.0312	0.0213	0.0187	0.0219	0.0178	0.0115	0.0081
-1	7	**0.0527**	0.0389	0.0379	0.028	0.0244	0.0288	0.0235	0.0158	0.011
			0.0482	0.0464	0.0366	0.0318	0.0375	0.0303	0.0212	0.0151
-1	3	0.0482	0.0311	0.0304	0.0293	0.0263	0.0296	0.0244	0.0254	0.019
-1	7	**0.0521**	0.0389	0.0379	0.0374	0.0331	0.0373	0.0313	0.0333	0.0247
			0.0489	0.0473	0.0471	0.0415	0.0469	0.0392	0.0422	0.0321
1	3	0.0473	0.0301	0.0294	0.0299	0.0267	0.0298	0.025	0.0258	0.0194
1	7	**0.0522**	0.038	0.0366	0.038	0.0338	0.0379	0.0315	0.0331	0.0251
			0.0481	0.0461	0.0482	0.0424	0.0475	0.0396	0.0419	0.0321

The "Pearson, Spearman" column gives type I error rates using a $t_{(n-2)}$ critical point, with Pearson first and Spearman underneath. The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A2. Type I Error Rates for Left-Tail Test, 0.01 level of significance

		RHO = 0			RHO = .5		RHO = .7		RHO = .9	
Skewness	Kurtosis	Pearson, Spearman	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	**0.0113**	0.0039	0.0049	0.0008	0.0011	0.0009	0.001	0.0021	0.0019
3	25	**0.0118**	0.0069	0.0079	0.0021	0.0023	0.0028	0.0027	0.0063	0.0049
			0.0115	**0.0123**	0.0049	0.0047	0.0077	0.0067	**0.0142**	**0.011**
-3	25	**0.0114**	0.0033	0.0043	0.0006	0.0008	0.001	0.001	0.0023	0.002
-3	25	**0.0119**	0.0063	0.0073	0.0019	0.002	0.0028	0.0026	0.0061	0.0047
			0.0117	**0.0124**	0.0051	0.0049	0.008	0.0068	**0.0149**	**0.0113**
-1	7	0.0105	0.0023	0.0033	0.0016	0.002	0.0016	0.0017	0.0017	0.0016
-1	7	**0.0117**	0.0052	0.0063	0.0037	0.004	0.0036	0.0035	0.004	0.0032
			0.0108	**0.0118**	0.009	0.0088	0.0086	0.0074	0.0097	0.0073
1	7	0.0105	0.0022	0.0032	0.0015	0.0019	0.0015	0.0017	0.0017	0.0015
1	7	**0.0112**	0.0051	0.0059	0.004	0.0042	0.0039	0.0038	0.0041	0.0033
			0.0108	**0.0118**	0.0086	0.0084	0.0088	0.0077	0.0091	0.0069
1	3	0.009	0.0017	0.0024	0.0019	0.0023	0.0018	0.002	0.002	0.0018
1	3	**0.0116**	0.0041	0.0048	0.0044	0.0047	0.0044	0.0043	0.0046	0.0038
			0.0094	0.0102	0.0099	0.0096	0.0102	0.0089	**0.011**	0.0083
-1	3	0.0084	0.0016	0.0023	0.0017	0.0021	0.0019	0.0021	0.0023	0.002
-1	3	**0.0115**	0.0039	0.0047	0.0042	0.0046	0.0046	0.0044	0.0055	0.0045
			0.0087	0.0097	0.0096	0.0094	0.0101	0.0089	**0.0123**	0.0097
-3	25	0.0087	0.0015	0.0024	0.0012	0.0014	0.0006	0.0007	5E-05	5E-05
-1	3	**0.0123**	0.0039	0.0049	0.0026	0.0029	0.0019	0.0018	0.0001	0.0001
			0.009	0.0097	0.0065	0.0063	0.0048	0.0041	0.0004	0.0003
3	25	0.0085	0.0019	0.0026	0.0009	0.0011	0.0007	0.0008	6E-05	5E-05
1	3	**0.0121**	0.0041	0.0049	0.0025	0.0025	0.0018	0.0017	0.0002	0.0002
			0.0088	0.0095	0.0056	0.0054	0.0045	0.004	0.0003	0.0002
3	25	**0.0111**	0.0028	0.0038	0.0009	0.0012	0.0008	0.0009	0.0003	0.0002
1	7	**0.0114**	0.0058	0.0068	0.0023	0.0025	0.0022	0.0021	0.0009	0.0006
			0.0116	**0.0125**	0.0059	0.0057	0.0057	0.005	0.0024	0.0018
-3	25	**0.0109**	0.0029	0.0036	0.0011	0.0014	0.0008	0.0009	0.0003	0.0003
-1	7	**0.0114**	0.0058	0.0068	0.0029	0.0031	0.0022	0.0021	0.0009	0.0007
			0.0112	**0.012**	0.0063	0.0061	0.0055	0.0048	0.0024	0.0017
-1	3	0.0096	0.002	0.0029	0.0017	0.002	0.0016	0.0018	0.0015	0.0013
-1	7	**0.0119**	0.0047	0.0055	0.0039	0.0042	0.0042	0.004	0.0036	0.0029
			0.0099	**0.0107**	0.0086	0.0084	0.0091	0.008	0.0077	0.006
1	3	0.0094	0.0019	0.0027	0.0017	0.0021	0.0016	0.0017	0.0011	0.001
1	7	**0.0117**	0.0043	0.0052	0.0038	0.0041	0.0039	0.0037	0.0026	0.0021
			0.0097	0.0105	0.009	0.0088	0.0089	0.0078	0.007	0.0054

The "Pearson, Spearman" column gives type I error rates using a $t_{(n-2)}$ critical point, with Pearson first and Spearman underneath. The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A3. Type I Error Rates for Right-Tail Test, 0.05 level of significance

		RHO = 0			RHO = .5		RHO = .7		RHO = .9	
Skewness	Kurtosis	Pearson, Spearman	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	**0.0635**	0.0479	0.0474	**0.1168**	**0.1171**	**0.1419**	**0.142**	**0.1671**	**0.1666**
3	25	0.0511	**0.0555**	**0.0544**	**0.1308**	**0.1303**	**0.1579**	**0.1571**	**0.1835**	**0.1822**
			0.0642	**0.0626**	**0.1458**	**0.1447**	**0.1746**	**0.1733**	**0.2006**	**0.1985**
-3	25	**0.0654**	0.05	0.0494	**0.1179**	**0.1181**	**0.1431**	**0.1432**	**0.1664**	**0.1661**
-3	25	**0.0524**	**0.0573**	**0.0564**	**0.132**	**0.1316**	**0.1578**	**0.1571**	**0.1826**	**0.1816**
			0.0662	**0.0645**	**0.1465**	**0.1454**	**0.1742**	**0.1729**	**0.1998**	**0.198**
-1	7	**0.0528**	0.0362	0.0357	0.0508	0.051	**0.0587**	**0.0588**	**0.0674**	**0.0672**
-1	7	**0.0532**	0.0441	0.0429	**0.0616**	**0.0612**	**0.0699**	**0.0694**	**0.0799**	**0.0787**
			0.0538	**0.0517**	**0.0737**	**0.0728**	**0.0828**	**0.0817**	**0.0936**	**0.092**
1	7	**0.0533**	0.0356	0.0348	0.0511	0.0514	**0.0598**	**0.0599**	**0.0683**	**0.0681**
1	7	0.0512	0.0442	0.043	**0.0614**	**0.0611**	**0.0707**	**0.0702**	**0.0804**	**0.0795**
			0.0542	**0.0523**	**0.0737**	**0.0728**	**0.0827**	**0.0816**	**0.0937**	**0.0922**
1	3	**0.0539**	0.0353	0.0347	0.0431	0.0433	0.0461	0.0462	0.0497	0.0495
1	3	**0.0532**	0.0442	0.0428	**0.0525**	**0.0522**	**0.0566**	**0.0563**	**0.0601**	**0.0591**
			0.055	**0.0528**	**0.064**	**0.0633**	**0.0685**	**0.0676**	**0.0725**	**0.0708**
-1	3	**0.0535**	0.0357	0.035	0.0424	0.0424	0.0469	0.047	0.0495	0.0492
-1	3	**0.0529**	0.0443	0.0431	**0.0525**	**0.0523**	**0.0569**	**0.0565**	**0.0604**	**0.0598**
			0.0544	**0.0524**	**0.0635**	**0.0625**	**0.0694**	**0.0684**	**0.0728**	**0.0714**
-3	25	**0.0569**	0.0394	0.0389	**0.0666**	**0.0668**	**0.0774**	**0.0775**	**0.0943**	**0.0937**
-1	3	**0.0526**	0.0475	0.0465	**0.0783**	**0.078**	**0.0915**	**0.0909**	**0.1151**	**0.1136**
			0.0578	**0.0559**	**0.0921**	**0.091**	**0.1078**	**0.1065**	**0.139**	**0.136**
3	25	**0.0582**	0.0401	0.0396	**0.0666**	**0.0668**	**0.0796**	**0.0797**	**0.0968**	**0.0964**
1	3	**0.0524**	0.0494	0.048	**0.0794**	**0.079**	**0.0931**	**0.0926**	**0.1168**	**0.115**
			0.0591	**0.0573**	**0.0937**	**0.0927**	**0.1096**	**0.1082**	**0.1412**	**0.1383**
3	25	**0.0576**	0.0404	0.0399	**0.0781**	**0.0784**	**0.0925**	**0.0926**	**0.1068**	**0.1063**
1	7	**0.0533**	0.0486	0.0474	**0.0906**	**0.0902**	**0.1061**	**0.1055**	**0.124**	**0.1226**
			0.0585	**0.0567**	**0.1044**	**0.1032**	**0.122**	**0.1207**	**0.1431**	**0.1412**
-3	25	**0.0585**	0.0409	0.0403	**0.0773**	**0.0776**	**0.0925**	**0.0926**	**0.1081**	**0.1078**
-1	7	**0.0532**	0.0491	0.048	**0.0897**	**0.0893**	**0.1065**	**0.1059**	**0.1256**	**0.1243**
			0.0591	**0.0575**	**0.1042**	**0.103**	**0.1234**	**0.122**	**0.1444**	**0.1423**
-1	3	**0.0523**	0.0344	0.0337	0.0464	0.0467	0.051	0.0511	**0.0561**	**0.0558**
-1	7	**0.0523**	0.043	0.0418	**0.0565**	**0.0562**	**0.0618**	**0.0613**	**0.0677**	**0.0667**
			0.0533	0.051	**0.0681**	**0.0674**	**0.0743**	**0.0731**	**0.0812**	**0.0797**
1	3	**0.0521**	0.0349	0.0345	0.0466	0.0467	0.0507	0.0507	**0.0562**	**0.0558**
1	7	**0.0516**	0.0431	0.0419	**0.0571**	**0.0568**	**0.0607**	**0.0602**	**0.0669**	**0.0661**
			0.0529	0.051	**0.0686**	**0.0677**	**0.0732**	**0.0721**	**0.0798**	**0.0781**

The "Pearson, Spearman" column gives type I error rates using a $t_{(n-2)}$ critical point, with Pearson first and Spearman underneath. The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha + t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A4. Type I Error Rates for Right-Tail Test, 0.01 level of significance

