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Preserving Useful Info While Reducing Noise of Physiological Signals by Using Wavelet Analysis

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PRESERVING USEFUL INFO WHILE REDUCING NOISE OF PHYSIOLOGICAL SIGNALS BY USING WAVELET ANALYSIS

by

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DEDICATION

This thesis is dedicated to my aunt, Christina Ng, who has always given me spiritual and financial support. She brought me to the United States, a place that changed my life. She took the time to give me a ride to school every day before I could drive by myself. She never once fell back on the excuse that she could not take me to school because it was not even 6 o’clock in the morning. She just did it because she loves me. Thank you, Aunt Christina!

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ABSTRACT

Wavelet analysis is a powerful mathematical tool commonly used in signal processing applications, such as image analysis, image compression, image edge detection, and communications systems. Unlike traditional Fourier analysis, wavelet analysis allows for multiple resolutions in the time and frequency domains; it can preserve time information while decomposing a signal spectrum over a range of frequencies. Wavelet analysis is also more suitable for detecting numerous transitory characteristics, such as drift, trends, abrupt changes, and beginnings and ends of events. These characteristics are often the most important and critical part of some non-stationary signals, such as physiological signals.

The thesis focuses on a formal analysis of using wavelet transform for noise filtering. The performance of the wavelet analysis is simulated on a variety of patient samples of Arterial Blood Pressure (ABP 14 sets) and Electrocardiography (ECG 14 sets) from the Mayo Clinic at Jacksonville. The performance of the Fourier analysis is also simulated on the same patient samples for comparison purpose. Additive white Gaussian noise (AWGN) is generated and added to the samples for studying the AWGN effect on physiological signals and both analysis methods. The algorithms of finding the optimal level of approximation and calculating the threshold value of filtering are created and different ways of adding the details back to the approximation are studied. Wavelet analysis has the ability to add or remove certain frequency bands with threshold selectivity from the original signal. It can effectively preserve the spikes and humps,

which are the information that is intended to be kept, while de-noising physiological signals. The simulation results show that the wavelet analysis has a better performance than Fourier analysis in preserving the transitory information of the physiological signals.

Chapter 1

INTRODUCTION

In recent years, physiological signals, such as arterial blood pressure (ABP), and electrocardiography (ECG) are getting more and more attention. They are recorded to assist in the diagnosis of diseases, such as coronary heart disease (CHD) and diabetes mellitus (DM) [1], or to monitor the progress of patients undergoing therapeutic procedures. If these signals are disrupted, interpretation or diagnosis becomes difficult and could give rise to false alarms, misdiagnosis or inappropriate treatment. This can be especially detrimental in the case of interference mimicking certain pathological conditions. Misdiagnosis could possibly threaten the health of patients or lead to legal action for malpractice [2]. Thus, the accuracy of physiological signals is critical. However, physiological signals are often corrupted by various noises, such as additive white Gaussian noise (AWGN) and high-frequency muscle contraction. These noises need to be effectively removed while preserving the important information. As such, a study of noise filtering on physiological signals is necessary.

Different techniques and methods can be used for noise filtering. As modern computer hardware is becoming less expensive and more robust, wavelet analysis is getting more attention in the use of digital signal processing. It has been successfully applied in many applications, such as image compression, communications systems, and noise filtering.

The thesis focuses on the noise filtering performance of the wavelet analysis on the physiological signals that are provided by Mayo Clinic at Jacksonville.

1.1 Physiological Signals

Physiological signals like bio-potentials such as electrocardiography (ECG) and physical quantities such as arterial blood pressure (ABP) are the two most commonly used diagnostic tools that measure and record the electrical activity of the heart. Although both ECG and ABP can be used for heart conditions diagnosis, they have different signal characteristics.

1.1.1 ECG Signals

The ECG signal represents the changes in electrical potential during the cardiac cycle as recorded between surface electrodes on the body. The characteristic shape of this signal is the result of an action potential that propagates within the heart and causes the contraction of the various portions of the cardiac muscle. This internal excitation starts at the sinus node that acts as a pacemaker and then spreads to the atria; this generates the characteristic P wave in the ECG (figure 1). The cardiac excitation then reaches the ventricles (ventricular depolarization), giving rise to the characteristic QRS complex. Once the ventricles have been completely stimulated (ST segment of the ECG), they re-polarize with respect to the T wave of the ECG. The automatic detection and timing of these waves is important for diagnostic purposes. The crucial step in the analysis of ECG is the detection of the QRS complex [3].

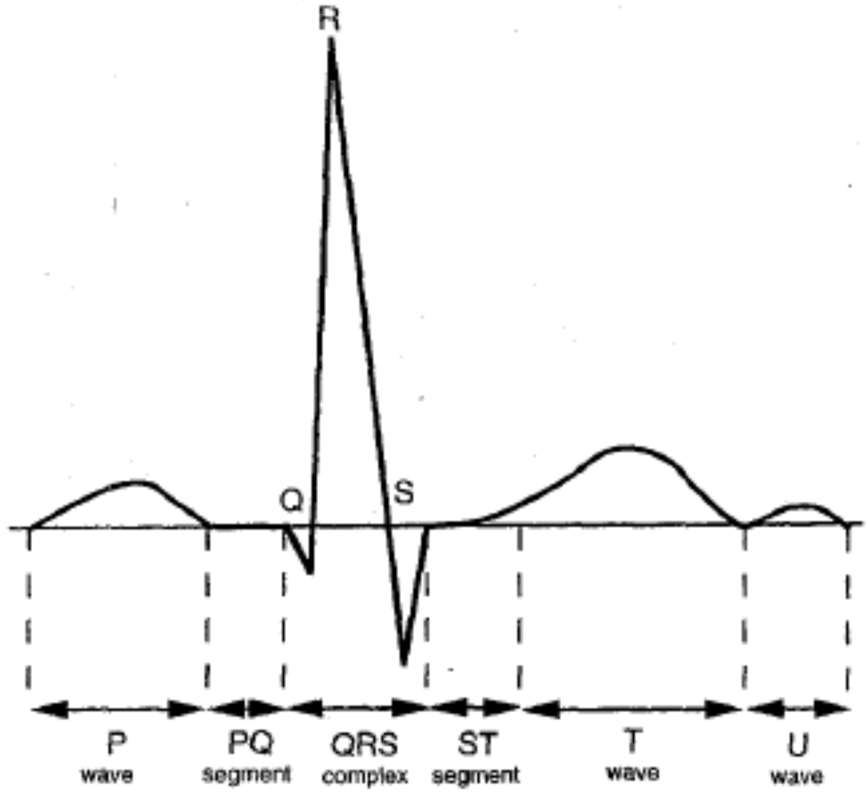

Figure 1. Schematic representation and labeling of the ECG (one cardiac cycle) [3]

1.1.2 ABP Signals

Unlike ECG, ABP waveforms (figure 2) provide mechanical information on cardiovascular circulation. It is appropriate to relate the systolic wave, the tidal wave and the diastolic wave with the behaviors of myocardial systole, pulse wave reflection and vasomotion, respectively [4].

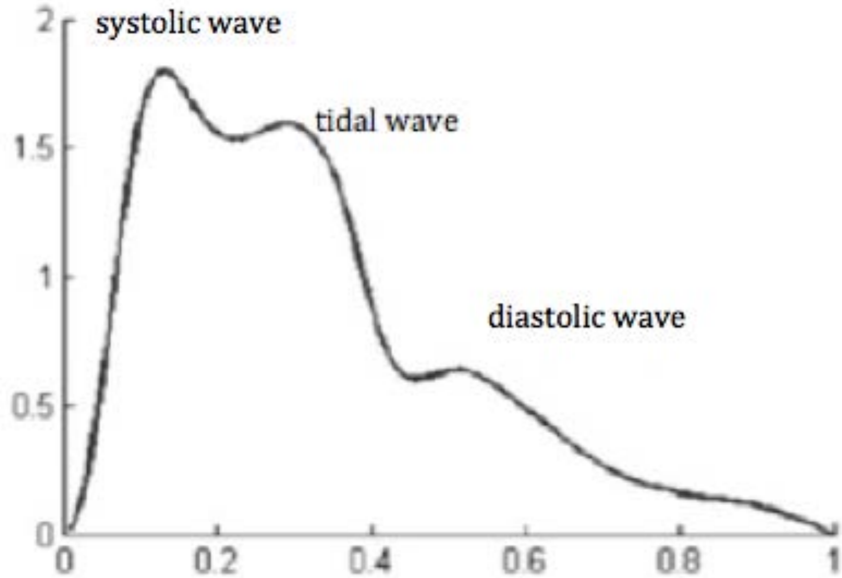

Figure 2. Synthesized normal ABP signal [4]

Although ECG and ABP have different waveforms, they both are often small signals with low frequency. For example, the frequency range of ECG signals ranges roughly from 0.01 Hz to 100 Hz. Therefore, after converting a physiological signal into an electrical signal with the corresponding type of sensor or transducer, an analog front-end circuit is often needed to filter and amplify the signal before digitizing it for further processing [5]. Although this allows the signals to be processed, enhanced, stored, and analyzed in the digital domain, noise can be also generated and added to the signals during this process.

1.2 Types of Noise Sources

Getting clean and accurate physiological signals has always been a challenging problem for both engineers and scientists, because some noise is unavoidable. The two basic types of noise are a result of external as well as internal sources.

1.2.1 External Noise

External noise sources are ubiquitous. For example, in some cases, ECG is recorded under exercise conditions, but the signals are often corrupted by extraneous disturbances due to muscular activity, such as electromyographic (EMG) noise and respiration. The EMG noise is random in nature and has a frequency content existing over a wide range. Other external noises, such as the electrosurgical and instrumentation noise are similar in nature to the EMG noise. Apart from this, one of the major sources of interference is from power-lines, also known as ac noise. Compared to other external noises, the effects of power-line interference are relatively easy to reduce through the use of proper shielding and suitably designed notch filters [6].

1.2.2 Internal Noise

Internal noise results from the thermal motion of electrons in conductors, random emission, and diffusion or recombination of charged carriers in electronic devices. Proper care can reduce the effect of internal noise but can never eliminate it [7]. Like many other electronic devices, ECG and ABP monitors may acquire internal noise from amplifiers and loads [8]. If a quartz-crystal resonator is used in the physiological signal monitor, the parallel capacitance of the resonator can increase the amplitude-frequency coupling and drastically modify both amplitude and phase spectra of the internal noise [9].

Additive White Gaussian Noise

Additive white Gaussian noise (AWGN) is one of the most common types of internal noise. It has a constant power spectral density, and is a good model for many communications systems. In the realistic environment, the noise level of AWGN is usually at around 30 dB SNR for

physiological signals [10]. Like many other noises, AWGN can distort or even destroy a signal. In electrical engineering, a filter is commonly used to remove or eliminate the effects of noise. It can selectively pass or block certain portions of the signal and noise. Butterworth filter is one of the widely used filtering techniques.

1.3 Butterworth Filter

The Butterworth filter is a type of signal processing filter designed to have a maximally flat frequency response. An ideal lowpass Butterworth filter characteristic has a constant gain of unity up to the cut-off frequency. Then the gain drops suddenly to zero for frequency greater than the cut-off frequency.

The magnitude of the frequency response of a typical Butterworth filter $|H(j\omega)|$ is given by:

$$|H(j\omega)| = \frac{1}{\sqrt{1+(j\omega / j\omega_c)^{2N}}} \qquad (1)$$

where

N = order of filter

ω = the frequency of the input signal

ω_c = cut-off frequency

In practice, as N goes to infinity (figure 3), the lowpass Butterworth filter response approaches the ideal response [11]. Although this ideal response is unattainable, it is not the biggest drawback. The main disadvantage of the Butterworth filter on physiological signals is

that it requires tremendous effort to preserve scattered information at different frequency bands. Also, the locations of the information are changing from signal to signal or even from time to time. Therefore, a better method for removing noise from physiological signals is desired.

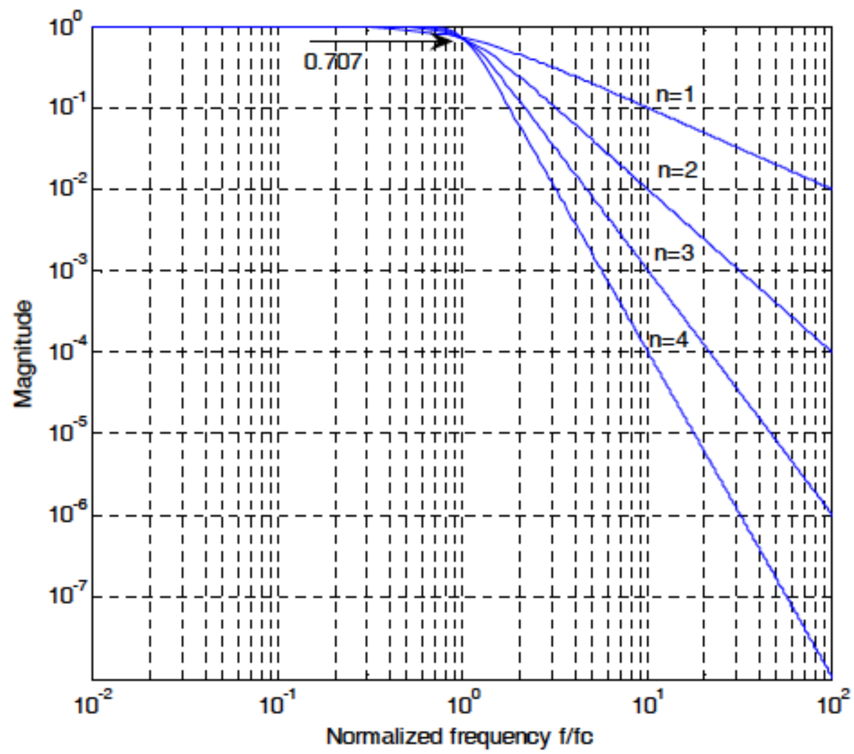

Figure 3. Frequency response of a Butterworth filter of order N [12]

1.4 Wavelet Transform

Wavelet analysis provides a new method for removing noise from signals that complements the classical methods of Fourier analysis, such as Butterworth filters [13]. The decomposition process itself is called Wavelet Transform (WT). WT is a powerful signal-processing tool. It transforms a time-domain waveform into time-frequency domain and

estimates the signal in the time and frequency domains simultaneously. WT is classified into continuous wavelet transform (CWT) and discrete wavelet transform (DWT).

1.4.1 Continuous Wavelet Transform (CWT)

CWT uses wavelets, or "small waves," which are functions with limited energy and zero average, as shown in equation (2).

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \qquad (2)$$

In CWT, a specific wavelet, generally referred to as the mother wavelet $\psi(\mathrm{t})$, is initially selected. Dilated (stretched) and translated (shifted in time) versions of the mother wavelet are then generated. Dilation is denoted by the scale parameter *a* (a positive real number) while time translation is adjusted through *b* (a real number), as given in equation (3).

$$\psi_{a,b}(\mathrm{t}) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad (3)$$

where the quantity $\frac{1}{\sqrt{a}}$ is a normalizing factor that ensures that the energy of $\psi(\mathrm{t})$ remains independent of *a* and *b* [14], i.e.,

$$\int_{-\infty}^{+\infty} |\psi_{a,b}(t)|^2 dt = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt \qquad (4)$$

In other words, the normalizing factor is to ensure that the transformed signal will have the same energy at every scale.

CWT is the convolution of a signal $f(t)$ with a specific mother wavelet $\psi_{a,b}(t)$ at a scale *a* and time translation *b*; this procedure is then repeated for different values of *a* and *b*.

Mathematically, CWT can also be expressed as the dot product of $f(t)$ and $\psi_{a,b}(t)$, as given in equation (5).

$$W(a,b) = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{a}} \psi^{*}\left(\frac{t-b}{a}\right) dt = \left\langle f(t) \bullet \psi_{a,b}(t) \right\rangle \qquad (5)$$

where, the * denotes the complex conjugate.

More explicitly, a wavelet at scale *a* = 0.1 and time translation *b* = 0 is multiplied by a signal $f(t)$ using equation (5) to calculate a value of the transformation. The same wavelet at scale *a* = 0.1 is then shifted to the right by increasing the value of time translation *b* to calculate another value of the transformation; the calculation procedure is repeated for every value of time translation *b* over all times of the signal. Once the wavelet reaches the end of the signal, the scale *a* is increased (mother wavelet is stretched) and then the convolution procedures are repeated over all times again. The starting value and the increment value of scale *a* can be changed from case to case. However, the value of scale *a* of the wavelets functions does not go beyond 1 for resulting the high frequency components of the signal.

A wavelet functions $\psi(\mathrm{t})$ can only obtain information of scale *a* < 1 corresponding to high frequencies. In order to obtain the low-frequency information of the original signal $f(t)$, a scaling function $\phi(t)$(father wavelet) is used to determine the wavelet coefficients for scale *a* > 1. The scaling function can also be scaled and translated as the wavelet function, as shown in equation (6).

$$\phi_{a,b}(t) = \frac{1}{\sqrt{a}}\phi\left(\frac{t-b}{a}\right) \qquad (6)$$

Similar to the wavelet function, the low-frequency approximation of $f(t)$ can be computed by using dot product of the signal $f(t)$ and the particular scaling function $\phi_{a,b}(t)$; it also can be computed by using equation (7) [15].

$$W(a,b) = \int_{-\infty}^{+\infty} f(t)\frac{1}{\sqrt{a}}\phi^{*}\left(\frac{t-b}{a}\right)dt \qquad (7)$$

1.4.2 Discrete Wavelet Transform (DWT)

The CWT performs a multi-resolution analysis by changing the scale of the wavelet functions, but the discrete wavelet transform (DWT) performs the multi-resolution analysis differently. It uses filter banks for the construction of the multi-resolution time-frequency plane [16].

The filter bank is commonly used in applications for transforming an input signal into a time-frequency domain representation. In DWT, the filter bank consists of a two-channel succession of lowpass and highpass discrete-time filters, which separate a signal into frequency bands [16], [17]. As mentioned in last section, the scaling function is more suitable for obtaining the low-frequency information, and the wavelet function is more suitable for obtaining the high-frequency information. Therefore, the scaling function and the wavelet function act like a lowpass filter and a highpass filter respectively.

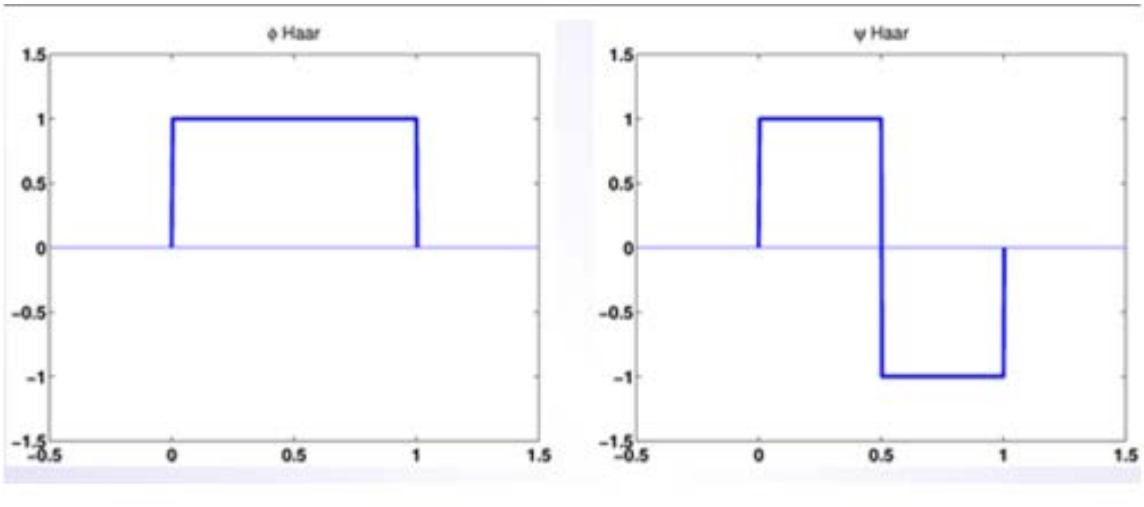

Figure 4. Haar scaling and wavelet functions [18].

To illustrate the concepts, for example, a simple signal x = [3 6 7 5] is used to perform a DWT. A Haar wavelet (figure 4), the first and simplest wavelet, is selected. A lowpass filter $\left[\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ and a highpass filter $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ are derived from the Haar scaling function and wavelet function respectively [19]. The low-frequency "approximation" and the high-frequency "detail" of the signal x can be calculated by convoluting the lowpass filter and the highpass filter respectively with the signal. The convolutions of the lowpass filter and the highpass filter with the signal are shown as follows:

$\left[\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right] \otimes$ x = [2.1213 6.3640 9.1924 8.48533.5355] → Approximation coefficients

$\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right] \otimes$ x = [-2.1213 -2.1213 -0.7071 1.4142 3.5355] → Detail coefficients

In DWT, a signal is split into an approximation (low-frequency components of the signal) and a detail (high-frequency components of the signal); this process is called wavelet decomposition. According to Nyquist's rule, half of the samples can be eliminated after filtering. The odd numbered coefficients are then discarded. The resulting approximation coefficients and detail coefficients come out to be [6.3640 8.4853] and [-2.1213 3.5355] respectively. The approximation is then itself split into a second-level (order) of approximation and detail, and the process is repeated to third-level, fourth-level, so on. Each time, the length of the approximation and the detail is halved from the previous level. Higher levels result in more approximations and details of the signal as shown in figure 5 [20], [21].

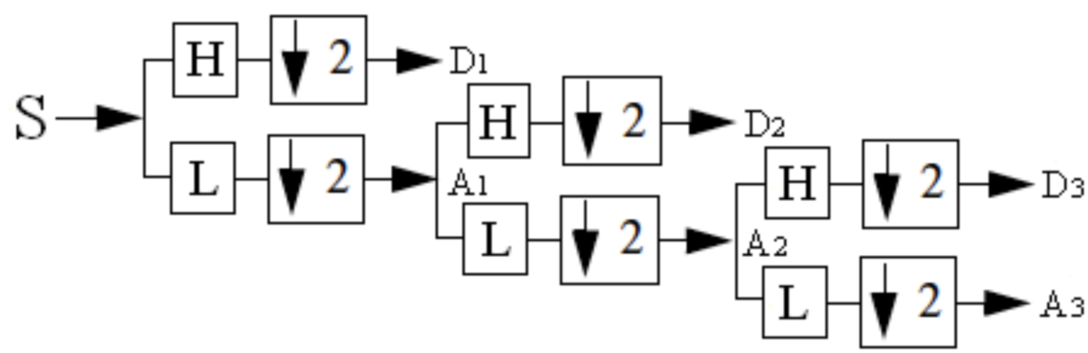

Figure 5. A 3-level filter bank tree for computing the discrete wavelet transform

Because the approximation and detail coefficients are followed by downsampling, in order to reconstruct the original signal S, the approximations and details need to be upsampled first. The original signal can then be reconstructed by combining all the levels of detail and the highest level of the approximation as $S = A_3 + D_3 + D_2 + D_1$.

1.4.3 Choice of Wavelet

The Haar wavelet is chosen in the example because of its simplicity. For other applications, the choice of wavelet is based on the shape of the signal $f(t)$ itself, since there is no ultimate way to select the wavelet. In the thesis, Daubechies family (figure 6) is chosen considering the similarities of the QRS complex in the ECG to the wavelet basis in this family [22].

Daubechies Family Wavelets

Daubechies is one of the most commonly used (mother) wavelets. The names of the Daubechies family wavelets are written as dbN, where N is the order, and db the "surname" of the wavelet. The first order of a Daubechies wavelet is the same as a Haar wavelet function (figure 4).

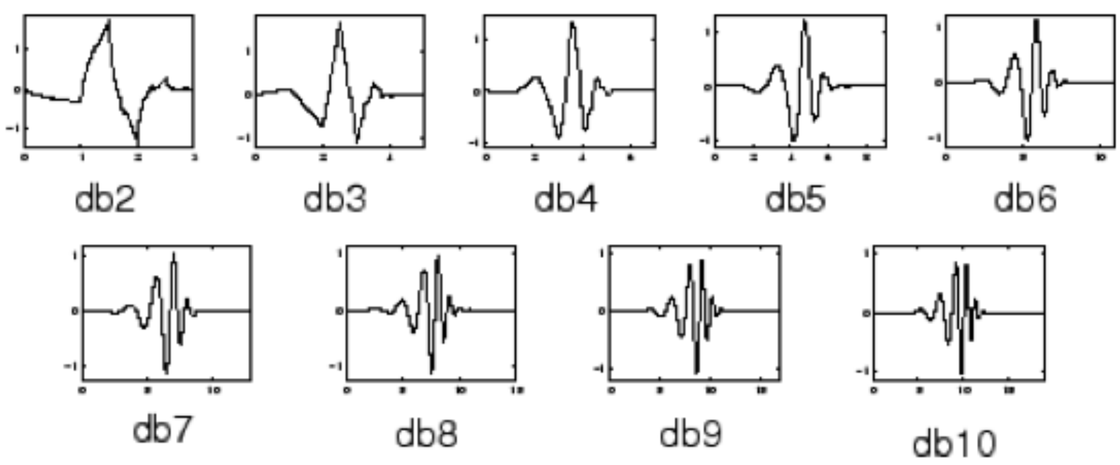

Figure 6. The Daubechies family wavelets [20]

1.4.4 Wavelet Analysis

Wavelet analysis is a relatively new mathematical tool that can extract information from many different kinds of data and has a much better performance on non-stationary signals such as physiological signals [22] than other common methods, such as Fourier analysis.

Fourier analysis is one of the most popular mathematical tools in signal processing. It is well known for its ability to separate different frequency bands, but it has a serious drawback in preserving the time characteristics. However, wavelet analysis can provide both time and frequency information [23].

For example, a simple signal with two transients is shown in figure 7. The signal has relatively higher frequency transient on the right.

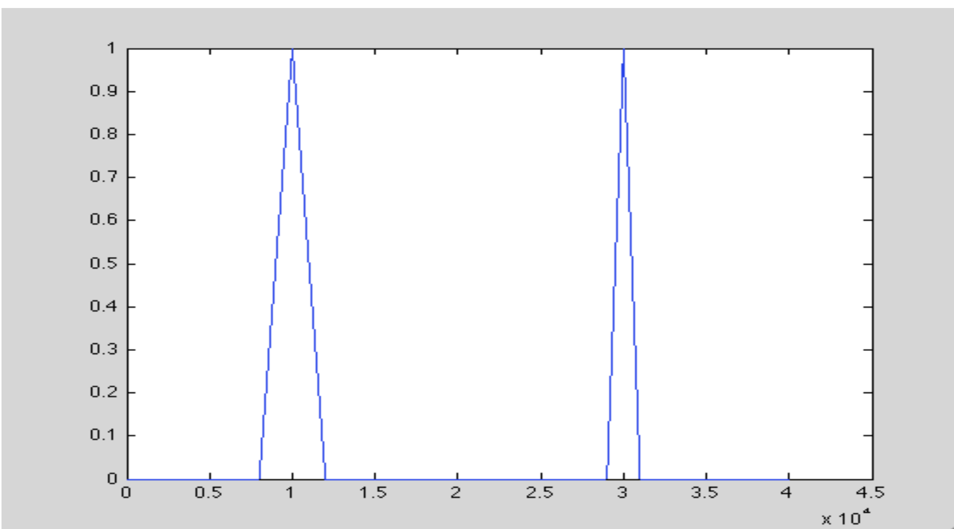

Figure 7. Example signal $f(t)$

In order to extract the high-frequency information from the signal, the Haar wavelet function $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ is selected instead of the scaling function. After the convolution of the Haar wavelet function with the example signal, the result is shown in figure 8. The result shows the transient on the right has a greater magnitude on the higher frequency transient after the transformation. It also shows that the time information, which is the location of the transients, can be preserved. Therefore, wavelet analysis is suitable for analyzing different frequencies and preserving the time information.