			RHO = 0		RHO = .5		RHO = .7		RHO = .9	
Skewness	Kurtosis	Pearson, Spearman	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	0.0242	0.0098	0.012	0.0345	0.0408	0.0454	0.0524	0.0546	0.0629
3	25	0.0113	0.0159	0.0176	0.0499	0.0545	0.0644	0.0697	0.0772	0.0832
			0.0248	0.026	0.0703	0.0738	0.0884	0.0915	0.1062	0.1098
-3	25	0.0246	0.01	0.0122	0.0347	0.0409	0.0446	0.052	0.0555	0.0639
-3	25	0.0116	0.016	0.0178	0.0504	0.0555	0.0643	0.0698	0.0778	0.0837
			0.025	0.0263	0.0714	0.0748	0.0888	0.0926	0.1055	0.1092
-1	7	0.0128	0.0032	0.0044	0.0058	0.0075	0.0081	0.0103	0.0092	0.0117
-1	7	0.0117	0.0069	0.0081	0.0112	0.0135	0.0147	0.017	0.017	0.0195
			0.0132	0.0141	0.0218	0.0238	0.0266	0.0286	0.0301	0.0321
1	7	0.0134	0.0033	0.0045	0.0063	0.0086	0.0081	0.0104	0.0096	0.0123
1	7	0.0126	0.007	0.0082	0.0123	0.0142	0.0151	0.0173	0.0176	0.0202
			0.0139	0.0148	0.0216	0.0233	0.0262	0.028	0.0308	0.0325
1	3	0.0118	0.0025	0.0035	0.0039	0.0054	0.004	0.0057	0.0052	0.0068
1	3	0.0115	0.0056	0.0068	0.0084	0.0101	0.0089	0.0106	0.0104	0.0122
			0.0122	0.0131	0.0165	0.018	0.0177	0.0191	0.0196	0.0211
-1	3	0.0129	0.0028	0.0039	0.0037	0.0053	0.0042	0.0057	0.005	0.0067
-1	3	0.0118	0.0066	0.0078	0.0079	0.0097	0.0088	0.0106	0.0105	0.0123
			0.0132	0.0141	0.0159	0.0173	0.0179	0.0194	0.0201	0.0216
-3	25	0.0153	0.0042	0.0056	0.0092	0.0121	0.0108	0.0142	0.0083	0.0108
-1	3	0.0115	0.0088	0.0102	0.0176	0.021	0.0207	0.0239	0.0175	0.0207
			0.0156	0.0166	0.031	0.0332	0.0365	0.039	0.035	0.0378
3	25	0.0149	0.004	0.0054	0.0092	0.0121	0.0108	0.014	0.0085	0.0115
1	3	0.0117	0.0082	0.0095	0.0171	0.0199	0.0208	0.024	0.0182	0.0216
			0.0154	0.0164	0.0297	0.0321	0.0365	0.0392	0.0363	0.0393
3	25	0.0168	0.0051	0.0064	0.0133	0.0165	0.0171	0.021	0.0177	0.0222
1	7	0.0117	0.0091	0.0109	0.0224	0.0257	0.0284	0.0323	0.0314	0.0352
			0.0172	0.0183	0.037	0.0395	0.0456	0.0486	0.0519	0.0549
-3	25	0.0165	0.0049	0.0064	0.0135	0.0171	0.0172	0.0215	0.0167	0.0212
-1	7	0.0117	0.0092	0.0107	0.0232	0.0262	0.0292	0.0327	0.0299	0.0343
			0.0168	0.0179	0.0375	0.0398	0.0466	0.0496	0.0509	0.0539
-1	3	0.0127	0.0031	0.0043	0.0042	0.0056	0.0054	0.0069	0.0064	0.0084
-1	7	0.0122	0.0068	0.008	0.0086	0.0105	0.0107	0.0127	0.0125	0.0146
			0.013	0.0141	0.0175	0.019	0.0205	0.0221	0.0232	0.0248
1	3	0.013	0.0029	0.0042	0.0044	0.0061	0.0056	0.0075	0.0064	0.0086
1	7	0.0121	0.0066	0.0078	0.0091	0.0108	0.0114	0.0136	0.013	0.0151
			0.0133	0.0143	0.0173	0.0191	0.0208	0.0226	0.0241	0.0256

The "Pearson, Spearman" column gives type I error rates using a $t_{(n-2)}$ critical point, with Pearson first and Spearman underneath. The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A5. Type I Error Rates for Two-Tail Test, 0.05 level of significance

		RHO = 0			RHO = .5		RHO = .7		RHO = .9	
Skewness	Kurtosis	Pearson, Spearman	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	**0.0648**	0.0376	0.0395	**0.0757**	**0.0789**	**0.0974**	**0.0997**	**0.1223**	**0.1222**
3	25	**0.0532**	0.0499	0.0508	**0.0946**	**0.0961**	**0.122**	**0.1218**	**0.152**	**0.1484**
			0.0659	**0.0658**	**0.1193**	**0.1184**	**0.1525**	**0.1486**	**0.1879**	**0.1793**
-3	25	**0.064**	0.0368	0.0389	**0.0773**	**0.0806**	**0.0998**	**0.1022**	**0.1235**	**0.1232**
-3	25	**0.0535**	0.0494	0.0504	**0.0976**	**0.0989**	**0.1251**	**0.1249**	**0.1523**	**0.1481**
			0.0652	**0.065**	**0.1224**	**0.1217**	**0.1561**	**0.1525**	**0.1863**	**0.1782**
-1	7	**0.0543**	0.0253	0.0274	0.0306	0.032	0.0352	0.036	0.0425	0.0416
-1	7	**0.0539**	0.0378	0.0389	0.045	0.0452	0.051	0.0506	**0.0592**	**0.0564**
			0.0554	**0.0554**	**0.0646**	**0.0629**	**0.0729**	**0.0701**	**0.0823**	**0.0766**
1	7	**0.0534**	0.0254	0.0272	0.0314	0.033	0.0369	0.0377	0.0418	0.0414
1	7	**0.0544**	0.0376	0.0387	0.045	0.0457	**0.0526**	**0.0519**	**0.0599**	**0.0569**
			0.0545	**0.0544**	**0.0651**	**0.0638**	**0.0741**	**0.071**	**0.0822**	**0.0763**
1	3	0.0513	0.0233	0.025	0.0268	0.0281	0.0301	0.03	0.0324	0.031
1	3	**0.054**	0.0353	0.0363	0.0407	0.0406	0.0443	0.0429	0.0477	0.0443
			0.0526	**0.0524**	**0.0601**	**0.0582**	**0.0637**	**0.0604**	**0.0691**	**0.0626**
-1	3	**0.0524**	0.0238	0.0256	0.0273	0.0285	0.029	0.0291	0.032	0.031
-1	3	**0.0555**	0.0361	0.037	0.0401	0.0401	0.0436	0.0422	0.0472	0.0439
			0.0537	**0.0536**	**0.06**	**0.058**	**0.0643**	**0.0606**	**0.0687**	**0.0619**
-3	25	**0.0547**	0.0265	0.0284	0.0379	0.04	0.0432	0.0448	0.037	0.0397
-1	3	**0.0556**	0.0388	0.0398	**0.0535**	**0.0545**	**0.061**	**0.0613**	**0.0546**	**0.0565**
			0.0557	**0.0556**	**0.0743**	**0.0734**	**0.0836**	**0.0819**	**0.0788**	**0.0788**
3	25	**0.0541**	0.0259	0.0279	0.0374	0.0395	0.0429	0.0446	0.0388	0.0415
1	3	**0.0552**	0.0384	0.0393	**0.0533**	**0.0539**	**0.0608**	**0.0606**	**0.0571**	**0.0584**
			0.0553	**0.0551**	**0.074**	**0.0734**	**0.0837**	**0.0821**	**0.0805**	**0.0805**
3	25	**0.0571**	0.0293	0.0311	0.0451	0.0473	**0.055**	**0.057**	**0.0559**	**0.0579**
1	7	**0.0545**	0.0415	0.0426	**0.0612**	**0.0622**	**0.0739**	**0.0742**	**0.0752**	**0.0754**
			0.0581	**0.058**	**0.0827**	**0.0818**	**0.098**	**0.0958**	**0.1**	**0.0978**
-3	25	**0.0566**	0.0302	0.0321	0.0448	0.0474	**0.055**	**0.057**	**0.0551**	**0.0572**
-1	7	**0.0543**	0.0425	0.0435	**0.0618**	**0.0627**	**0.0742**	**0.0743**	**0.0747**	**0.0747**
			0.0578	**0.0577**	**0.0833**	**0.0826**	**0.0978**	**0.0956**	**0.0989**	**0.0967**
-1	3	**0.0516**	0.0235	0.0251	0.0275	0.0289	0.0298	0.0301	0.0317	0.0311
-1	7	**0.0536**	0.0353	0.0365	0.0413	0.0413	0.0438	0.043	0.0463	0.0444
			0.0528	**0.0527**	**0.0606**	**0.0589**	**0.0642**	**0.0612**	**0.0676**	**0.0626**
1	3	**0.0518**	0.0234	0.0254	0.0282	0.0293	0.031	0.0317	0.0316	0.0311
1	7	**0.0556**	0.0358	0.037	0.042	0.0422	0.0454	0.0444	0.0465	0.0443
			0.0529	**0.0529**	**0.0612**	**0.0597**	**0.0663**	**0.0629**	**0.0674**	**0.0621**

The "Pearson, Spearman" column gives type I error rates using a $t_{(n-2)}$ critical point, with Pearson first and Spearman underneath. The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha + t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A6. Type I Error Rates for Two-Tail Test, 0.01 level of significance