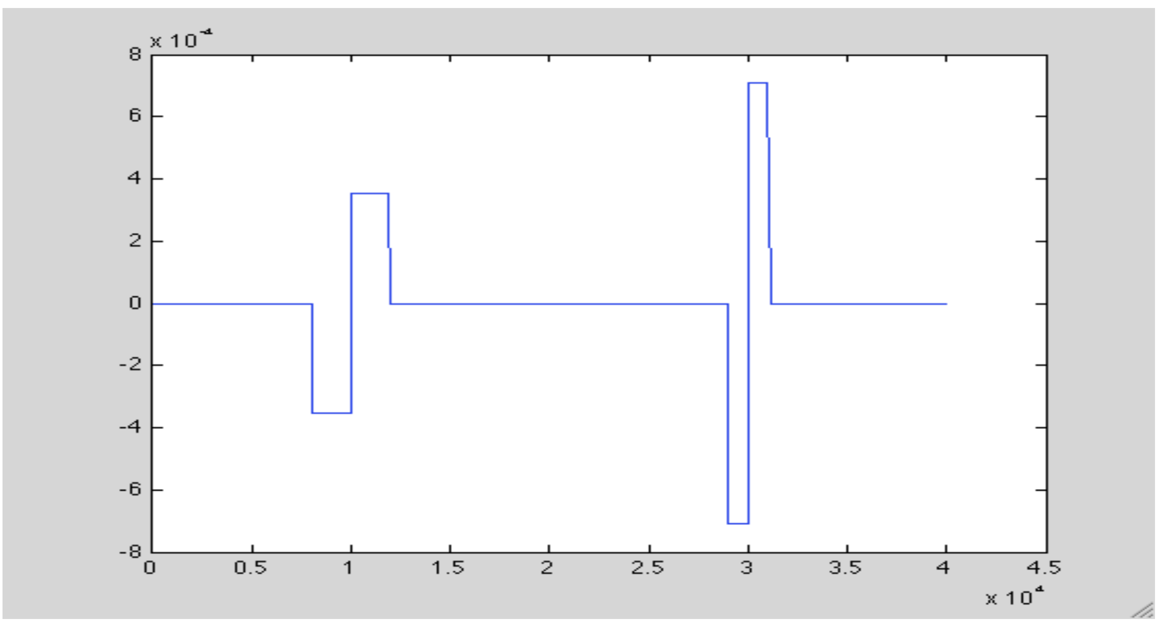

Figure 8. The convolution of Haar wavelet function with example signal $f(t)$

As shown in the example, wavelet transform has the ability to differentiate the frequencies. In wavelet analysis, a signal is divided into different small frequency bands; signal analysis can be performed on each small frequency band individually. This can give a micro-scoping view of the interested portion of the signal, making wavelet analysis

a good signal-processing tool. Noise filtering is one of the most common signal processing applications that use wavelet analysis. In the thesis, wavelet analysis is used to de-noise physiological signals which were provided by Mayo Clinic at Jacksonville.

1.5 Patient Samples used in the Analysis

In the study, all the data samples were collected from liver transplant patients before, during, or after surgery at Mayo Clinic. Due to information privacy, no personally identifiable information will be disclosed from the sample data. There are a total of 14 ECG and ABP signals. Data may have already been upscaled or downscaled before the noise-filtering testing. Since scaling does not change the signal-to-noise ratio of the sample signals, the de-noising performance of both Fourier analysis and wavelet analysis can still be tested on the scaled signals.

1.6 Thesis Organization

Chapter 1 provides a summary of the background and introduction to concepts, such as the physiological signals ECG and ABP and the different kinds of noise associated with them. It also covers the fundamental concepts of both Fourier Analysis and Wavelet Analysis.

Chapter 2 describes the importance of preserving the transitory information associated with physiological signals, such as ECG and ABP. It also covers the disadvantages of using a traditional Fourier filter, Butterworth filter, on the noise filtering of those signals.

Chapter 3 discusses the methodology that is illustrated as the most important part of the thesis. The algorithms of finding the optimal level of approximation and calculating the threshold value of filtering for ECG & ABP signals are discussed and presented.

Chapter 4 provides the results from the experiments. It shows wavelet analysis is suitable for analyzing different frequencies and preserving the time information on those physiological signals.

Chapter 5 states the conclusions for the work. It also suggests possible directions for future research.

Chapter 2

PROBLEM STATEMENT

The accuracy of physiological signals is very important for monitoring a person's health, especially in the intensive care unit (ICU); however, the signals are often severely corrupted by noise, artifacts and missing data, which could lead to high false alarm rates (sometimes as high as 90% for some alarm types) from ICU monitors [24] or even misdiagnosis.

In order to resolve the problems, understanding the physical meaning of the physiological signals becomes important. Blood circulation generates one of the most common physiological signals in human body. It carries the nutritional supply of the human body, and distributes it to all parts of body via heart systole and diastole. Pulse fluctuations in human arteries respond to physiological rhythms of their heartbeats, and therefore, when a heart works abnormally, some corresponding symptoms in pulse signals, ABP, would appear [25]. These changes in pulse signals could be very useful for diagnosing heart related diseases. Along with the ABP, ECG is also a widely accepted diagnosis tool in clinical validations of heart related diseases [26].

Figure 9 is one of the recorded ECG signals from the data samples provided by Mayo Clinic at Jacksonville. Depending on the preset threshold values and algorithms of the alarm, the arrow-pointing spike could trigger a false alarm because of the noise. A noisy

signal could also affect the determination of the ECG characteristics, such as T wave in this case. Therefore, a de-noising method of the physiological signals is valuable.

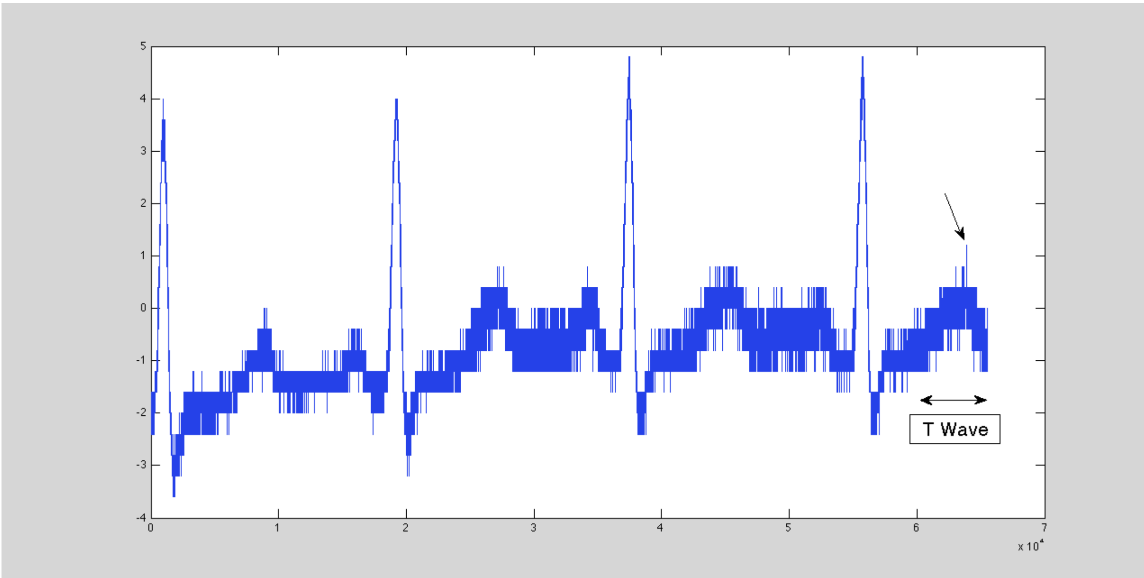

Figure 9. ECG signal in time domain

Skilled physicians traditionally analyze the physiological signals within the time domain, like the ECG signal shown in figure 9. However, pathological conditions may not always be obvious in the original time domain signal. In order to get that information in frequency domain and to filter the noises from the physiological signals, mathematical tools are needed for analysis.

Fourier analysis is one of the most famous and popular methods that engineers and mathematicians use today. It is well known for its capability of transforming signals into frequency domain, but it has a serious drawback that time characteristics will become less obvious [23]. That is, the time information would not be effectively retained from a

Fourier transform of a signal. This drawback is not so obvious and important if the signal is a stationary signal, whose properties do not change much over time. Unfortunately, physiological signals are always non-stationary signals. Fourier analysis is not suitable for detecting transitory characteristics, such as drift, trends, and abrupt changes. These characteristics are often the most important part of the signal; therefore, a method that can better handle and preserve these characteristics is desired.

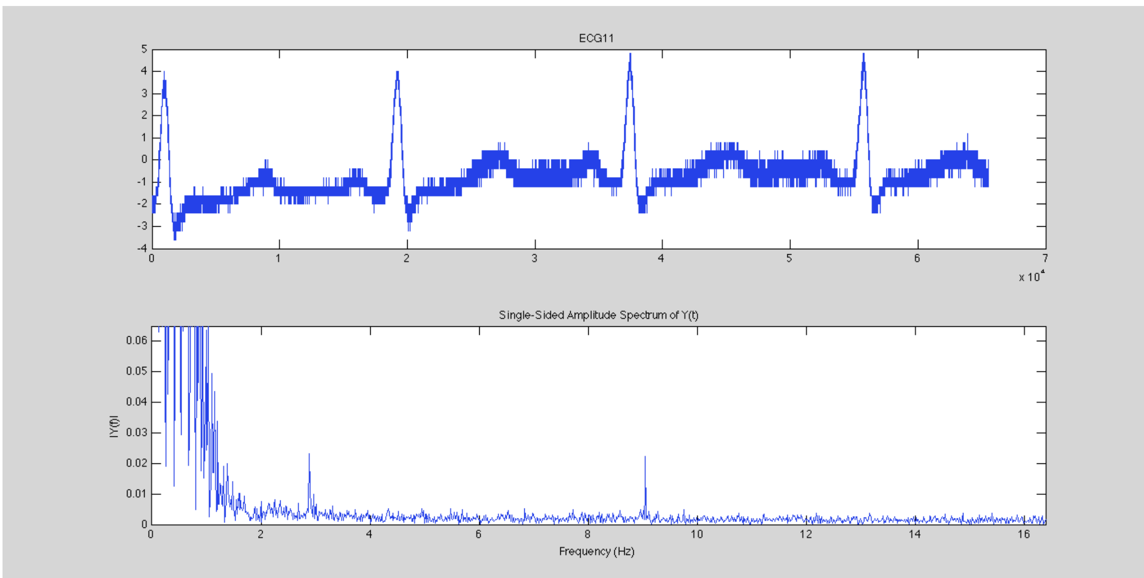

Figure 10. ECG signal (no. 11) with its frequency spectrum

Figure 10 shows an ECG signal with its frequency spectrum. In order to determine the signal characteristics, such as the QRS complex more accurately, signal processing is needed to filter out the noise from the signal. Figure 11 shows the same signal (no. 11), the filtered signal, and the frequency spectrum of the filtered signal. In this example, a 2nd order lowpass Butterworth filter with 10 Hz cut-off frequency is chosen. Although the ECG signal characteristics are more readable and determinable on the filtered signal,

the spike located at around 9 Hz is suppressed and all the information above the cut-off frequency is removed on the frequency spectrum. Even though many physiological signal noises are high frequency noise, such as muscle contraction, power line interferences, and Gaussian noise, but some high frequency information could also be valuable for diagnosing purposes. Having all the high frequency information removed may not be a good idea when de-noising physiological signals. A suitable method for filtering the physiological signals should have the ability to selectively preserve or remove certain frequency bands by certain magnitude.

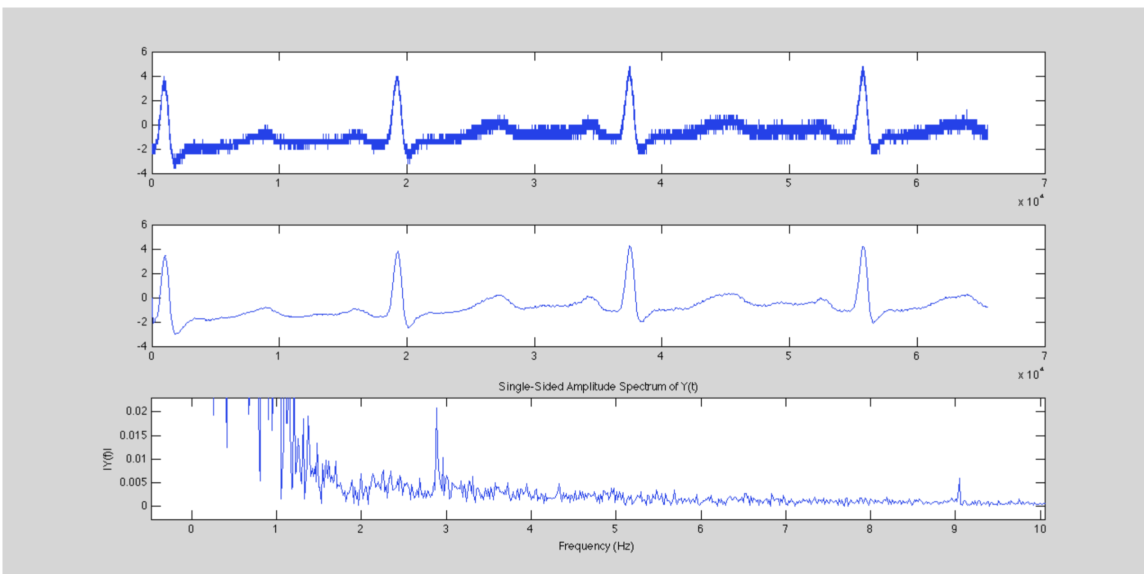

Figure 11. Butterworth filtered signal (no. 11) with its freq. spectrum

Like the ECG signal characteristics, the detection of the tidal wave could be a crucial step in the analysis of ABP signals. Figure 12 shows an ABP signal with its filtered signal by using a 2nd order Butterworth lowpass filter. Similar to the previous example, the Butterworth filter does a good job of smoothing a noisy signal; however, the arrow-

pointing hump is also flattened. This is a serious drawback of using a Butterworth filter on physiological signals, since the humps and certain high-frequency details are important for diagnosing heart conditions. Furthermore, a bandpass Butterworth filter cannot be used to remove only certain frequency band signals because this information is unknown or unpredictable for non-stationary signals. Additionally, the time information of each frequency bands is unclear after the Fourier transformation.

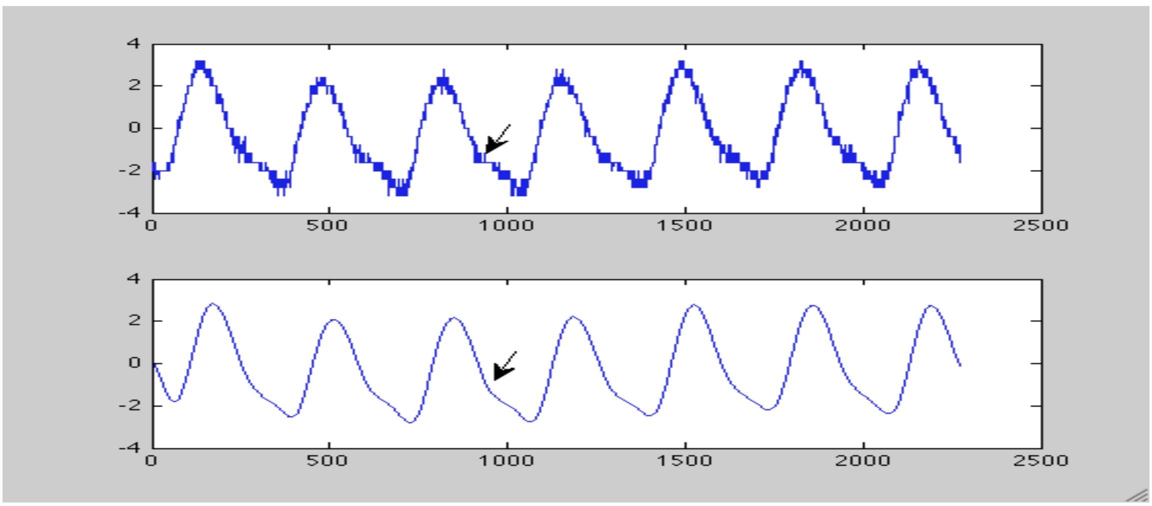

Figure 12. Butterworth filtered ABP signal

Wavelet analysis is a relatively new mathematical tool that can provide both time and frequency information, and overcome the limitation of Fourier analysis on non-stationary signals, such as physiological signals [23]. Multi-resolution wavelet analysis has been widely applied to many fields, especially to biomedical signals, such as brain wave signal processing and ECG analysis. Since wavelet transformation provides desirable characteristics in time-frequency signal processing, it is suitable for analyzing the time varying characteristic of the non-stationary signal such as ECG and ABP [22]. Hence, it is

important to study the wavelet analysis of the physiological signals and its performance of the noise filtering. In the proposed thesis, the wavelet analysis and its noise filtering performance will be analyzed by using Matlab computing software.

In wavelet analysis, a signal is divided into an approximation and a detail. The approximation is then divided into a second-level approximation and detail, and the process is repeated on the third-level, fourth-level, and so on. A suitable level of approximation is best for physiological signal monitoring, because it gives a cleaner and shorter signal. However, if the level of approximation is over the optimal level, the signal will be distorted as shown in figure 13. Therefore, an algorithm of determining the optimal level of wavelet decomposition will be studied and created.

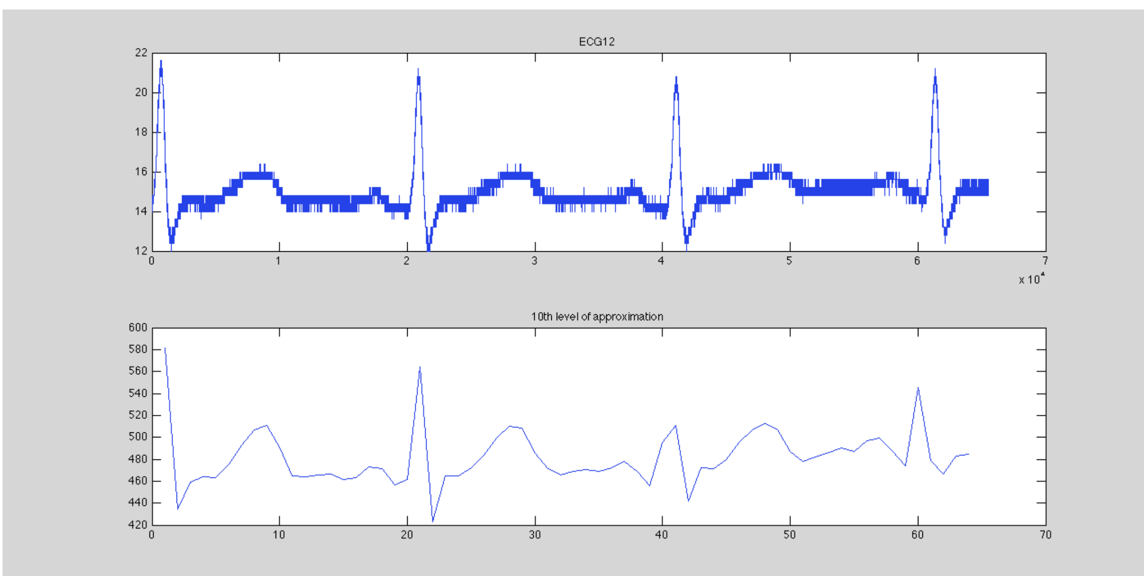

Figure 13. ECG signal (no. 12) with its distorted 10^{th} level of approximation

Wavelet analysis also has the ability to selectively remove only certain frequency bands from a signal. The threshold can be applied on each frequency band to cut off certain magnitudes. These two great features make the wavelet analysis to be one of the most powerful and suitable analytical methods for the physiological signals. In the study, an algorithm that can calculate the threshold value has been created to help physicians more systemically and effectively adjust the viewable information from case by case.

As the accuracy of physiological signals is getting more and more attention, it is valuable to study the de-noising performance of wavelet analysis on ECG and ABP. In the proposed thesis, the ability of preserving the time information, the algorithm of finding the optimal level of approximation, different ways of adding the details back to the approximation, and the algorithm that can calculate the threshold value systemically and effectively are designed and presented.

Chapter 3

METHODOLOGY

The focus of the thesis is to study the effect of noise filtering on physiological signals using wavelet analysis. This is accomplished by performing wavelet analysis on sample data, ECG and ABP, which are provided by Mayo Clinic at Jacksonville. A number of sample data will be used; all the sample data are recorded from liver transplant patients before, during, or after transplant surgery.

The wavelet analysis and the noise filtering on physiological signals are simulated on Matlab computing software. Through the simulation, the performances of both Fourier analysis and wavelet analysis has been studied and compared, the optimal level of wavelet decomposition has been determined, and the algorithm that can calculate the threshold value more adjustably and efficiently has been created.

3.1 Compare the Performances of Fourier and Wavelet Analysis

In this study, the de-noising performances of both Fourier analysis and wavelet analysis are simulated by using Matlab computing software. Since the Butterworth filter is a widely used filtering technique in Fourier analysis, it is chosen for the purpose of comparison. Like many other Fourier filters, the first step of applying the Butterworth filter is to transform the signals into frequency domain. The next step is to study the frequency spectrum; the cut-off frequency of the Butterworth filters will then be determined. Since most noise in

physiological signals are high frequency noise; the point where the low frequency bands die-out will be selected as the cut-off frequency for the Butterworth lowpass filter. Sometimes, a clear die-out point may be difficult to determine, thus different cut-off frequencies will also be selected to study the effect to the signals.

On the other hand, the same ECG or ABP signals have been transformed by using DWT. Different levels of approximation and detail have been generated. The representation and meaning of the approximations and the details have been studied. The sample signals are first transformed from time-domain to frequency-domain individually. To compare the de-noising performances of both Fourier Analysis and Wavelet Analysis, the approximations have been used to de-noise the same sample signals. Also, high frequency details have been added back to the approximation for studying the effect of the de-noising performance.

3.2 AWGN Test

The de-noising performances of Fourier analysis and wavelet analysis are first tested on the given sample signals during the first stage of experiment. Additive white Gaussian noise (AWGN) is then generated and added to the sample signals. Based on the research, AWGN of 30 dB signal-to-noise (SNR) is used in this testing. The 30 dB SNR is a typical AWGN level for physiological signals in the realistic environment [10]. The main purpose of this experiment is to test the de-noising performance of Fourier analysis and wavelet analysis on physiological signals under the AWGN; the performances of both analytical methods are then compared.

3.3 Level of Decomposition

Wavelet analysis has the ability to divide a signal into different levels of composition; theoretically a signal can have infinite levels of approximation and detail. Depending on the application, sometimes only the approximation is needed. Since the details are the high-frequency components, they only impart flavor or nuance to a signal. For physiological signals, most of the noise is high-frequency noise; therefore, discarding the details can result in a cleaner signal. Although choosing a higher level of approximation will result in a smoother signal, over decomposition could also remove the low-frequency components and lead to signal distortion. To avoid this, an algorithm of determining the optimal level of decomposition needs to be created.

In this study, the wavelet decomposition and the representation of approximations and details for physiological signals are first studied. The interested physiological signals are then transformed into frequency domain by using the "fft" function in Matlab; the frequency spectrums of the signals are then studied. Different levels of approximation and detail are generated; the characteristics of each level of approximations and details are then studied. Through the studies, the algorithm of determining the optimal level of decomposition has been created.

3.4 Adding Back the Details

Although a cleaner signal can be obtained by discarding all the high-frequency components, many of the original signal's sharpest features can also be lost in the process. These sharpest

features could be very important for diagnosing heart related diseases, so it is wise to keep them. However, keeping all the sharpest features could result a signal that is close to the original signal, and it will defeat the purpose of de-noising. To remove only the unwanted portion of the details while keeping certain amount of the sharpest features, a threshold can be set on the resulting wavelet coefficient. Each level of detail represents different high-frequency bands; a certain level of detail can be maintained in order to easily identify the sharpest features at certain frequency bands.

3.4.1 Adjustable Threshold Values

In this study, an algorithm that can systemically calculate threshold values has been studied and created. In Wavelet Analysis, thresholds can be set on each levels of detail. This powerful feature can remove only certain portions of the frequency magnitude. The blue dotted lines in figure 14 represent a threshold value. The magnitude within the blue dotted lines is the part that needs to be removed. Although adjusting the threshold value by free hand can provide the maximum control of the setting, a more systematic method is desirable for signal analysis. The first step of creating the algorithm is to study all levels of detail. Several methods, such as the average energy E (equation 8) and the increment of the mean value $\bar{x}$ (equation 9), are also tried for calculating the threshold value more systemically. Eventually, an algorithm that uses percentages to calculate the threshold value has been created. With the created algorithm, the threshold value is a function of the percentage of the maximum values. Each level of detail consists of numerous frequency points, and these points have different magnitudes. The absolute values of each magnitude are first calculated, and then sorted from the maximum to the minimum values. Users can determine and fine-

tune the percentage of the maximum absolute values out of the total frequency points. Depends on the selected percentage, the threshold value is calculated by averaging the maximum absolute values. Smaller percentage values result in a higher threshold value, vice versa. The same procedure is then repeated for each level of detail.

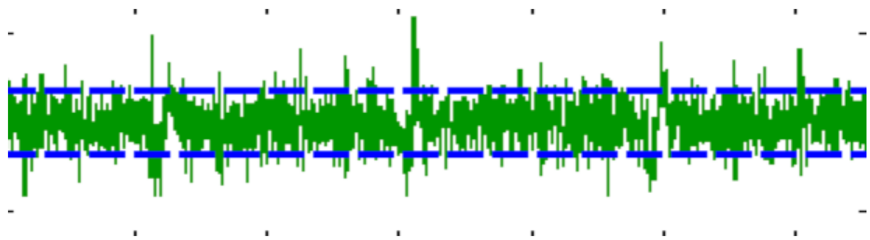

Figure 14. Adjustable Threshold Values

$$E = \int_{-\infty}^{+\infty} f^2(t)dt \qquad (8)$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (9)$$

3.4.2 Different Frequency Bands

In Wavelet Analysis, high frequency information is divided into different levels of detail, D1, D2, D3, D4, and so on (figure 15). Each level of detail represents different frequency bands. In this study, different numbers of different levels of detail are selected to construct the de-noised signal (figure 16); this allows the showing of only certain interested high-frequency bands in the original signal. For example, only the second level of detail is shown on the de-noised signal. Other high frequency information is removed. Along with the threshold

value, physicians have the maximum selectivity of de-noised signals over certain frequency information.