			RHO = 0		RHO = .5		RHO = .7		RHO = .9	
Skewness	Kurtosis	Pearson, Spearman	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	**0.0232**	0.0052	0.0078	**0.0172**	**0.0237**	**0.0237**	**0.0319**	**0.0297**	**0.0393**
3	25	**0.0125**	**0.0115**	**0.0146**	**0.0313**	**0.0376**	**0.0415**	**0.0489**	**0.052**	**0.0601**
			0.0238	**0.0265**	**0.054**	**0.0593**	**0.0688**	**0.074**	**0.0872**	**0.0916**
-3	25	**0.0229**	0.0054	0.0082	**0.0167**	**0.0232**	**0.0238**	**0.0318**	**0.0299**	**0.0399**
-3	25	**0.0121**	**0.0118**	**0.0147**	**0.0309**	**0.0366**	**0.0416**	**0.0489**	**0.053**	**0.0608**
			0.0235	**0.0263**	**0.0527**	**0.0576**	**0.0701**	**0.0754**	**0.0881**	**0.0923**
-1	7	**0.0132**	0.0015	0.0028	0.0019	0.0035	0.0026	0.0043	0.0036	0.0056
-1	7	**0.0124**	0.0046	0.0068	0.006	0.0083	0.0076	0.01	0.01	**0.0125**
			0.0136	**0.0158**	**0.0172**	**0.0197**	**0.0197**	**0.0219**	**0.0241**	**0.0255**
1	7	**0.012**	0.0014	0.0024	0.002	0.0035	0.003	0.0048	0.0034	0.0055
1	7	**0.0125**	0.0042	0.006	0.0063	0.0087	0.008	0.0103	0.0095	**0.0117**
			0.0124	**0.0149**	**0.0173**	**0.0198**	**0.0201**	**0.022**	**0.0233**	**0.0247**
1	3	**0.0106**	0.0011	0.0021	0.0014	0.0025	0.0015	0.0026	0.002	0.0033
1	3	**0.0123**	0.0035	0.0052	0.0044	0.0063	0.005	0.0067	0.0061	0.0075
			0.0111	**0.013**	**0.0137**	**0.0158**	**0.015**	**0.0167**	**0.0168**	**0.0176**
-1	3	**0.0109**	0.0012	0.002	0.0014	0.0025	0.0015	0.0025	0.0017	0.0029
-1	3	**0.0135**	0.0037	0.0054	0.0046	0.0064	0.005	0.0069	0.0059	0.0072
			0.0113	**0.0138**	**0.0137**	**0.0156**	**0.0152**	**0.0166**	**0.0165**	**0.017**
-3	25	**0.013**	0.0019	0.0033	0.003	0.005	0.004	0.0063	0.0023	0.0042
-1	3	**0.0125**	0.0053	0.0072	0.0084	**0.0116**	0.01	**0.0133**	0.0068	0.0099
			0.0135	**0.0157**	**0.0218**	**0.0249**	**0.0248**	**0.0278**	**0.0203**	**0.0235**
3	25	**0.0135**	0.0019	0.0032	0.0034	0.0057	0.0037	0.0064	0.0019	0.0038
1	3	**0.0118**	0.0053	0.0071	0.0089	**0.0119**	0.0101	**0.0134**	0.0072	0.0099
			0.0141	**0.016**	**0.0215**	**0.0245**	**0.0239**	**0.027**	**0.0201**	**0.0234**
3	25	**0.0163**	0.0023	0.0041	0.0054	0.0088	0.007	**0.0112**	0.006	0.0097
1	7	**0.0129**	0.0065	0.009	**0.0131**	**0.0166**	**0.0163**	**0.0203**	**0.0149**	**0.0196**
			0.0167	**0.0193**	**0.0279**	**0.0313**	**0.034**	**0.0374**	**0.0342**	**0.0377**
-3	25	**0.0157**	0.0023	0.0041	0.0056	0.0087	0.0077	**0.0112**	0.0066	0.0103
-1	7	**0.0124**	0.0065	0.0087	**0.0133**	**0.0167**	**0.0162**	**0.0201**	**0.0159**	**0.0202**
			0.0162	**0.0186**	**0.028**	**0.0312**	**0.034**	**0.0373**	**0.0343**	**0.0382**
-1	3	**0.0119**	0.0012	0.0024	0.0012	0.0026	0.0018	0.0032	0.002	0.0032
-1	7	**0.0128**	0.0042	0.0059	0.0049	0.0068	0.0059	0.0078	0.0062	0.008
			0.0123	**0.0144**	**0.0145**	**0.0166**	**0.0161**	**0.0177**	**0.017**	**0.0182**
1	3	**0.0121**	0.0011	0.0022	0.0015	0.0027	0.0019	0.0034	0.0021	0.0034
1	7	**0.0126**	0.0042	0.006	0.005	0.007	0.0064	0.0082	0.0063	0.0079
			0.0124	**0.0146**	**0.0148**	**0.0168**	**0.0159**	**0.0178**	**0.0168**	**0.0182**

The "Pearson, Spearman" column gives type I error rates using a $t_{(n-2)}$ critical point, with Pearson first and Spearman underneath. The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A7. Power Results for Left-Tail Test when ρ = 0.7, 0.05 level of significance

Skewness	Kurtosis	RHO = 0.7		RHO = 0.5		RHO = 0.4		RHO = 0.3		RHO = 0.2		RHO = 0.1	
		r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	0.0326	0.0264	0.1658	0.1442	0.2699	0.2422	0.3906	0.3583	0.5195	0.4871	0.648	0.6175
3	25	0.0427	0.0347	0.1964	0.1734	0.3099	0.2793	0.4354	0.4012	0.5617	0.53	0.6858	0.658
		0.0546	0.0451	0.23	0.2033	0.3493	0.3183	0.4781	0.4445	0.6026	0.5702	0.7195	0.6934
-3	25	0.032	0.0257	0.1633	0.1429	0.2661	0.2392	0.3891	0.3565	0.5213	0.4886	0.6489	0.6194
-3	25	0.0415	0.034	0.1948	0.1705	0.3067	0.276	0.4338	0.4	0.5646	0.532	0.6875	0.6583
		0.0541	0.044	0.2283	0.2021	0.3473	0.3153	0.4756	0.442	0.6048	0.5736	0.7212	0.6951
-1	7	0.0289	0.0241	0.1612	0.1424	0.2676	0.2402	0.3909	0.3577	0.5185	0.4848	0.639	0.6059
-1	7	0.0376	0.0307	0.1919	0.1685	0.3074	0.2771	0.4374	0.4023	0.5628	0.5292	0.6809	0.6495
		0.0471	0.0395	0.2257	0.1986	0.35	0.316	0.4824	0.4466	0.606	0.5719	0.7195	0.6891
1	7	0.0297	0.0247	0.161	0.1409	0.2699	0.2421	0.391	0.3587	0.5172	0.4833	0.6357	0.6046
1	7	0.0381	0.0314	0.1917	0.1682	0.3104	0.2791	0.4366	0.4021	0.5626	0.528	0.6784	0.646
		0.0479	0.04	0.2245	0.1983	0.3526	0.319	0.4821	0.446	0.6061	0.5718	0.7179	0.6865
1	3	0.0331	0.0277	0.1696	0.1494	0.2782	0.2509	0.3985	0.3669	0.5229	0.4891	0.6369	0.6059
1	3	0.0421	0.035	0.1998	0.1767	0.3182	0.2877	0.443	0.4086	0.5669	0.5333	0.6776	0.6469
		0.0529	0.0441	0.2335	0.2069	0.3586	0.3263	0.488	0.452	0.6114	0.576	0.7166	0.6859
-1	3	0.0328	0.0276	0.1706	0.1501	0.2765	0.2493	0.3986	0.3667	0.5219	0.4883	0.6393	0.6078
-1	3	0.0417	0.0347	0.2013	0.1777	0.3157	0.2857	0.4428	0.4089	0.5671	0.5327	0.6803	0.6492
		0.0524	0.0438	0.2344	0.2081	0.358	0.3242	0.4881	0.4522	0.6096	0.5763	0.7184	0.6879

The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A7. Power Results for Left-Tail Test when ρ = 0.7, 0.05 level of significance, continued…

Skewness	Kurtosis	RHO = 0.7		RHO = 0.5		RHO = 0.4		RHO = 0.3		RHO = 0.2		RHO = 0.1	
		r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
-3	25	0.019	0.0155	0.1476	0.1285	0.2544	0.2286	0.3818	0.3503	0.5112	0.4778	0.6333	0.6034
-1	3	0.0254	0.0203	0.1759	0.1542	0.2936	0.2634	0.4253	0.3925	0.5555	0.5219	0.6744	0.6434
		0.0332	0.0267	0.2066	0.1819	0.3353	0.302	0.4689	0.4346	0.5987	0.5646	0.7112	0.6828
3	25	0.0195	0.0156	0.1461	0.127	0.2539	0.2279	0.38	0.349	0.5123	0.4794	0.6366	0.6065
1	3	0.0256	0.0208	0.1746	0.1522	0.2923	0.263	0.4239	0.3905	0.557	0.5225	0.6778	0.6468
		0.0335	0.027	0.206	0.181	0.3315	0.3004	0.4671	0.4327	0.5998	0.5664	0.7147	0.6859
3	25	0.0212	0.0171	0.1471	0.1275	0.2533	0.2254	0.3773	0.3457	0.5138	0.4805	0.6382	0.6062
1	7	0.0281	0.0226	0.1768	0.154	0.2918	0.2627	0.4222	0.388	0.5594	0.5246	0.6793	0.6481
		0.0367	0.0296	0.2087	0.183	0.3328	0.3001	0.4672	0.4316	0.6028	0.5686	0.7169	0.6879
-3	25	0.0218	0.0174	0.1479	0.1286	0.2554	0.2277	0.3798	0.3478	0.5134	0.4797	0.6397	0.6078
-1	7	0.0291	0.0234	0.1778	0.155	0.2948	0.2648	0.4243	0.3905	0.5595	0.5247	0.6795	0.6497
		0.0374	0.0305	0.2094	0.1843	0.3361	0.3029	0.4694	0.4341	0.6023	0.5686	0.7171	0.6876
-1	3	0.0291	0.0245	0.1642	0.1444	0.2718	0.2446	0.3955	0.363	0.52	0.4871	0.6361	0.6045
-1	7	0.0371	0.0308	0.1942	0.1711	0.311	0.2809	0.4399	0.4062	0.5654	0.531	0.6771	0.6463
		0.0468	0.0391	0.2268	0.2008	0.3519	0.3193	0.4851	0.4491	0.6096	0.5748	0.7167	0.6849
1	3	0.0278	0.0229	0.1659	0.1458	0.2692	0.2417	0.3942	0.3621	0.5174	0.4841	0.6356	0.6032
1	7	0.0363	0.0297	0.1964	0.1731	0.3092	0.2787	0.4385	0.4053	0.5636	0.5288	0.6768	0.6458
		0.0463	0.0383	0.2289	0.2031	0.3503	0.3177	0.4835	0.4482	0.6067	0.5729	0.7159	0.6856

The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A8. Power Results for Left-Tail Test when ρ = 0.7, 0.01 level of significance

Skewness	Kurtosis	RHO = 0.7		RHO = 0.5		RHO = 0.4		RHO = 0.3		RHO = 0.2		RHO = 0.1	
		r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
3	25	0.0008	0.0009	0.0102	0.0117	0.0243	0.027	0.0514	0.0564	0.0989	0.1069	0.1715	0.1838
3	25	0.0027	0.0025	0.0269	0.0261	0.0582	0.0565	0.1111	0.1078	0.1906	0.1867	0.2978	0.2928
		0.0077	0.0067	0.0619	0.0549	0.118	0.1071	0.2027	0.1865	0.3126	0.2933	0.4401	0.4193
-3	25	0.001	0.0011	0.0105	0.0117	0.0247	0.0273	0.0525	0.0574	0.0991	0.1073	0.1732	0.1852
-3	25	0.0027	0.0027	0.0268	0.0258	0.0588	0.057	0.1105	0.1077	0.1906	0.1867	0.2987	0.2933
		0.0078	0.0066	0.0614	0.0547	0.1207	0.1093	0.2018	0.1863	0.3126	0.293	0.4413	0.4199
-1	7	0.0016	0.0017	0.0153	0.0166	0.0344	0.0373	0.0692	0.0738	0.1199	0.1272	0.1927	0.2037
-1	7	0.0036	0.0035	0.0343	0.0332	0.0688	0.0671	0.1261	0.1235	0.2038	0.1999	0.3038	0.2988
		0.0085	0.0074	0.0667	0.0604	0.1243	0.114	0.2093	0.1954	0.3132	0.295	0.4312	0.4122
1	7	0.0015	0.0016	0.0157	0.0172	0.0352	0.0381	0.0686	0.0737	0.1198	0.1278	0.195	0.2057
1	7	0.0038	0.0037	0.0331	0.0323	0.0702	0.0684	0.1254	0.1228	0.2047	0.2013	0.3054	0.3008
		0.009	0.0079	0.0648	0.059	0.1262	0.1159	0.2088	0.1946	0.316	0.2984	0.4319	0.4122
1	3	0.0018	0.0021	0.0186	0.0199	0.0379	0.0408	0.0733	0.0788	0.1259	0.1336	0.1988	0.2103
1	3	0.0046	0.0046	0.0384	0.0375	0.0754	0.0737	0.1335	0.1309	0.2127	0.2092	0.3101	0.3055
		0.0104	0.0093	0.0735	0.0672	0.1346	0.1237	0.2196	0.2055	0.3232	0.3058	0.4368	0.4169
-1	3	0.0016	0.0018	0.0184	0.0198	0.0377	0.0409	0.0748	0.0801	0.1275	0.1358	0.1995	0.2108
-1	3	0.0043	0.0041	0.0381	0.0372	0.0759	0.0742	0.1352	0.1326	0.2138	0.21	0.3117	0.3069
		0.0102	0.009	0.0723	0.066	0.1331	0.123	0.2189	0.2046	0.323	0.3058	0.4379	0.4182

The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha+t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix Table A8. Power Results for Left-Tail Test when ρ = 0.7, 0.01 level of significance, continued…

Skewness	Kurtosis	RHO = 0.7		RHO = 0.5		RHO = 0.4		RHO = 0.3		RHO = 0.2		RHO = 0.1	
		r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F	r_L	r_F
-3	25	0.0008	0.0009	0.0122	0.0134	0.0296	0.0322	0.0636	0.0682	0.1152	0.1228	0.1933	0.2045
-1	3	0.0019	0.0019	0.0273	0.0265	0.0627	0.0612	0.1195	0.1167	0.2017	0.1981	0.3062	0.3012
		0.005	0.0045	0.0564	0.0513	0.1161	0.107	0.203	0.1886	0.3112	0.2934	0.4346	0.415
3	25	0.0006	0.0007	0.0119	0.013	0.0301	0.0327	0.0629	0.0676	0.1137	0.1211	0.1909	0.2022
1	3	0.0019	0.0018	0.0271	0.0263	0.0631	0.0613	0.1207	0.118	0.2001	0.1963	0.3042	0.2994
		0.005	0.0045	0.0569	0.0513	0.1178	0.108	0.2028	0.1886	0.3111	0.2932	0.4347	0.4149
3	25	0.0011	0.0011	0.0114	0.0125	0.0282	0.0305	0.0586	0.0633	0.1101	0.1174	0.1831	0.1938
1	7	0.0022	0.0021	0.0259	0.0251	0.0598	0.0583	0.1134	0.1104	0.1936	0.1898	0.2962	0.2917
		0.0054	0.0047	0.055	0.0496	0.1133	0.1033	0.1968	0.1822	0.3055	0.2872	0.4298	0.4093
-3	25	0.0009	0.001	0.0116	0.0128	0.0289	0.0313	0.0594	0.0642	0.1104	0.1178	0.1854	0.196
-1	7	0.0023	0.0023	0.0275	0.0267	0.0613	0.0595	0.1152	0.1128	0.1934	0.1894	0.2979	0.2932
		0.0056	0.0049	0.0563	0.0509	0.1154	0.1055	0.1984	0.1836	0.3066	0.2872	0.4291	0.4099
-1	3	0.0016	0.0018	0.0166	0.0179	0.0371	0.0401	0.0716	0.0769	0.1233	0.1313	0.1975	0.2086
-1	7	0.0038	0.0037	0.0354	0.0344	0.0725	0.0707	0.1298	0.1272	0.2089	0.2054	0.309	0.3044
		0.0088	0.0079	0.0687	0.0624	0.1305	0.1201	0.2123	0.1982	0.3177	0.3004	0.4365	0.4164
1	3	0.0016	0.0017	0.0165	0.0179	0.0367	0.04	0.071	0.0763	0.1252	0.1331	0.1989	0.2094
1	7	0.004	0.0039	0.0351	0.0342	0.0735	0.0718	0.1291	0.1267	0.2112	0.2074	0.3077	0.3031
		0.0094	0.0082	0.0681	0.0623	0.1306	0.1208	0.214	0.1999	0.319	0.3015	0.4358	0.4167

The "r_L" and "r_F" results are calculated using the critical values $t_{(n-2,\alpha)}$, $(z_\alpha + t_{(n-2,\alpha)})/2$, and z_α, as first, second, and third number in the respective population's row.