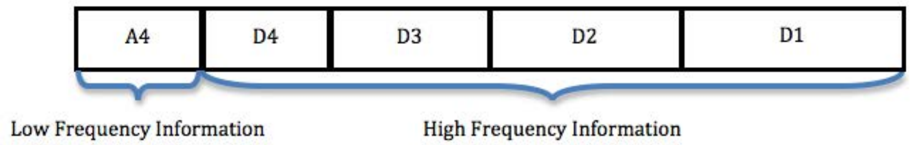

Figure 15. Different levels of detail

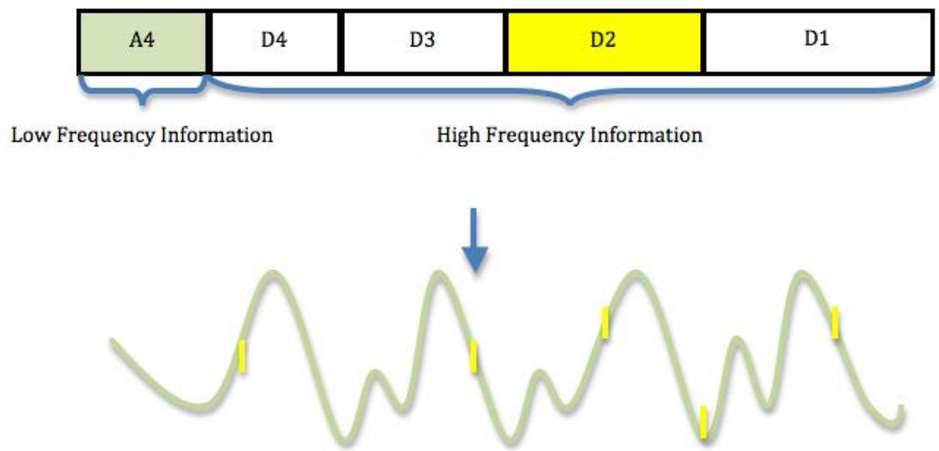

Figure 16. Adding back only certain details to create a de-noised signal

In the next chapter, experiments are conducted by following the methodologies that mentioned above. The de-noising performances of both analytical methods will be compared and analyzed. The advantages and disadvantages of Wavelet Analysis on the de-noising physiological signals will be found on the experiment results.

Chapter 4

ANALYSIS OF EXPERIMENT RESULTS

4.1 Compare the Performances of Fourier and Wavelet Analysis

This study focused on the differences between Fourier Analysis and Wavelet Analysis in de-noising performance on physiological signals. Fourier Analysis is a very common analytical method that has long been used for signal processing and analysis in electrical engineering; therefore, it was selected for comparison with the Wavelet Analysis. These two analytical methods were tested on the physiological signals that were provided by Mayo Clinic at Jacksonville. Before conducting the comparison, a total of 14 sets of ECG and ABP signals were de-noised by using each method individually.

4.1.1 De-noising Performance of Fourier Analysis

As stated, Butterworth is one of the most commonly used filtering techniques in Fourier Analysis. Like many other filtering techniques, cut-off frequency needs to first be determined based on the signal characteristics and the unwanted portions of the original signal. An average human heart beats or contracts about 72 times per minute, giving about a 1.2 Hz cut-off frequency in both ECG and ABP in general. However, patients with heart disease may have a much higher heartbeat rate. Also, since heartbeat rate changes from time to time and from patient to patient, using a fixed cut-off frequency for all physiological signals will be unrealistic and inaccurate. Because of these factors, in order to determine the cut-off frequency accurately, the signal needs to be first

transformed from time-domain to frequency-domain. In Matlab, the Fast Fourier Transform function can transform signal from time-domain to frequency spectrum.

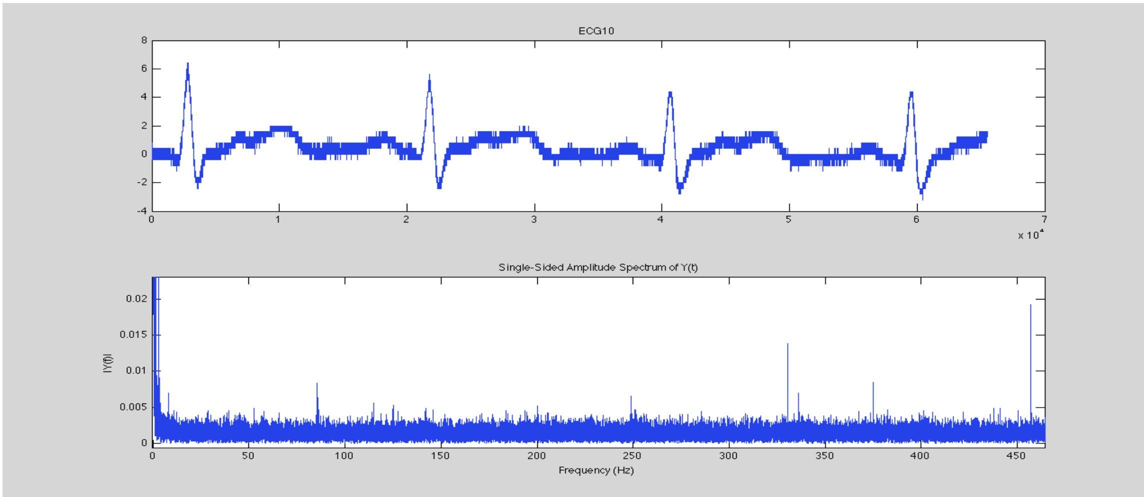

Figure 17. ECG (no. 10) with its frequency spectrum

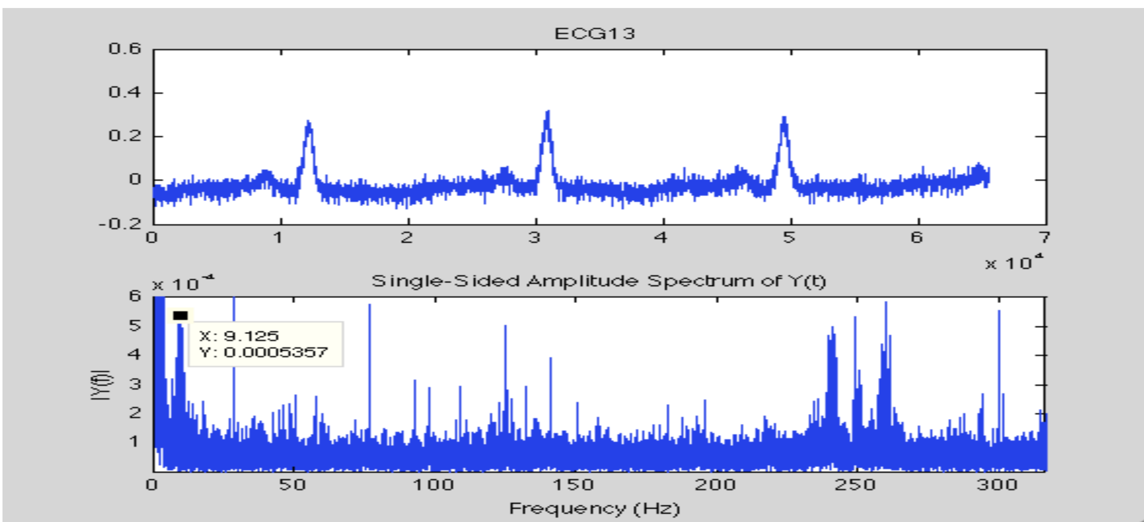

Figure 18. ECG (no. 13) with its frequency spectrum

In figure 17, the cut-off frequency was easy to determine; it could be set anywhere between 5-10 Hz. However, in some cases, the cut-off frequency might be relatively

difficult to determine. For example, in figure 18, should the cut-off frequency be set before or after the hump at around 9 Hz? If the cut-off frequency was set before the hump, there was a risk of over-filtering as a result. On the other hand, if the cut-off frequency was set after the hump, the signal might be still noisy. The problem might not be obvious by only looking at the filtered signals in time-domain; however, the problem could be easily seen on the zoomed in frequency spectrums in figures 19, 20, and 21.

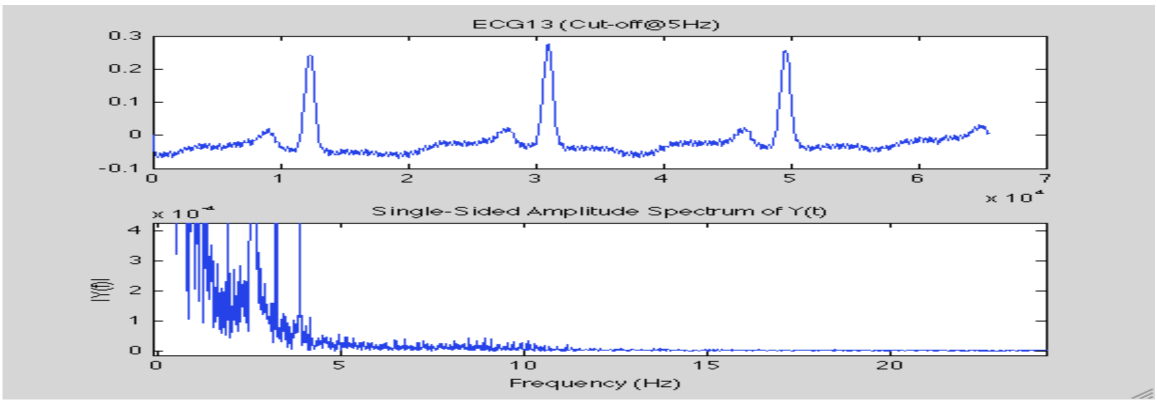

Figure 19. Butterworth filtered signal@5 Hz cut-off (no. 13) with its freq. spectrum

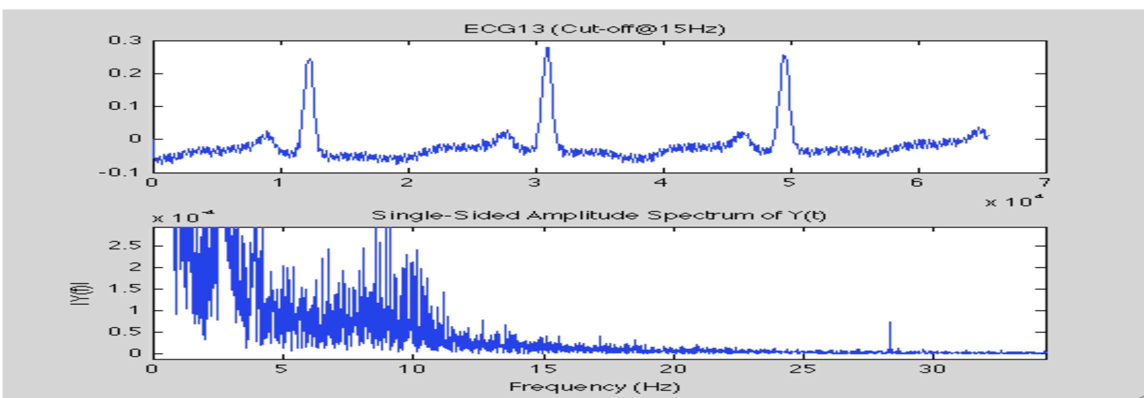

Figure 20. Butterworth filtered signal@15 Hz cut-off (no. 13) with its freq. spectrum

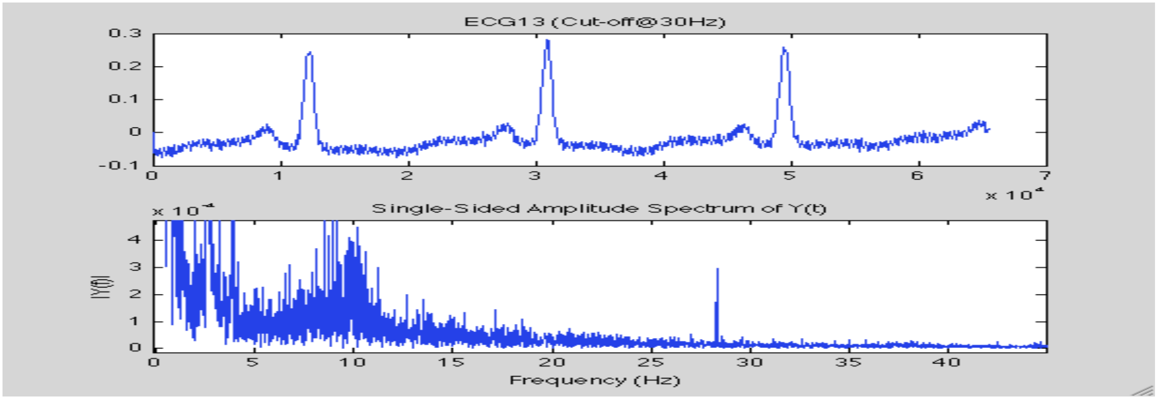

Figure 21. Butterworth filtered signal@30 Hz cut-off (no. 13) with its freq. spectrum

As shown, the de-noised signals in the time-domain were almost the same but different in the frequency-domain; all of the information beyond the cut-off frequency was suppressed. In fact, information such as humps and spikes at different frequency locations in figure 18 should only be removed if they were noise, because some of the humps and spikes could contain important information for diagnosing heart conditions. Thus, with the exception of the noise, all the other information should be well preserved when doing the noise filtering. Although a bandpass filter could be used to remove only certain frequency band signals in Fourier Analysis, it was not suitable for the physiological signals as important information and noise were unknown or unpredictable.