Appendix B. Fortran Program, Type I error

```
! file nonnormal rv ge.for
        Use numerical_libraries
        implicit real*8 (a-h,o-z)
        integer size,simsize,set,RUN
        parameter (simsize=100000)

        real*8 y1(10),y2(10),ro,y1r(10),y2r(10)
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        real*8 probfst,probpfsz,probfsav,probfpt,probfpz,probfpav,
     *problst,problsz,problsav,problpt,problpz,problpav,probst,probpt

        integer gotpt, gotst,gotlpt,gotlpz,gotlpav,gotfpt,gotfpz,gotfpav,
     *gotlst,gotlsz,gotlsav,gotfst,gotfsz,gotfsav,sumpt,sumst,
     *sumlst,sumlsz,sumlsav,sumlpt,sumlpz,sumlpav,sumfst,sumfsz,sumfsav,
     *sumfpt,sumfpz,sumfpav

        REAL*8  y1bar,y1var,y1std,y2bar,y2var,y2std
        REAL*8  y1rbar,y1rvar,y1rstd,y2rbar,y2rvar,y2rstd
        real*8 rhohatp,rhohats,targUP,alpha
        real*8 z1(10),z2(10),res(32)

        integer nssize,iseed
        iseed=123457

        open (unit=9,file='F:\June 11\results
     *\left results ALPHA01.txt')
        write(9,*),'Left Tail'
        write(9,*),'Alpha=0.01'

        open (unit=10,file='F:\June 11\results
     *\left results ALPHA05.txt')
        write(10,*),'Left Tail'
        write(10,*),'Alpha=0.05'

        data yvar1,yvar2/1.0d+00,1.0d+00/


        DO 10000 RUN =1,12

        IF (RUN .EQ. 1) THEN
                YSKEW1 = 3.0D+00
                YKURT1 = 25.0D+00
```

```
        YSKEW2 = 3.0D+00
        YKURT2 = 25.0D+00
ENDIF

IF (RUN .EQ. 2) THEN
        YSKEW1 = -3.0D+00
        YKURT1 = 25.0D+00
        YSKEW2 = -3.0D+00
        YKURT2 = 25.0D+00

ENDIF

IF (RUN .EQ. 3) THEN
        YSKEW1 = -1.0D+00
        YKURT1 = 7.0D+00
        YSKEW2 = -1.0D+00
        YKURT2 = 7.0D+00
ENDIF

IF (RUN .EQ. 4) THEN
        YSKEW1 = 1.0D+00
        YKURT1 = 7.0D+00
        YSKEW2 = 1.0D+00
        YKURT2 = 7.0D+00
ENDIF

IF (RUN .EQ. 5) THEN
        YSKEW1 = 1.0D+00
        YKURT1 = 3.0D+00
        YSKEW2 = 1.0D+00
        YKURT2 = 3.0D+00
ENDIF

IF (RUN .EQ. 6) THEN
        YSKEW1 = -1.0D+00
        YKURT1 = 3.0D+00
        YSKEW2 = -1.0D+00
        YKURT2 = 3.0D+00
ENDIF

IF (RUN .EQ. 7) THEN
        YSKEW1 = -3.0D+00
        YKURT1 = 25.0D+00
        YSKEW2 = -1.0D+00
        YKURT2 = 3.0D+00
ENDIF
```

```
      IF (RUN .EQ. 8) THEN
            YSKEW1 = 3.0D+00
            YKURT1 = 25.0D+00
            YSKEW2 = 1.0D+00
            YKURT2 = 3.0D+00
      ENDIF

      IF (RUN .EQ. 9) THEN
            YSKEW1 = 3.0D+00
            YKURT1 = 25.0D+00
            YSKEW2 = 1.0D+00
            YKURT2 = 7.0D+00
      ENDIF

      IF (RUN .EQ. 10) THEN
            YSKEW1 = -3.0D+00
            YKURT1 = 25.0D+00
            YSKEW2 = -1.0D+00
            YKURT2 = 7.0D+00
      ENDIF

      IF (RUN .EQ. 11) THEN
            YSKEW1 = -1.0D+00
            YKURT1 = 3.0D+00
            YSKEW2 = -1.0D+00
            YKURT2 = 7.0D+00
      ENDIF

      IF (RUN .EQ. 12) THEN
            YSKEW1 = 1.0D+00
            YKURT1 = 3.0D+00
            YSKEW2 = 1.0D+00
            YKURT2 = 7.0D+00
      ENDIF


c     find coefficients
      skew=yskew1
      skurt=ykurt1

      call coef(skew,skurt,a1,b1,c1,d1)

      skew=yskew2
      skurt=ykurt2
```

```
      call coef(skew,skurt,a2,b2,c2,d2)


      do 2 ia = 1, 30
            res(ia) = 0.0d+00
2     CONTINUE

      set = 0

c     loop to perform entire thing for two alpha levels, one samp size, 4 ro
values

      do 1000 set = 1,8

      sumpt = 0
      sumst = 0

      sumlpt = 0
      sumlpz = 0
      sumlpav = 0

      sumfpt = 0
      sumfpz= 0
      sumfpav = 0




      alpha = 0.0d+00
      nssize=0
      ro = 0.0d+00

      if (set .LE. 4)
   *then
            alpha = .05d+00
            nssize =10
      endif

      if (set .GT. 4)
   *then
            alpha =.01d+00
            nssize =10
      endif

      if (set .EQ. 1 .OR. set .EQ. 5) then
            ro = 0.0d+00
```

```
            roz = ro
            go to 111
         endif
         if (set .EQ. 2 .OR. set .EQ. 6) ro = 0.5d+00
         if (set .EQ. 3 .OR. set .EQ. 7) ro = 0.7d+00
         if (set .EQ. 4 .OR. set .EQ. 8) ro = 0.9d+00


c        calculate ro of the std normal vars

         call calcroz(roz)
111      call rnset(id)


         do 100 i = 1, simsize

                 do 200 j=1,nssize
                         y1(j)=0.0d+00
                         y2(j)=0.0d+00
                         z1(j)=0.0d+00
                         z2(j)=0.0d+00
200              continue
         y1bar=0.0D+00
         y1var=0.0D+00
         y1std=0.0D+00
         y2bar=0.0D+00
         y2var=0.0D+00
         y2std=0.0D+00

         RHOHATP=0.0D+00

c        generate data
         call genbinorm(roz,nssize,z1,z2)

                 do 1 i1=1,nssize
                         y1(i1)=a1+b1*z1(i1)+c1*z1(i1)**2+d1*z1(i1)**3
                         y2(i1)=a2+b2*z2(i1)+c2*z2(i1)**2+d2*z2(i1)**3

1                continue
c        print*,'y1',y1,'y2',y2

c        calculate sample statistics for the data

         call smpstat(y1,y1bar,y1var,y1std)
         call smpstat(y2,y2bar,y2var,y2std)
```

```
C       CALCULATE Pearson and Spearman ESTIMATEs FOR CORRELATION
        call pearson(y1,y2,y1bar,y1std,y2bar,y2std,rhohatP)

        if (ro .eq. 0.0d+00) then
                CALL dRANKS (Nssize, y1, 0.0d+00, 0,0,y1r)
                CALL dRANKS (Nssize, y2, 0.0d+00, 0,0,y2r)
                call smpstat(y1r,y1rbar,y1rvar,y1rstd)
                call smpstat(y2r,y2rbar,y2rvar,y2rstd)
                call pearson(y1r,y2r,y1rbar,y1rstd,y2rbar,y2rstd,rhohatS)
c               PRINT*,'P',RHOHATP,'S',RHOHATS
        endif

c       calculate RL and RF and return got variable for each of t,z, and avg of the
two statistics
c       Pearson

        gotpt = 0
        gotst = 0

        gotlpt = 0
        gotlpz = 0
        gotlpav = 0

        gotfpt = 0
        gotfpz = 0
        gotfpav = 0

        if (ro .eq. 0.0d+00) then
                CAll calc(rhohatP,gotpt)
                CAll calc(rhohatS,gotst)
        endif

                CALL CALCL(ro,rhohatP,rlp,gotlpt,gotlpz,gotlpav)
                CALL CALCf(ro,rhohatP,rfp,gotfpt,gotfpz,gotfpav)


        sumpt = sumpt+gotpt
        sumst = sumst+gotst

C       print*,'sumpt',sumpt,'sumst',sumst
        sumlpt = sumlpt+gotlpt
        sumlpz = sumlpz+gotlpz
        sumlpav = sumlpav+gotlpav
```

```
      sumfpt = sumfpt +gotfpt
      sumfpz = sumfpz +gotfpz
      sumfpav = sumfpav +gotfpav



100   continue
c     sim loop

c     Calculate probabilities for each of the different dist'ns

      if (ro .eq. 0.0d+00) then
      probpt = dfloat(sumpt) / dfloat(simsize)
      probst = dfloat(sumst) / dfloat(simsize)
C           PRINT*,'PROBPT',PROBPT,'PROBST',PROBST
      endif

      problpt= dfloat(sumlpt) / dfloat(simsize)
      problpz= dfloat(sumlpz) / dfloat(simsize)
      problpav= dfloat(sumlpav) / dfloat(simsize)

      probfpt= dfloat(sumfpt) / dfloat(simsize)
      probfpz= dfloat(sumfpz) / dfloat(simsize)
      probfpav= dfloat(sumfpav) / dfloat(simsize)

c     PRINT*,PROBPT,PROBST,PROBLPT,PROBLPZ,PROBLPAV
c     PRINT*,PROBFPT,PROBFPZ,PROBFPAV

    res(1) = alpha
      if (ro .eq. 0.0d+00) then
            res(3) = problpt
            res(4) = problpav
            res(5) = problpz
            res(6) = probfpt
            res(7) = probfpav
            res(8) = probfpz
            res(31) = probpt
            res(32) = probst
      endIF
      if (ro .eq. 0.5d+00) then
            res(9) = problpt
            res(10) = problpav
            res(11) = problpz
            res(12) = probfpt
```

```
            res(13) = probfpav
            res(14) = probfpz
      endIF
      if (ro .eq. 0.7d+00) then
            res(15) = problpt
            res(16) = problpav
            res(17) = problpz
            res(18) = probfpt
            res(19) = probfpav
            res(20) = probfpz
      endIF
      if (ro .eq. 0.9d+00) then
            res(21) = problpt
            res(22) = problpav
            res(23) = problpz
            res(24) = probfpt
            res(25) = probfpav
            res(26) = probfpz
      endIF
      res(27) = yskew1
      res(28) = ykurt1
      res(29) = yskew2
      res(30) = ykurt2


      DUM=0.0D+00

9000  format(F3.0,1X,F3.0,5x, F10.8, 2x,F10.8, 2x,
     *F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8)
8000  format(24x, F10.8, 2x,F10.8, 2x,
     *F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8)



      IF (SET .EQ. 4 .OR. SET .EQ. 8) THEN
C     T
            IF (ALPHA .EQ. 0.01D+00) THEN
            WRITE(9,9000),res(27),res(28),res(31),res(3),res(6),res(9)
     *    ,res(12),res(15),res(18),res(21),res(24)
C     Z
            WRITE(9,9000),res(29),res(30),res(32),res(4),res(7),res(10)
     *    ,res(13),res(16),res(19),res(22),res(25)
C     AVG
            WRITE(9,8000),res(5),res(8),res(11),res(14)
     *,res(17),res(20),res(23),res(26)
            WRITE(9,*),' '
```

```
              ENDIF



              IF (ALPHA .EQ. 0.05D+00) THEN
C       T
              WRITE(10,9000),res(27),res(28),res(31),res(3),res(6),res(9)
     *   ,res(12),res(15),res(18),res(21),res(24)
C       Z
              WRITE(10,9000),res(29),res(30),res(32),res(4),res(7),res(10)
     *   ,res(13),res(16),res(19),res(22),res(25)
C       AVG
              WRITE(10,8000),res(5),res(8),res(11),res(14)
     *,res(17),res(20),res(23),res(26)
              WRITE(10,*),' '
              ENDIF

        ENDIF



1000    continue

10000   CONTINUE
        stop

        end
c       end main program





C       CALCULATE SAMPLE STATISTICS FOR BOTH Y1 AND Y2
        subroutine smpstat(y,xbar,var,std)
        implicit real*8 (a-h,o-z)

        REAL*8 val(nssize), y(nssize), VAR , XBAR, S, STD
        integer j,j2
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        XBAR = 0.0d+00
      VAR = 0.0d+00
      DO 110 J = 1 , Nssize
        val(j) = y(j)
       XBAR = XBAR + VAL(J)
```

```
 110  CONTINUE
    XBAR = XBAR/dfloat(Nssize)

        s = 0.0d+00

    DO 210 J2 = 1 , Nssize
      S = VAL(J2) - XBAR
      VAR = VAR + S*S
 210  CONTINUE
    VAR = VAR/(dfloat(Nssize)-1.0d+00)

        STD = dsqrt(VAR)

        end
c       end subroutine smpstat



c       Pearson subroutine to calculate correlation estimate
        subroutine pearson(x,y,xbar,xstd,ybar,ystd,rhoP)
        implicit real*8 (a-h,o-z)
        real*8 top, bot, x(nssize), y(nssize)
        real*8 xbar, ybar, xstd, ystd, rhoP
        integer j310
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        top=0.0d+00
        bot=0.0d+00
        rhop = 0.0d+00

        do 310 j310 = 1,Nssize
                top = top+(x(j310)-xbar)*(y(j310)-ybar)
  310   continue

                bot = (float(nssize)-1.0d+00)*xstd*ystd
                rhoP = top/bot


        end
c       end subroutine pearson


c        subroutine to calculate if got spearman/pearson
        subroutine calc(est,gott)
        implicit real*8 (a-h,o-z)
        INTEGER nssize,gott
```

```
      real*8 top,bot,teststat,est
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

C     print*,'estimate',est

      top = est*dsqrt(dfloat(nssize)-2.0D+00)
      bot = dsqrt(1.0d+00-est**2.0d+00)
      teststat = top / bot
C     print*,'teststat',teststat
      if (alpha .eq. .05d+00) then
                    if (teststat.lt. -1.86d+00)  gott =1
             else gott =0
      endif

      if (alpha .eq. .01d+00)  then
             if (teststat .lt. -2.896d+00)  gott =1
             else gott =0
      endif


C     print*,'gott',gott
      end
c     end calc


c      subroutine to calculate rL
      subroutine calcL(trurho, rhohat,rl,gotlt, gotlz, gotlav)
      implicit real*8 (a-h,o-z)
      INTEGER nssize,in,gotlt,gotlz,gotlav
   REAL*8  b,sign,parth,parti,ro,start,probL,rL,r
      real*8 n,roinc,parta,partb,partc,partd,parte
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

      iN=NSsize-4
      n=dfloat(in)
      Z=rhohat
      gotlt=0
      gotlz =0
      gotlav = 0

      roINC=trurho


      nobs=dfloat(nssize)

      parta=dsqrt(n)
```

```
partb=(1.0d+00-roINC*z)/(1.0d+00-roINC**2)

partc= (z-roINC)/(1.0d+00-z**2)

u=parta*((partb)**(1.5d+00))*partc

partd = 1.0d+00-roINC*z

parte=dsqrt(1.0d+00-roINC**2)*dsqrt(1.0d+00-z**2)

r1=(partd)/(parte)
r2=dlog(r1)
b = z-roinc
sign = dsign(b,b) / dabs(b)

r=sign*dsqrt(2.0d+00*n*r2)


rL=r+(dlog(u/r))/r

if (alpha .eq. .05d+00 .and. nssize .eq. 10) then

        if (rl .lt. -1.86d+00)  gotlt =1
        else gotlt =0

        if (rl .lt. -1.645d+00)  gotlz =1
        else gotlz =0

        if (rl .lt. -1.7525d+00)  gotlav =1
        else gotlav =0
endif

if (alpha .eq. .01d+00 .and. nssize  .eq. 10)  then

        if (rl .lt. -2.896d+00)  gotlt =1
        else gotlt =0

        if (rl .lt. -2.326d+00)  gotlz =1
        else gotlz =0

        if (rl .lt. -2.611d+00)  gotlav =1
        else gotlav =0

endif
```

```
      end
c     end calcL

c     Calculate the RF variable
      subroutine calcF(trurho, rhohat,rf,gotFt,gotfz,gotfav)
      implicit real*8 (a-h,o-z)
      INTEGER nssize,in,gotft,gotfz,gotfav
   REAL*8  sign,parth,parti,ro,start,probF
      real*8 n,roinc,partg,partf,rF,rhohat,trurho
      external dnordf
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

      iN=NSsize-4
      n=dfloat(in)
      Z=rhohat
      gotft=0
      gotfz = 0
      gotfav = 0
      roINC=trurho


      nobs=dfloat(nssize)

      d = nobs-3.0d+00
      parti = (1.0d+00+roINC)/(1.0d+00-roINC)
      partg = dlog(parti)
      partf = dlog((1.0d+00+z)/(1.0d+00-z))
      parth = roINC/(2.0d+00*(nobs-1.0d+00))

      rF=(.5d+00*(partf)-.5d+00*(partg)-parth)*dsqrt(d)



      if (alpha .eq. .05d+00 .and. nssize .eq. 10) then
            if (rf .lt. -1.86d+00)  gotft =1
            else gotft =0

            if (rf .lt. -1.645d+00)  gotfz =1
            else gotfz =0

            if (rf .lt. -1.7525d+00)  gotfav =1
            else gotfav =0
      endif

      if (alpha .eq. .01d+00 .and. nssize  .eq. 10)  then
```

```
          if (rf .lt. -2.896d+00)  gotft =1
          else gotft =0

          if (rf .lt. -2.326d+00)  gotfz =1
          else gotfz =0

          if (rf .lt. -2.611d+00)  gotfav =1
          else gotfav =0

      endif




      end
c     end calcF

c     ** calculate the Fleishman coefficients in order to obtain univariate
c     non-normal variables. input the desired skewness and durtoses and return
c     the coefficients a, b, c, d
c     Fleishman power transformation is y=a+bz+cz^2+dz^3
c     see continuous multivariate distribution by kotz... page 36+
      Subroutine coef(sskew,sskurt,a,b,c,d)
      implicit real*8 (a-h,o-z)
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

      external fcn,dneqnf,umach
      real*8 fnorm,x(3),xguess(3)
      data xguess/0.5d+00,0.50d+00,0.5d+00/
      integer nout
      errrel=0.00000001
      itmax=10000
c     print*,'in coef subroutine skew and skurt:',skew,skurt
      call umach(2,nout)
      Call DNEQNF(fcn,errrel,3,itmax,xguess,x,fnorm)
      b=x(1)
      c=x(2)
      d=x(3)
      a=-c
      return
      end


c     ** functions of the Fleishman's method
```

```
c       generate uniform deviates
        subroutine fcn(x,f,n)
        implicit real*8 (a-h,o-z)
        real*8 x(3), f(3)
        integer n
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

c       print*,'skew skurt in fcn:',skew,skurt
        f(1)=x(1)**2+6.0d+00*x(1)*x(3)+2.0d+00*x(2)**2+15.0d+00*x(3)**2
     *-1.0d+00
        f(2)=2.0d+00*x(2)*(x(1)**2+24.0d+00*x(1)*x(3)+105.0d+00*x(3)**2
     *+2.0d+00)-skew
        f(3)=24.0d+00*(x(1)*x(3)+x(2)**2*(1.0d+00+x(1)**2
     *+28.0d+00*x(1)*x(3))+x(3)**2*(12.0d+00+48.0d+00*x(1)*x(3)
     *+141.0d+00*x(2)**2+225.0d+00*x(3)**2))-skurt
        return
        end


c  calcroz calculate the ro of the 2 standard normal random variables

c  ro is the true linear correlation desired for the 2 non-normal rv

        Subroutine calcroz(roz)
        implicit real*8 (a-h,o-z)
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        external f,dzreal
        integer imax,nroot
          parameter (nroot=3)
          real*8 eps,errabs,errrel,eta
          real*8 f,x(nroot),xguess(nroot),d(nroot)
          integer info(nroot)

          eps=1.0e-8
          errabs=1.0e-8
          errrel=1.0d-8
          eta=1.0e-4
          itmax=5000
          data xguess/0.5d+00,0.5d+00,0.5d+00/
          call dzreal(f,errabs,errrel,eps,eta,nroot,itmax,xguess,x,info)
          diff=1.0d+00


        if (x(10) .ge. -1.0d+00 .and. x(1) .le. 1.0d+00) roz = x(1)
          return
```

```
      end

c     This double precision function to calculate the the cubic roots
c     of the roz
      Double Precision Function f(x)
      Implicit Real*8 (A-H, O-Z)
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

      real*8 x
      coe0=-ro
      coe1=b1*b2+3.0d+00*b1*d2+3.0d+00*b2*d1+9.0d+00*d1*d2
      coe2=2.0d+00*c1*c2
      coe3=6.0d+00*d1*d2
      f=coe3*x**3+coe2*x**2+coe1*x+coe0
      return
      end

c     this subroutine generates bivariate standard normal random variates
c     with nssize observations and correlation roz
      Subroutine genbinorm(roz,nsize,z1,z2)
      Implicit Real*8 (A-H, O-Z)
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize
      integer nsize,k,id,ldr,ldrsig,i,j
      real*8 cov(2,2),r(nsize,2),rsig(2,2),z1(nsize),z2(nsize)
      external dchfac,drnmvn,rnset
      real*8 roz
      k=2
      ldrsig=2
      ldr=nsize
      cov(1,1)=1.0d+00
      cov(2,2)=1.0d+00
      cov(1,2)=roz
      cov(2,1)=roz
c     cov(1,2)=roz*1*1 (for standard normal both std dev are 1)
      call dchfac(k,cov,2,1.0e-8,irank,rsig,ldrsig)

      call drnmvn(nsize,k,rsig,ldrsig,r,ldr)

      do 20 i20=1,nsize
                z1(i20)=r(i20,1)
                z2(i20)=r(i20,2)

20    continue
      return
      end
```