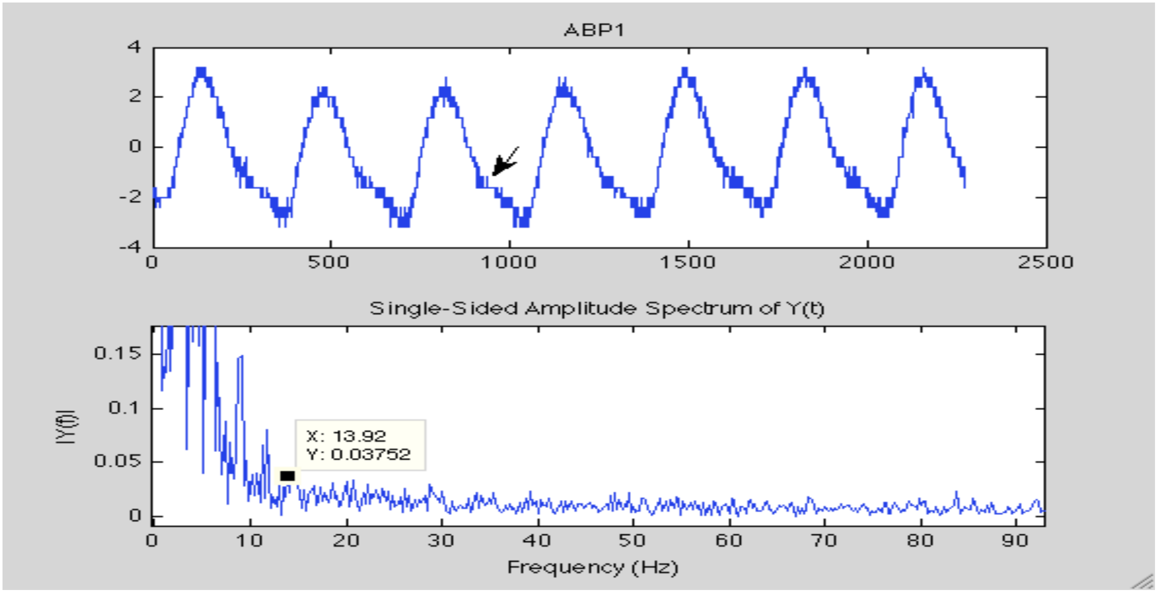

Figure 22. ABP signal (no. 1) with its freq. spectrum

Similar to the high-frequency details, sharp features such as the notch indicated by the arrow in figure 22 could be critical for diagnosing heart diseases. This information should be preserved when de-noising the signal. According to the frequency spectrum in figure 22, a 15 Hz cut-off frequency was selected to build the Butterworth filter. Although the ABP signal was less noisy after the filtering (shown in figure 23), the sharp features including the notch were flattened or gone. Even though increasing the cut-off frequency could bring the sharp features back, determining a suitable cut-off frequency to save the "notch" but not the noise would then be a challenging problem. If the cut-off frequency was set too high, it could even defeat the purpose of de-noising the signal.

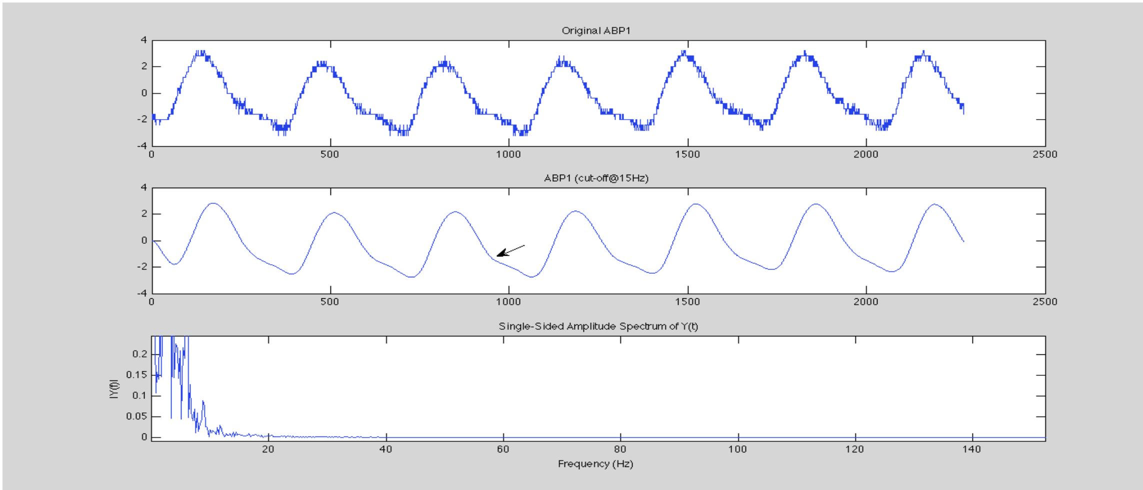

Figure 23. Butterworth filtered ABP signal (no. 1) with its freq. spectrum

4.1.2 De-noising Performance of Wavelet Analysis

Wavelet Analysis is a relatively new analytical method in electrical engineering. Like Fourier Analysis, it is one of the most popular de-noising methods that engineers and scientists use today. To study the de-noising performance of the wavelet analysis, the same 14 sets of physiological signals, ECG and ABP, were tested.

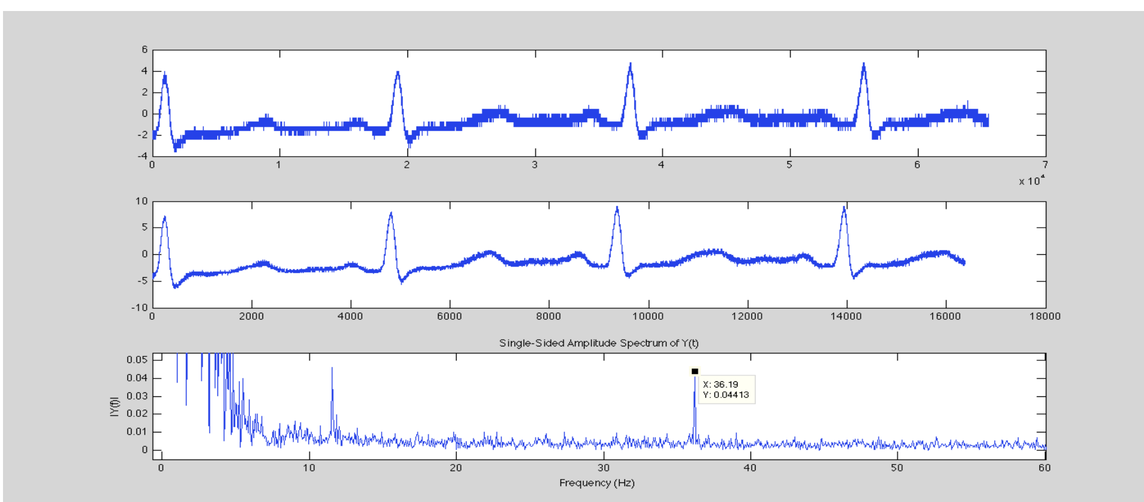

Figure 24. Wavelet filtered ECG signal (no. 11) with the frequency spectrum

In wavelet analysis, signals are treated as a combination of details and approximations. Details are the high frequency contents and approximations are the low frequency contents. Thus, approximations can act the same as the filtered signal after lowpass Butterworth filter. Figure 24 showed the same ECG signal as figure 11 with its filtered signal. The filtered signal was de-noised by using wavelet analysis in 2nd level of approximation. As shown in the figure, the filtered signal (middle) was less noisy than the original signal (top). Since the filtered signal was the 2nd level of approximation, the length of the signal was also compressed. The length of the approximation could be easily calculated; it was equal to the length of the original signal divided by 2 to the power of the level of the approximation. The original signal length was equal to 65536, so the length of the approximation = 65536/2^2 = 16384. This result showed that the wavelet analysis had the ability to de-noise as well as compress a signal at the same time.

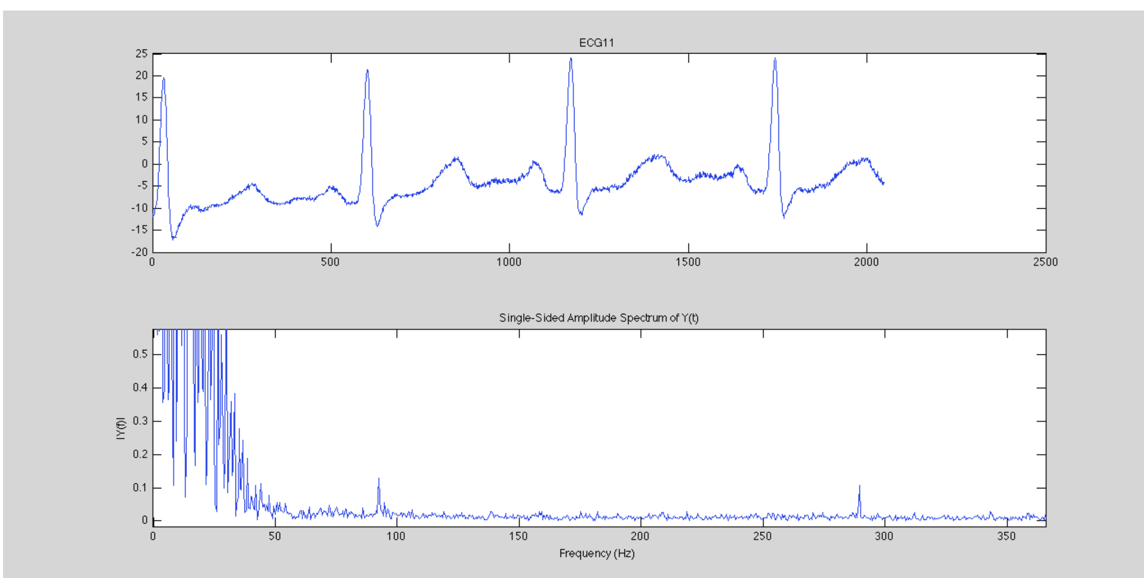

Figure 25. 5th level of approximation of ECG signal (no.11) with its frequency spectrum

Using a higher level of approximation could easily decrease the noise level of the signal. Figure 25 showed the same ECG signal (no. 11) in 5^{th} level of approximation. The length of the filtered signal = 65536/2^5 = 2048. This was only 3.1% of the length of the original signal.

In figures 24 and 25, two spikes were shown on each frequency spectrum. These two spikes were the same as in figure 11. As shown, the locations of the spikes were shifted in approximations. That shifting was due to the compression effect (down sampling) from Wavelet Transform. The original positions of the spikes could be easily calculated by taking the current location divided by 2 to the power of the order of approximation. In figure 25, the original locations of the spikes were equal to 92.29 Hz/2^5 = 2.88 Hz and 289.6/2^5 = 9.05 Hz respectively. In fact, not only the locations of the spikes were shifted but also the whole frequency band was stretched. This provided a zoom-in view of the frequency information if a number of humps or spikes were on the frequency spectrum, because the distances between the humps or spikes were increased.

Excellent de-noising performance was not only shown in ECG signals, but also in ABP signals. For example, a wavelet de-noised ABP signal (no.1) and its frequency spectrum were shown in figure 26. The signal was de-noised by using a 2^{nd} level of approximation. Unlike Butterworth filtering, the notch indicated by the arrow was well preserved after the noise filtering. The difference could be easily seen if both de-noised signals were compared side-by-side. In figure 27, the wavelet de-noised ABP signal (bottom) preserved the notches

and sharpest features. Also, the length of the wavelet de-noised signal was much shorter. This could greatly help decrease the storage size.

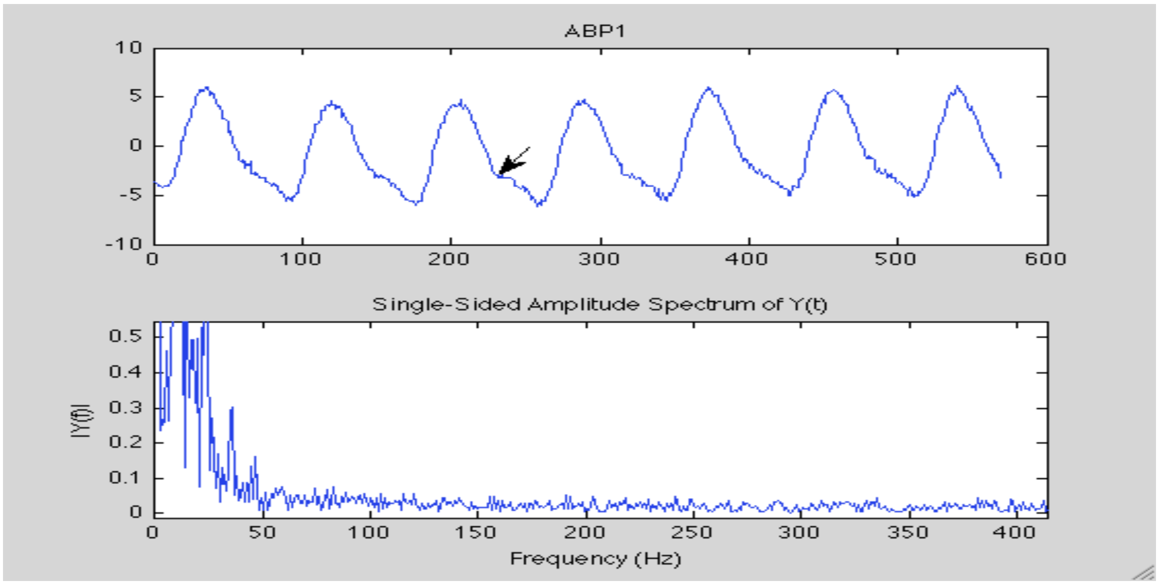

Figure 26. 2nd level of approximation of ABP signal (no.1) with its frequency spectrum

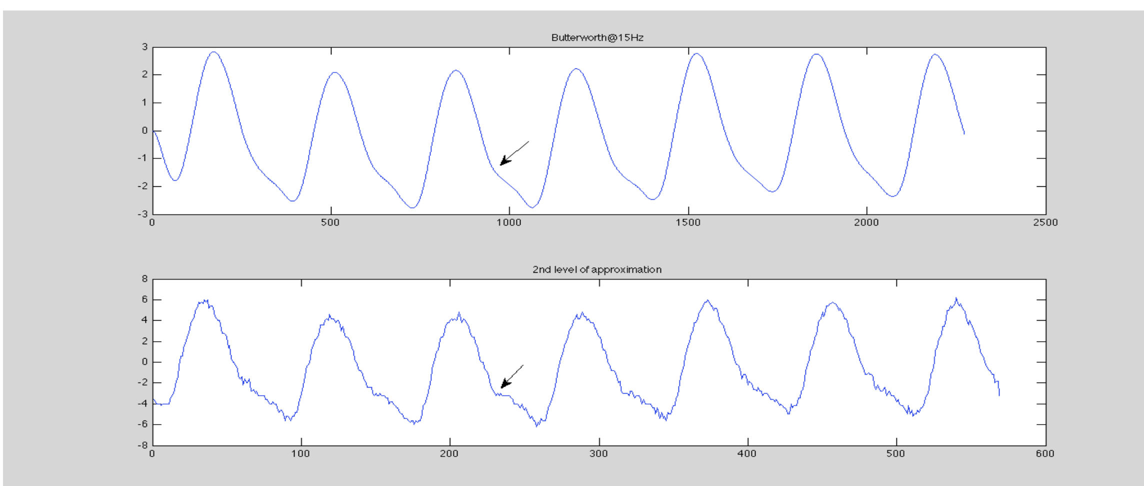

Figure 27. ABP signal (no.1) de-noised by Butterworth and Wavelet Analysis

As mentioned before, high frequency contents could contain important information for diagnosing heart conditions. However, the information was always covered by high frequency noise in physiological signals. Therefore, critical information was always removed with the noise during the noise filtering process. Since wavelet analysis had the ability to select the magnitude of frequency, important information such as humps and spikes could be preserved while de-noising the signals.