Appendix C. Fortran Program, Left-Tail Power

```
! file nonnormal rv ge.for
        Use numerical_libraries
        implicit real*8 (a-h,o-z)
        integer size,simsize,set,RUN,ia
        parameter (simsize=100000)

        real*8 y1(10),y2(10),ro,rocalc
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        real*8 probfst,probpfsz,probfsav,probfpt,probfpz,probfpav,
     *problst,problsz,problsav,problpt,problpz,problpav

        integer gotlpt,gotlpz,gotlpav,gotfpt,gotfpz,gotfpav,
     *gotlst,gotlsz,gotlsav,gotfst,gotfsz,gotfsav,
     *sumlst,sumlsz,sumlsav,sumlpt,sumlpz,sumlpav,sumfst,sumfsz,sumfsav,
     *sumfpt,sumfpz,sumfpav

        REAL*8  y1bar,y1var,y1std,y2bar,y2var,y2std
        real*8 rhohatp,rhohats,targUP,alpha
        real*8 z1(10),z2(10),res(42)

        integer nssize,iseed
        iseed=123457

        open (unit=9,file='F:\June 11\results
     *\left power results ro = .7 ALPHA01 B.txt')
        write(9,*),'Left Tail Power'
        write(9,*),'Alpha = 0.01'

        open (unit=10,file='F:\June 11\results
     *\left power results ro = .7 ALPHA05 B.txt')
        write(10,*),'Left Tail Power'
        write(10,*),'Alpha = 0.05'

        data yvar1,yvar2/1.0d+00,1.0d+00/


        DO 10000 RUN =1,12

        IF (RUN .EQ. 1) THEN
                YSKEW1 = 3.0D+00
                YKURT1 = 25.0D+00
                YSKEW2 = 3.0D+00
```

```
        YKURT2 = 25.0D+00
ENDIF

IF (RUN .EQ. 2) THEN
        YSKEW1 = -3.0D+00
        YKURT1 = 25.0D+00
        YSKEW2 = -3.0D+00
        YKURT2 = 25.0D+00

ENDIF

IF (RUN .EQ. 3) THEN
        YSKEW1 = -1.0D+00
        YKURT1 = 7.0D+00
        YSKEW2 = -1.0D+00
        YKURT2 = 7.0D+00
ENDIF

IF (RUN .EQ. 4) THEN
        YSKEW1 = 1.0D+00
        YKURT1 = 7.0D+00
        YSKEW2 = 1.0D+00
        YKURT2 = 7.0D+00
ENDIF

IF (RUN .EQ. 5) THEN
        YSKEW1 = 1.0D+00
        YKURT1 = 3.0D+00
        YSKEW2 = 1.0D+00
        YKURT2 = 3.0D+00
ENDIF

IF (RUN .EQ. 6) THEN
        YSKEW1 = -1.0D+00
        YKURT1 = 3.0D+00
        YSKEW2 = -1.0D+00
        YKURT2 = 3.0D+00
ENDIF

IF (RUN .EQ. 7) THEN
        YSKEW1 = -3.0D+00
        YKURT1 = 25.0D+00
        YSKEW2 = -1.0D+00
        YKURT2 = 3.0D+00
ENDIF
```

```
      IF (RUN .EQ. 8) THEN
            YSKEW1 = 3.0D+00
            YKURT1 = 25.0D+00
            YSKEW2 = 1.0D+00
            YKURT2 = 3.0D+00
      ENDIF

      IF (RUN .EQ. 9) THEN
            YSKEW1 = 3.0D+00
            YKURT1 = 25.0D+00
            YSKEW2 = 1.0D+00
            YKURT2 = 7.0D+00
      ENDIF

      IF (RUN .EQ. 10) THEN
            YSKEW1 = -3.0D+00
            YKURT1 = 25.0D+00
            YSKEW2 = -1.0D+00
            YKURT2 = 7.0D+00
      ENDIF

      IF (RUN .EQ. 11) THEN
            YSKEW1 = -1.0D+00
            YKURT1 = 3.0D+00
            YSKEW2 = -1.0D+00
            YKURT2 = 7.0D+00
      ENDIF

      IF (RUN .EQ. 12) THEN
            YSKEW1 = 1.0D+00
            YKURT1 = 3.0D+00
            YSKEW2 = 1.0D+00
            YKURT2 = 7.0D+00
      ENDIF


c     find coefficients
      skew=yskew1
      skurt=ykurt1

      call coef(skew,skurt,a1,b1,c1,d1)

      skew=yskew2
      skurt=ykurt2
      call coef(skew,skurt,a2,b2,c2,d2)
```

```
      do 2 ia = 1, 42
            res(ia) = 0.0d+00
2     CONTINUE

      set = 0

c     loop to perform entire thing for two alpha levels, one samp size, 6 ro
values

      do 1000 set = 1,12

      sumlpt = 0
      sumlpz = 0
      sumlpav = 0

      sumfpt = 0
      sumfpz= 0
      sumfpav = 0




      alpha = 0.0d+00
      nssize=10
      rocalc = 0.70d+00

      if (set .LE. 6) alpha = .05d+00


      if (set .GT. 6) alpha =.01d+00


      if (set .EQ. 1 .OR. set .EQ. 7) ro = 0.70d+00
      if (set .EQ. 2 .OR. set .EQ. 8) ro = 0.5d+00
      if (set .EQ. 3 .OR. set .EQ. 9) ro = 0.4d+00
      if (set .EQ. 4 .OR. set .EQ. 10) ro = 0.3d+00
      if (set .EQ. 5 .OR. set .EQ. 11) ro = 0.2d+00
      if (set .EQ. 6 .OR. set .EQ. 12) ro = 0.1d+00

c     calculate ro of the std normal vars



      call calcroz(roz)
```

```
111   call rnset(id)
c     print*,'set',set
c     print*,'dist1    skew1',yskew1,'kurt1',ykurt1
c     print*,'coeff1',a1,b1,c1,d1
c     print*,'dist2    skew2',yskew2,'kurt2',ykurt2
c     print*,'coeff2',a2,b2,c2,d2
c     print*,'ro',ro
c     print*,'roz',roz


      do 100 i = 1, simsize

            do 200 j=1,nssize
                  y1(j)=0.0d+00
                  y2(j)=0.0d+00
                  z1(j)=0.0d+00
                  z2(j)=0.0d+00
200         continue
      y1bar=0.0D+00
      y1var=0.0D+00
      y1std=0.0D+00
      y2bar=0.0D+00
      y2var=0.0D+00
      y2std=0.0D+00

      RHOHATP=0.0D+00

c     generate data
      call genbinorm(roz,nssize,z1,z2)

            do 1 i1=1,nssize
                  y1(i1)=a1+b1*z1(i1)+c1*z1(i1)**2+d1*z1(i1)**3
                  y2(i1)=a2+b2*z2(i1)+c2*z2(i1)**2+d2*z2(i1)**3
c           print*,'y1',y1(i1)
c           print*,'y2',y2(i1)
1           continue


c     calculate sample statistics for the data

      call smpstat(y1,y1bar,y1var,y1std)
      call smpstat(y2,y2bar,y2var,y2std)

c     print*,'y1',y1
c     print*,'stats1',y1bar,y1var,y1std
c     print*,'y2',y2
```

```
c       print*,'stats2',y2bar,y2var,y2std


C       CALCULATE Pearson and Spearman ESTIMATEs FOR CORRELATION
        call pearson(y1,y2,y1bar,y1std,y2bar,y2std,rhohatP)

c       print*,'rhohatP',rhohatP

c       calculate RL and RF and return got variable for each of t,z, and avg of the
two statistics
c       Pearson



        gotlpt = 0
        gotlpz = 0
        gotlpav = 0

        gotfpt = 0
        gotfpz = 0
        gotfpav = 0

                CALL CALCL(rocalc,rhohatP,rlp,gotlpt,gotlpz,gotlpav)
                CALL CALCf(rocalc,rhohatP,rfp,gotfpt,gotfpz,gotfpav)
C       print*,'rlp',rlp,gotlpt,gotlpz,gotlpav
C       print*,'rfp',rfp,gotfpt,gotfpz,gotfpav

        sumlpt = sumlpt+gotlpt
        sumlpz = sumlpz+gotlpz
        sumlpav = sumlpav+gotlpav


        sumfpt = sumfpt +gotfpt
        sumfpz = sumfpz +gotfpz
        sumfpav = sumfpav +gotfpav



100     continue
c       sim loop

c       Calculate probabilities for each of the different dist'ns

C       print*,'sumL',sumlpt,sumlpz,sumlpav
```

```
    problpt= dfloat(sumlpt) / dfloat(simsize)
    problpz= dfloat(sumlpz) / dfloat(simsize)
    problpav= dfloat(sumlpav) / dfloat(simsize)

    probfpt= dfloat(sumfpt) / dfloat(simsize)
    probfpz= dfloat(sumfpz) / dfloat(simsize)
    probfpav= dfloat(sumfpav) / dfloat(simsize)



res(1) = alpha
    if (ro .eq. 0.70d+00) then
            res(3) = problpt
            res(4) = problpav
            res(5) = problpz
            res(6) = probfpt
            res(7) = probfpav
            res(8) = probfpz
    endIF
    if (ro .eq. 0.50d+00) then
            res(9) = problpt
            res(10) = problpav
            res(11) = problpz
            res(12) = probfpt
            res(13) = probfpav
            res(14) = probfpz
    endIF
    if (ro .eq. 0.40d+00) then
            res(15) = problpt
            res(16) = problpav
            res(17) = problpz
            res(18) = probfpt
            res(19) = probfpav
            res(20) = probfpz
    endIF
    if (ro .eq. 0.30d+00) then
            res(21) = problpt
            res(22) = problpav
            res(23) = problpz
            res(24) = probfpt
            res(25) = probfpav
            res(26) = probfpz
    endIF
    if (ro .eq. 0.20d+00) then
            res(27) = problpt
            res(28) = problpav
```

```
          res(29) = problpz
          res(30) = probfpt
          res(31) = probfpav
          res(32) = probfpz
      endIF
      if (ro .eq. 0.10d+00) then
          res(33) = problpt
          res(34) = problpav
          res(35) = problpz
          res(36) = probfpt
          res(37) = probfpav
          res(38) = probfpz
      endIF
      res(39) = yskew1
      res(40) = ykurt1
      res(41) = yskew2
      res(42) = ykurt2

9000  format(F3.0,1X,F3.0,5x, F10.8, 2x,F10.8,2x,
    *F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,
    *F10.8,2x,F10.8,2x,F10.8)
8000  format(12x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,
    *F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8,2x,F10.8)
C     print 9000,RES(1),res(27),res(28),res(3),res(6),res(9),res(12),
C    *res(15),res(18),res(21),res(24)


      IF (SET .EQ. 6 .OR. SET .EQ. 12) THEN
C     T
          IF (ALPHA .EQ. 0.01D+00) THEN
          WRITE(9,9000),res(39),res(40),res(3),res(6),res(9),res(12)
    *,res(15),res(18),res(21),res(24),res(27),res(30), res(33),res(36)
C     Z
          WRITE(9,9000),res(41),res(42),res(4),res(7),res(10),res(13)
    *,res(16),res(19),res(22),res(25),res(28),res(31),res(34),res(37)
C     AVG
          WRITE(9,8000),res(5),res(8),res(11),res(14)
    *,res(17),res(20),res(23),res(26),res(29),res(32),res(35),res(38)
          WRITE(9,*),' '
          ENDIF
```

```
              IF (ALPHA .EQ. 0.05D+00) THEN
C       T
              WRITE(10,9000),res(39),res(40),res(3),res(6),res(9),res(12)
     *,res(15),res(18),res(21),res(24),res(27),res(30), res(33),res(36)
C       Z
              WRITE(10,9000),res(41),res(42),res(4),res(7),res(10),res(13)
     *,res(16),res(19),res(22),res(25),res(28),res(31),res(34),res(37)
C       AVG
              WRITE(10,8000),res(5),res(8),res(11),res(14)
     *,res(17),res(20),res(23),res(26),res(29),res(32),res(35),res(38)
              WRITE(10,*),' '
              ENDIF

        ENDIF
c       if (ro .eq. 0.3d+00 .or. ro .eq. 0.2d+00) print*,problpt,probfpt



1000    continue

10000 CONTINUE
        stop

        end
c       end main program




C       CALCULATE SAMPLE STATISTICS FOR BOTH Y1 AND Y2
        subroutine smpstat(y,xbar,var,std)
        implicit real*8 (a-h,o-z)

        REAL*8 val(nssize), y(nssize), VAR , XBAR, S, STD
        integer j,j2
        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        XBAR = 0.0d+00
     VAR = 0.0d+00
     DO 110 J = 1 , Nssize
        val(j) = y(j)
       XBAR = XBAR + VAL(J)
 110  CONTINUE
     XBAR = XBAR/dfloat(Nssize)
```

```
        s = 0.0d+00

     DO 210 J2 = 1 , Nssize
       S = VAL(J2) - XBAR
       VAR = VAR + S*S
 210  CONTINUE
     VAR = VAR/(dfloat(Nssize)-1.0d+00)

        STD = dsqrt(VAR)

        end
c       end subroutine smpstat



c       Pearson subroutine to calculate correlation estimate
        subroutine pearson(x,y,xbar,xstd,ybar,ystd,rhoP)
        implicit real*8 (a-h,o-z)
        real*8 top, bot, x(nssize), y(nssize)
        real*8 xbar, ybar, xstd, ystd, rhoP
        integer j310

        common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

        top=0.0d+00
        bot=0.0d+00
        rhop = 0.0d+00

        do 310 j310 = 1,Nssize
                top = top+(x(j310)-xbar)*(y(j310)-ybar)
  310   continue

                bot = (float(nssize)-1.0d+00)*xstd*ystd
                rhoP = top/bot


        end
c       end subroutine pearson



c        subroutine to calculate rL
        subroutine calcL(trurho, rhohat,rl,gotlt, gotlz, gotlav)
        implicit real*8 (a-h,o-z)
        INTEGER nssize,in,gotlt,gotlz,gotlav
```

```
REAL*8  b,sign,parth,parti,ro,start,probL,rL,r
   real*8 n,roinc,parta,partb,partc,partd,parte

   common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize
   iN=NSsize-4
   n=dfloat(in)
   Z=rhohat
   gotlt=0
   gotlz =0
   gotlav = 0

   roINC=trurho


   nobs=dfloat(nssize)

   parta=dsqrt(n)

   partb=(1.0d+00-roINC*z)/(1.0d+00-roINC**2)

   partc= (z-roINC)/(1.0d+00-z**2)

   u=parta*((partb)**(1.5d+00))*partc

   partd = 1.0d+00-roINC*z

   parte=dsqrt(1.0d+00-roINC**2)*dsqrt(1.0d+00-z**2)

   r1=(partd)/(parte)
   r2=dlog(r1)
   b = z-roinc
   sign = dsign(b,b) / dabs(b)

   r=sign*dsqrt(2.0d+00*n*r2)


   rL=r+(dlog(u/r))/r

   if (alpha .eq. .05d+00 .and. nssize .eq. 10) then

          if (rl .lt. -1.86d+00)  gotlt =1
          else gotlt =0

          if (rl .lt. -1.645d+00)  gotlz =1
          else gotlz =0
```

```
            if (rl .lt. -1.7525d+00)  gotlav =1
            else gotlav =0
      endif

      if (alpha .eq. .01d+00 .and. nssize  .eq. 10)  then

            if (rl .lt. -2.896d+00)  gotlt =1
            else gotlt =0

            if (rl .lt. -2.326d+00)  gotlz =1
            else gotlz =0

            if (rl .lt. -2.611d+00)  gotlav =1
            else gotlav =0

      endif


      end
c     end calcL

c     Calculate the RF variable
      subroutine calcF(trurho, rhohat,rf,gotFt,gotfz,gotfav)
      implicit real*8 (a-h,o-z)
      INTEGER nssize,in,gotft,gotfz,gotfav
   REAL*8  sign,parth,parti,ro,start,probF
      real*8 n,roinc,partg,partf,rF,rhohat,trurho
      external dnordf

      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize
      iN=NSsize-4
      n=dfloat(in)
      Z=rhohat
      gotft=0
      gotfz = 0
      gotfav = 0
      roINC=trurho


      nobs=dfloat(nssize)

      d = nobs-3.0d+00
      parti = (1.0d+00+roINC)/(1.0d+00-roINC)
      partg = dlog(parti)
      partf = dlog((1.0d+00+z)/(1.0d+00-z))
      parth = roINC/(2.0d+00*(nobs-1.0d+00))
```

```
      rF=(.5d+00*(partf)-.5d+00*(partg)-parth)*dsqrt(d)


      if (alpha .eq. .05d+00 .and. nssize .eq. 10) then
            if (rf .lt. -1.86d+00)  gotft =1
            else gotft =0

            if (rf .lt. -1.645d+00)  gotfz =1
            else gotfz =0

            if (rf .lt. -1.7525d+00)  gotfav =1
            else gotfav =0
      endif

      if (alpha .eq. .01d+00 .and. nssize  .eq. 10)  then
            if (rf .lt. -2.896d+00)  gotft =1
            else gotft =0

            if (rf .lt. -2.326d+00)  gotfz =1
            else gotfz =0

            if (rf .lt. -2.611d+00)  gotfav =1
            else gotfav =0

      endif




      end
c     end calcF

c     ** calculate the Fleishman coefficients in order to obtain univariate
c     non-normal variables. input the desired skewness and durtoses and return
c     the coefficients a, b, c, d
c     Fleishman power transformation is y=a+bz+cz^2+dz^3
c     see continuous multivariate distribution by kotz... page 36+
      Subroutine coef(sskew,sskurt,a,b,c,d)
      implicit real*8 (a-h,o-z)

      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize
      external fcn,dneqnf,umach
```

```
      real*8 fnorm,x(3),xguess(3)
      data xguess/0.5d+00,0.50d+00,0.5d+00/
      integer nout
      errrel=0.00000001
      itmax=10000
c     print*,'in coef subroutine skew and skurt:',skew,skurt
      call umach(2,nout)
      Call DNEQNF(fcn,errrel,3,itmax,xguess,x,fnorm)
      b=x(1)
      c=x(2)
      d=x(3)
      a=-c
      return
      end


c     ** functions of the Fleishman's method
c     generate uniform deviates
      subroutine fcn(x,f,n)
      implicit real*8 (a-h,o-z)
      real*8 x(3), f(3)
      integer n

      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize
c     print*,'skew skurt in fcn:',skew,skurt
      f(1)=x(1)**2+6.0d+00*x(1)*x(3)+2.0d+00*x(2)**2+15.0d+00*x(3)**2
     *-1.0d+00
      f(2)=2.0d+00*x(2)*(x(1)**2+24.0d+00*x(1)*x(3)+105.0d+00*x(3)**2
     *+2.0d+00)-skew
      f(3)=24.0d+00*(x(1)*x(3)+x(2)**2*(1.0d+00+x(1)**2
     *+28.0d+00*x(1)*x(3))+x(3)**2*(12.0d+00+48.0d+00*x(1)*x(3)
     *+141.0d+00*x(2)**2+225.0d+00*x(3)**2))-skurt
      return
      end


c calcroz calculate the ro of the 2 standard normal random variables

c ro is the true linear correlation desired for the 2 non-normal rv

      Subroutine calcroz(roz)
      implicit real*8 (a-h,o-z)
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

      external f,dzreal
      integer imax,nroot
```

```
      parameter (nroot=3)
      real*8 eps,errabs,errrel,eta
      real*8 f,x(nroot),xguess(nroot),d(nroot)
      integer info(nroot)

      eps=1.0e-8
      errabs=1.0e-8
      errrel=1.0d-8
      eta=1.0e-4
      itmax=5000
      data xguess/0.5d+00,0.5d+00,0.5d+00/
      call dzreal(f,errabs,errrel,eps,eta,nroot,itmax,xguess,x,info)
C     print*,x

      diff=1.0d+00

     if (x(10) .ge. -1.0d+00 .and. x(1) .le. 1.0d+00) roz = x(1)

      return
      end


c    This double precision function to calculate the the cubic roots
c    of the roz
     Double Precision Function f(x)
     Implicit Real*8 (A-H, O-Z)
     common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

     real*8 x
     coe0=-ro
     coe1=b1*b2+3.0d+00*b1*d2+3.0d+00*b2*d1+9.0d+00*d1*d2
     coe2=2.0d+00*c1*c2
     coe3=6.0d+00*d1*d2
     f=coe3*x**3+coe2*x**2+coe1*x+coe0
     return
     end


c    this subroutine generates bivariate standard normal random variates
c    with nssize observations and correlation roz
     Subroutine genbinorm(roz,nsize,z1,z2)
```

```
      Implicit Real*8 (A-H, O-Z)
      common skew,skurt,a1,b1,c1,d1,a2,b2,c2,d2,ro,alpha,nssize

      integer nsize,k,id,ldr,ldrsig,i,j
      real*8 cov(2,2),r(nsize,2),rsig(2,2),z1(nsize),z2(nsize)
      external dchfac,drnmvn,rnset
      real*8 roz
      k=2
      ldrsig=2
      ldr=nsize
      cov(1,1)=1.0d+00
      cov(2,2)=1.0d+00
      cov(1,2)=roz
      cov(2,1)=roz
c     cov(1,2)=roz*1*1 (for standard normal both std dev are 1)
      call dchfac(k,cov,2,1.0e-8,irank,rsig,ldrsig)

      call drnmvn(nsize,k,rsig,ldrsig,r,ldr)
c     print*,((r(i,j),j=1,k),i=1,nsize)
      do 20 i20=1,nsize
                z1(i20)=r(i20,1)
                z2(i20)=r(i20,2)
c     print*,'z1',z1(i20),' z2',z2(i20)
20    continue
      return
      end
```