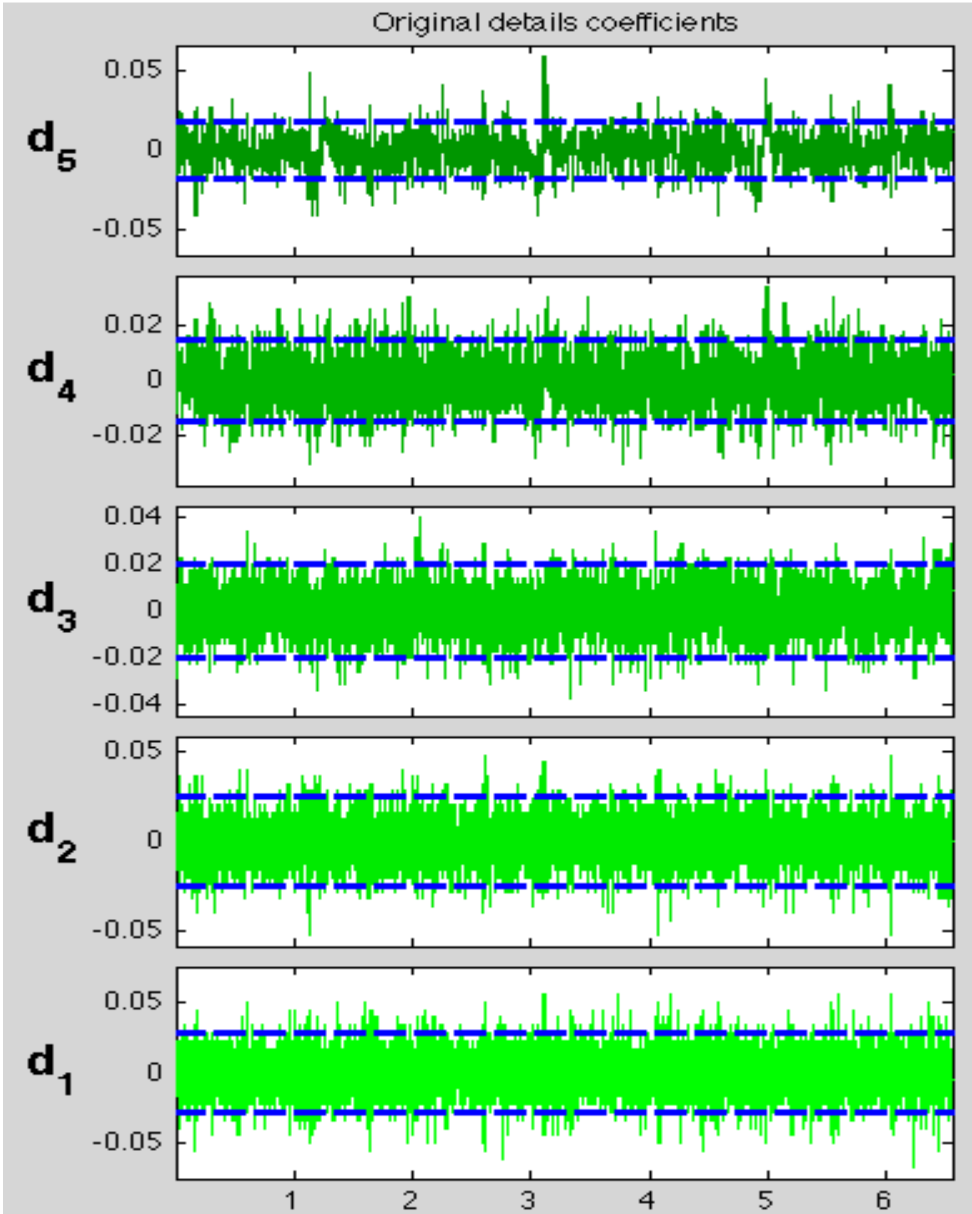

Figure 28. Different levels of detail of ECG signal with thresholds (no. 13)

Figure 28 showed five different levels of details of an ECG signal (no. 13). As shown, each level of detail had two blue dotted lines. These two blue dotted lines created a window. The magnitude within the window would be removed from that frequency band. In wavelet

analysis, the windows would be individually adjustable. This adjusting process in wavelet analysis is called thresholding.

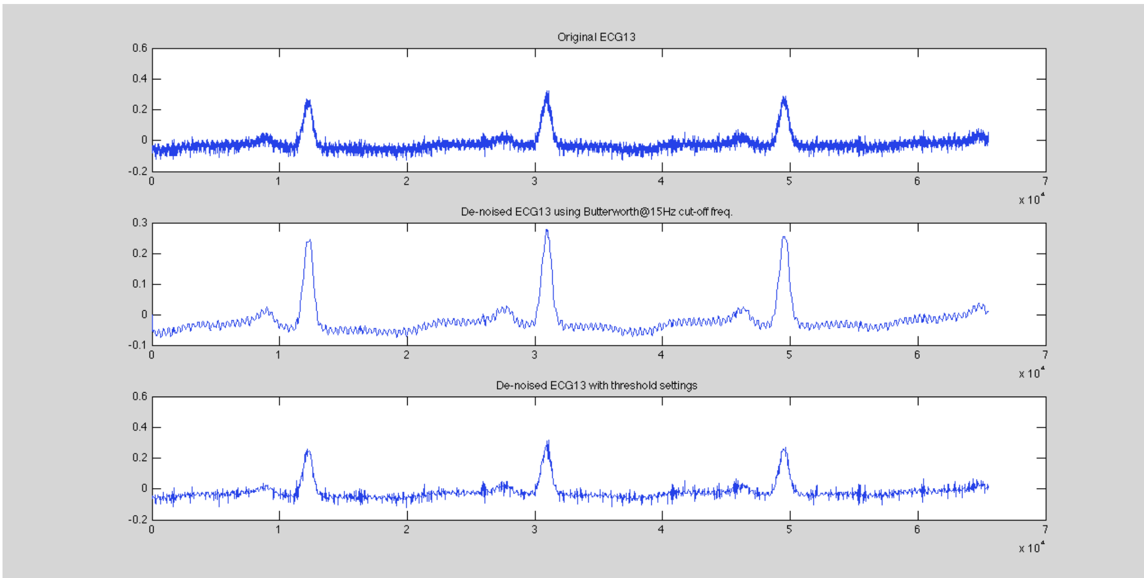

Figure 29. ECG signal (no. 13) and De-noised signals (Butterworth and Wavelet)

In figure 29, from top to bottom were the original ECG signal (no. 13), de-noised signal using Butterworth at 15 Hz cut-off frequency, and de-noised signal using wavelet analysis with thresholding applied. In this example, the threshold values were selected by naked eyes at 0.02193, 0.01258, 0.02155, 0.0283, and 0.03144 for D5, D4, D3, D2, and D1 respectively. It was intended to remove only the “common” magnitude from the frequency bands. The “common” magnitudes were the portions within the blue dotted lines that were shown in figure 28. These magnitudes were uniformly located throughout the frequency bands; therefore, they provided no valuable information but noise. The wavelet de-noised signal not only showed a much lower noise level than the original signal but also preserved the high frequency details. The difference could be clearly seen by comparing both frequency

spectrums. As shown in figure 30 and 31, the "common" magnitudes level dropped from around 1x10^-4 to only 0.5x10^-4. Fine-tuning the thresholds could even drop the noise level further.

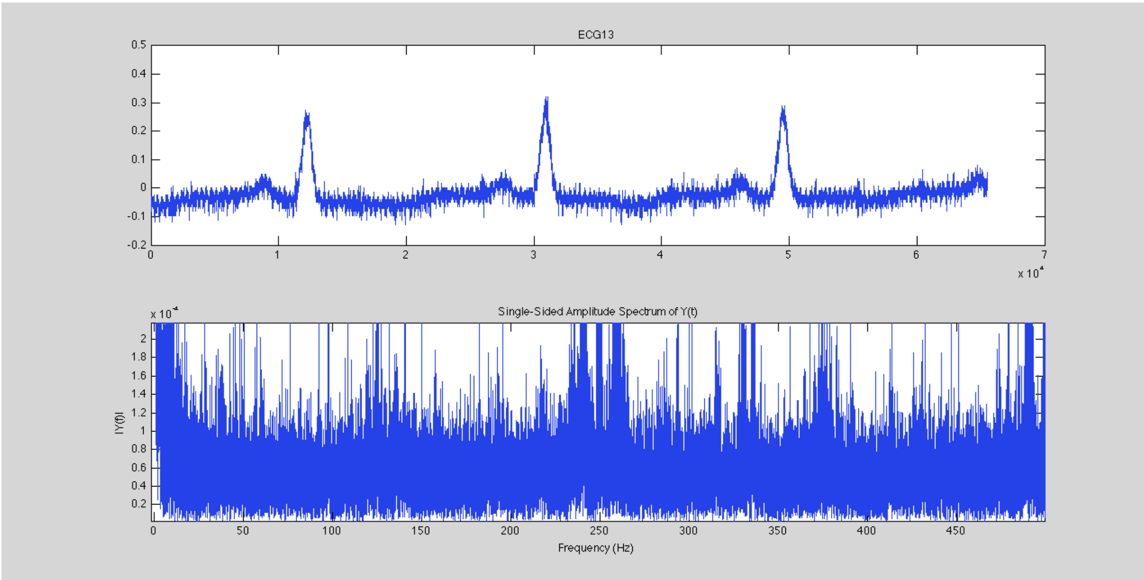

Figure 30. Original ECG signal (no. 13) with its frequency spectrum

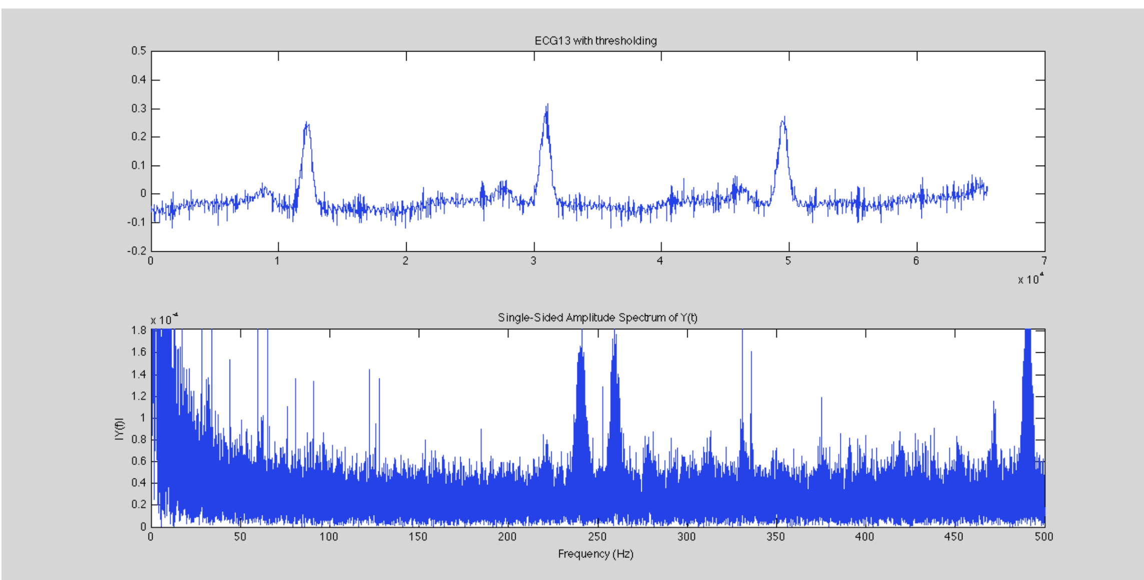

Figure 31. Wavelet De-noised ECG signal (no. 13) using thresholding with freq. spectrum.

4.2 AWGN Test

In many cases, the source of noise could directly affect the de-noising performance of any filtering methods. Since AWGN was one of the most common noises in physiological signals its effect was studied. In the study, a typical AWGN value, 30 dB SNR, would be generated and added to the signals. 30 dB SNR was treated as the worst-case scenario; any AWGN above this level should be limited by the hardware design.

Figure 33 showed an ECG signal (no. 10) with 30 dB SNR AWGN added. By comparing the noise added ECG signal with the original signal, no significant changes could be seen in time-domain and frequency-domain. To prove this, ten testing points were randomly taken along the frequency bands as follow:

	Original Signal (x 10^-3)	Noise Added Signal (x 10^-3)	Difference (%)
@58.59 Hz	0.00345	0.003614	4.75
@74.43 Hz	0.003651	0.003658	0.19
@102.7 Hz	0.003813	0.00384	0.71
@118.5 Hz	0.004179	0.004163	-0.62
@171.2 Hz	0.004112	0.004077	-0.15
@ 251.2 Hz	0.004702	0.004405	-6.32
@286.8 Hz	0.003468	0.003536	1.96
@316.4 Hz	0.003784	0.00395	4.39
@349.1 Hz	0.003837	0.003858	0.55

@421.8 Hz	0.004376	0.004267	-2.49

Table 1. Percentage difference between original signal and AWGN added signal

From the calculated percentage difference, the average frequency magnitude increased by only 0.3 %.