Bibliography:

Anderson, T.W. (1984), *An Introduction to Multivariate Statistical Analysis*, New York: John Wiley.

Daniels, H. E. (1954), Saddlepoint Approximations in Statistics, *Annals of Mathematical Statistics*, 25, 631-650.

Fan, X., Felsovalyi, A., Sivo, S., and Keenan, S. (2002), *SAS for Monte Carlo Studies*, North Carolina: SAS Publishing.

Fisher, R. A. (1915), Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population, *Biometrika*, 10, 4, 507-521.

Fleishman, A. (1978), A Method for Simulating Non-normal Distributions, *Psychometrika*, 43, 521 – 532.

Galton, F. (1886), Regression Towards Mediocrity in Hereditary Stature, *Journal of the Anthropological Institute,* 15, 246–263.

Huzurbazar, S. (1999), Practical Saddlepoint Approximations, *The American Statistician*, 53, 3, 225-232.

Jensen, J. (1995), *Saddlepoint Approximations*, New York: Oxford University Press Inc.

Johnson, M., Tietjen, G., Beckman, R. (1980), A New Family of Probability Distributions With Applications to Monte Carlo Studies, *Journal of the American Statistical Association*, 75, 370, 276-279.

Kotz, S., Balakrishnan, N., and Johnson, N. (2000), *Continuous Multivariate Distributions, Volume 1: Models and Applications,* Second Edition, New York: John Wiley and Sons, Inc.

Kutner, M., Nachtsheim, C., Neter, J., and Li, W. (2005), *Applied Linear Statistical Models*, Fifth Edition, New York: McGraw-Hill Irwin.

Pearson. K. (1896), Mathematical Contributions to the Theory of Evolution. III. Regression, Heredity and Panmixia, *Philosophical Transactions of the Royal Society A*, 187, 253-318.

Steyn, H. S. (1993), On the problem of more than one kurtosis parameter in multivariate analysis, *Journal of Multivariate Analysis*, 44, 1-22.

Stuart, A. and Ord, J. K. (1994), *Kendall's Advanced Theory of Statisitcs, Volume 1*, Sixth Edition, New York: Halsted Press.

Tadikamalla, P. (1980), On simulating Non-normal Distributions, *Psychometrika*, 45, 273-279.

Vale, C. and Maurelli, V. (1983) "Simulating Multivariate Nonnormal Distributions", Psychometrika, 48, 465–471.

Louanne M. Beversdorf

6211 Darwin Court, Apt 1101, Raleigh, NC 27612
727-271-3727
LouanneB@gmail.com

Education

2008 — **Master's degree: Mathematical Sciences**
Statistics Concentration
University of North Florida, Jacksonville, Florida
Dr. Pali Sen, Graduate Advisor

2003 — **Bachelor of Science**: Statistics Major, Mathematics Minor
University of North Florida, Jacksonville, Florida

2001 — **Associate in Arts**: Mathematics Education Focus
University of South Florida, Tampa, Florida

Professional Experience in Statistics

April 2004 - August 2006 — Nielsen Media Research, Statistical Research Dept.
Associate Research Statistician

May 2007-August 2007 — The Nielsen Company, Statistical Consultant Intern

Teaching Experience

August 2006 – May 2008

Graduate Assistant, University of North Florida

- Taught lecture sections of Intro Statistics and Finite Math
- Taught breakout sessions as assistant to professors
  Courses include: Finite Math, College Algebra, Intro Statistics for Business Majors, Intro Statistics for Social Science Majors
- Monitored computer lab and helped undergraduate students

May –Aug 2003 — Personal Tutor, Growing Desire Educational Services

- Met clients at their home or local library to tutor. Focus on Calculus, CLAST exam, and Introductory Statistics.

Spring 2002 — Tutor children at Safe Harbor Boys' Home in Jacksonville, FL

Teaching Experience Continued...

Feb. - Dec. 2003	Mathematics Instructor, Kumon Learning Centers - Math tutor for all levels of math, preschool through Calculus. Also graded papers, organized work for students.
2002	Personal Tutor, Private - Tutor other college students in Calculus, and Introductory Statistics
Spring 2000	Field experience, 15 hours classroom observation (required of all education majors)
1998- 2001	Teacher Assistant after school hours for 6^{th} grade English teacher

Honors and Activities

2003	SAS Programming II Class, SAS Certification Prep Online Class
2003-2004	Undergraduate Statistics Major with the Most Outstanding Senior Year, Department of Mathematics and Statistics, UNF
2001 – 2003	Vice President of the Math/Stat Club, UNF
2003	Dean's List
2002	Inducted into Pi Mu Epsilon, National Mathematics Honor Society
March 1999	President's Student Service Scholar Award

Skills

SAS Base Programmer Certification, 2004
Experience with SPSS, MATLAB, Maple, Fortran
SAS in CMS/MVS environments as well as PC SAS
Microsoft Office (Word, PowerPoint, Excel, Outlook)

Academic Research Experience

Aug 2007- Jun 2008	"Tests for Correlation on Nonnormal Bivariate Data" Graduate thesis under direction of Dr. Ping Sa, Professor, UNF
December 2003	Statistics Capstone, Factorial Design - Effects of height and weight of paper on the distance of flight for paper airplanes

Professional Research Experience

July 2007	College student television viewing The Nielsen Company - Audit: Match production numbers by researching possible processing methods of viewing information. Design SAS procedures to duplicate results. Documentation of methods for others to utilize.
January 2006	Logistic regression analysis with sample survey data Nielsen Media Research Inc. - Complete analysis of data including: validation of assumptions, model selection, interpretation of the model, presentation, and correct execution in SAS programming environment.
January 2005	Understanding the SURVEYSELECT procedure in SAS Nielsen Media Research Inc. - Extensive studying of the procedure's capabilities, presentation to colleagues persuading them to implement in everyday procedures, rewrite of many everyday sampling programs, eventual utilization by many colleagues in the Statistical Research Department
June 2005	Using Macros in SAS: Repeated ANOVA analysis Nielsen Media Research Inc. - Self taught the use of macros in SAS programming, streamlined many company programs with this concept, designed a program to perform a repeated ANOVA analysis. Used for model building (similar concept as stepwise selection). Allows combinations of continuous and categorical data. Categories can be ordinal or nominal.

Memberships

American Mathematical Society
American Statistical Association
Mathematical Association of America
SAS Certified Professionals

Hobbies

Piano/Keyboard and Songwriting
Photography

Community Service

1997 - present	Annual fundraising and volunteer work Children's Home Society of Florida
Spring 2002	Tutor children at Safe Harbor Boys' Home in Jacksonville, FL
1998 – 2000	Teacher assistant, Mrs. Janet Kengott, Gulf Middle School, New Port Richey, FL
2004	Fundraising event for Alzheimer's Foundation
March-April 2006	Walk America, fundraiser for March of Dimes

Academic References

Dr. Scott Hochwald, Department Chair
Department of Mathematics and Statistics
University of North Florida
shochwal@unf.edu
904-620-2653

Dr. Ping Sa, Professor
Department of Mathematics and Statistics
University of North Florida
psa@unf.edu
904-620-3729

Dr. Donna Mohr, Professor
Department of Mathematics and Statistics
University of North Florida
dmohr@unf.edu
904-620-2884

Dr. Pali Sen, Professor, Graduate Advisor
Department of Mathematics and Statistics
University of North Florida
psen@unf.edu
904-620-3724

Dr. William J. Wilson
Professor Emeritus of Statistics
bwilson@unf.edu
904-241-0923

Professional Reference

Susan Scholl Avanessian, Director , Statistical Research, The Nielsen Company
Susan.scholl@nielsen.com 813-366-4385