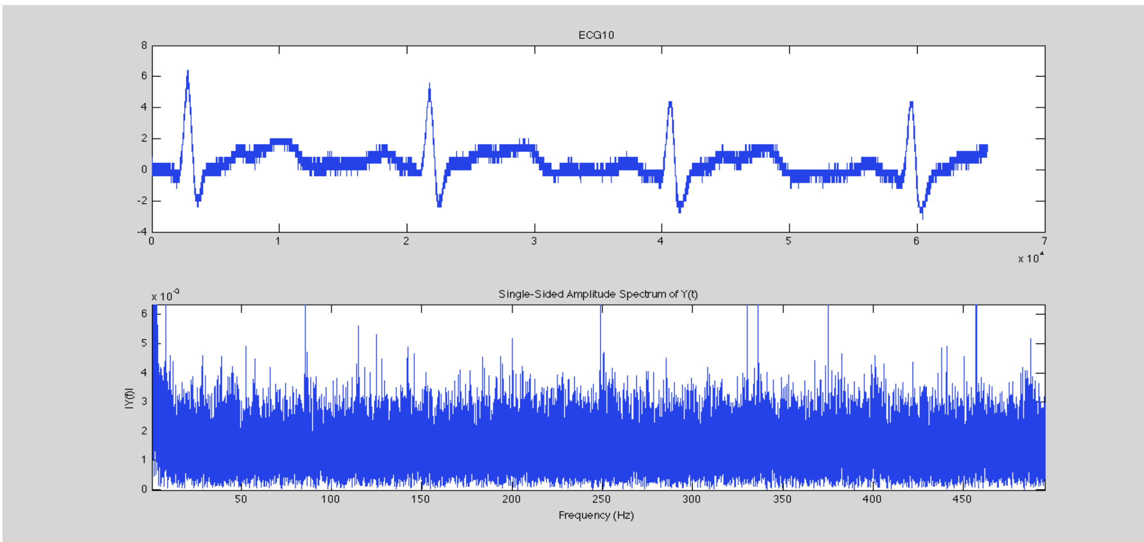

Figure 32. ECG signal (no. 10) with its frequency spectrum

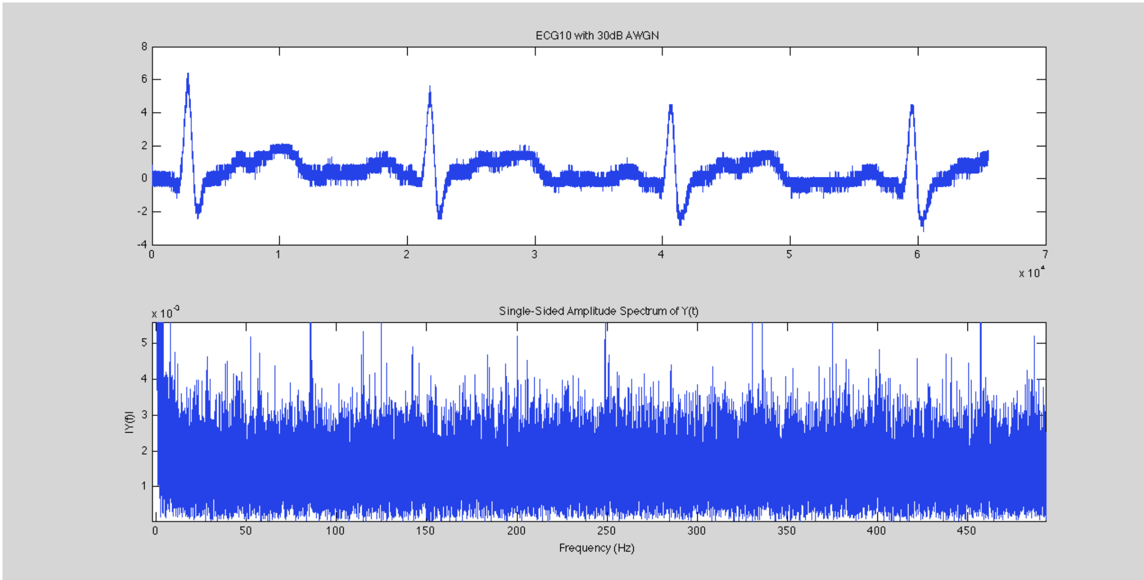

Figure 33. ECG signal (no. 10) with 30dB AWGN added and the freq. spectrum

Figures 34 and 35 showed an original ECG signal (no. 7) and the AWGN added signal respectively. Points were also taken from these two signals, and then compared. Since the frequency magnitude of the ECG signal no. 7 was relatively low by comparing to the ECG signal no. 10, the effect of the AWGN was expected to be greater. Based on the testing points, the average percentage difference was also greater than the previous example; however, it did not give an obvious effect to the signal in time-domain. The difference could only be seen clearly in the frequency-domain; some of the frequencies with low magnitude were covered by the AWGN.

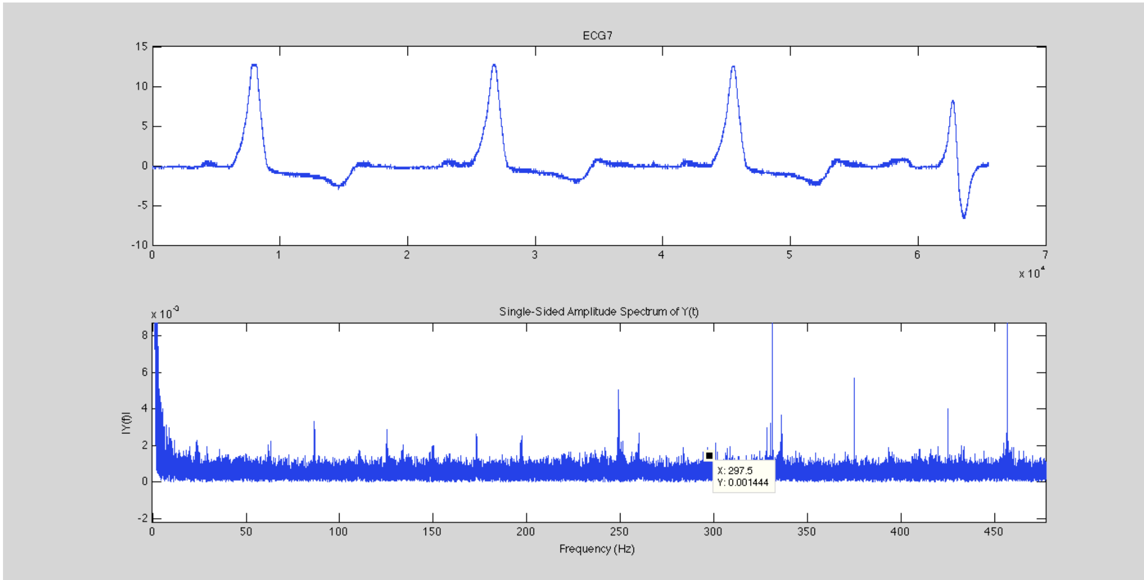

Figure 34. ECG signal (no.7) with its frequency spectrum

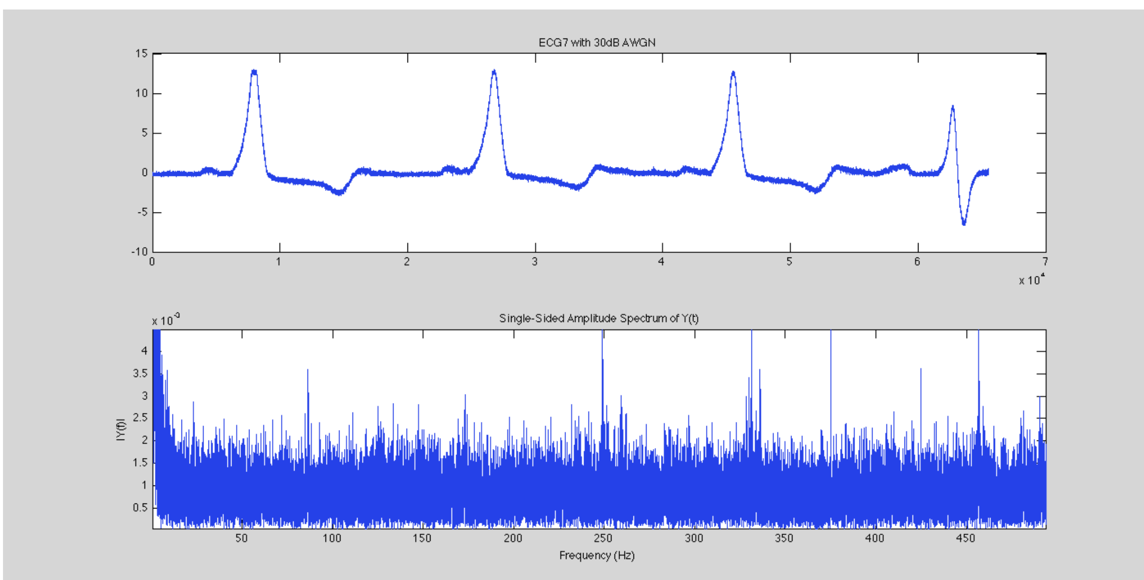

Figure 35. ECG signal (no. 7) with 30dB AWGN added and the freq. spectrum

The AWGN test was also performed on ABP signals. In figures 36 and 37, the results showed that the AWGN gave a great change to the ABP signal (no. 13). This time, the effect could easily be seen even in the time-domain. For example, the spikes in figure 36 at

about 340 Hz and 450 Hz were gone in figure 37; the two spikes were destroyed by the AWGN. The experimental results showed that the ABP signals had a lower tolerance to the AWGN than ECG signals.

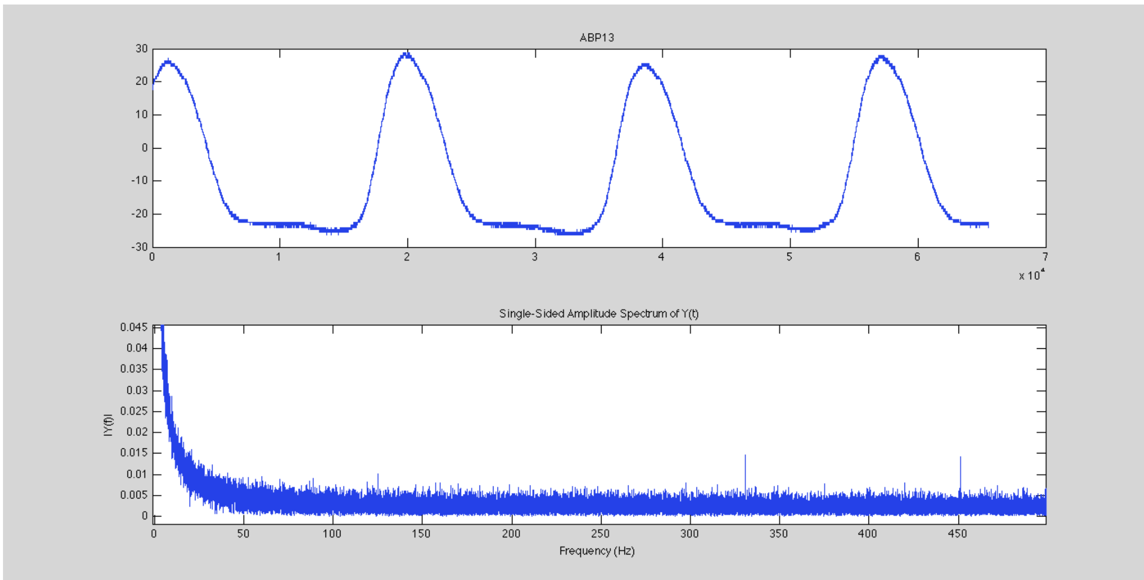

Figure 36. ABP signal (no. 13) with its frequency spectrum

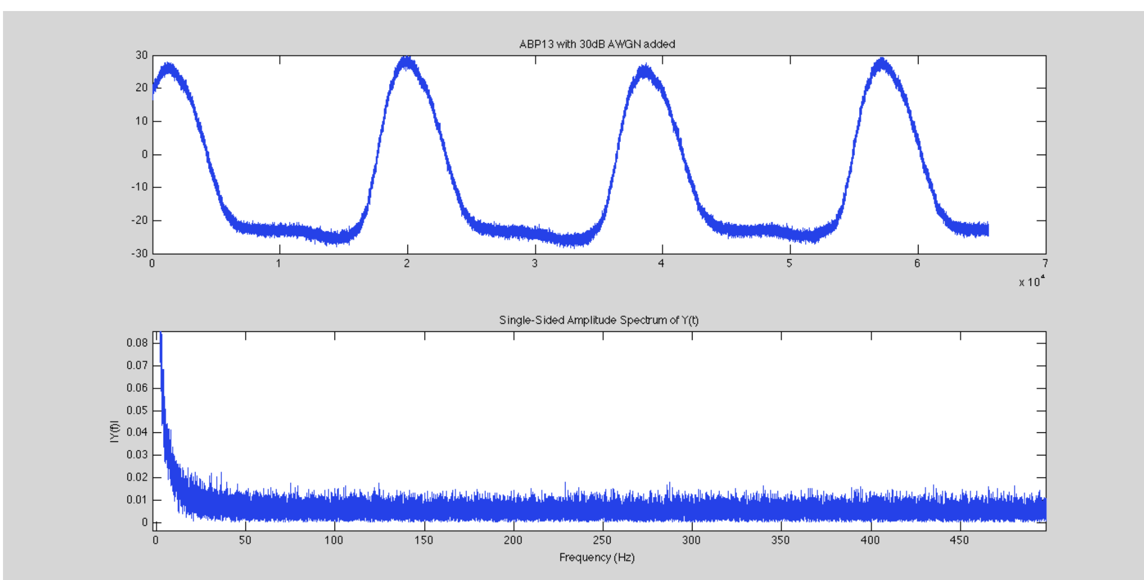

Figure 37. ABP signal (no. 13) with 30dB AWGN added and the freq. spectrum

Since the effect of the AWGN was small in ECG signals, the de-noising performance of both Fourier Analysis and Wavelet Analysis could be expectedly unchanged. To prove this, experiments were done and shown in figures 38 and 39. As seen, high frequency information could be kept after the noise filtering by using wavelet analysis; however, all the high frequency information was removed beyond the cut-off frequency when using the Butterworth filter.

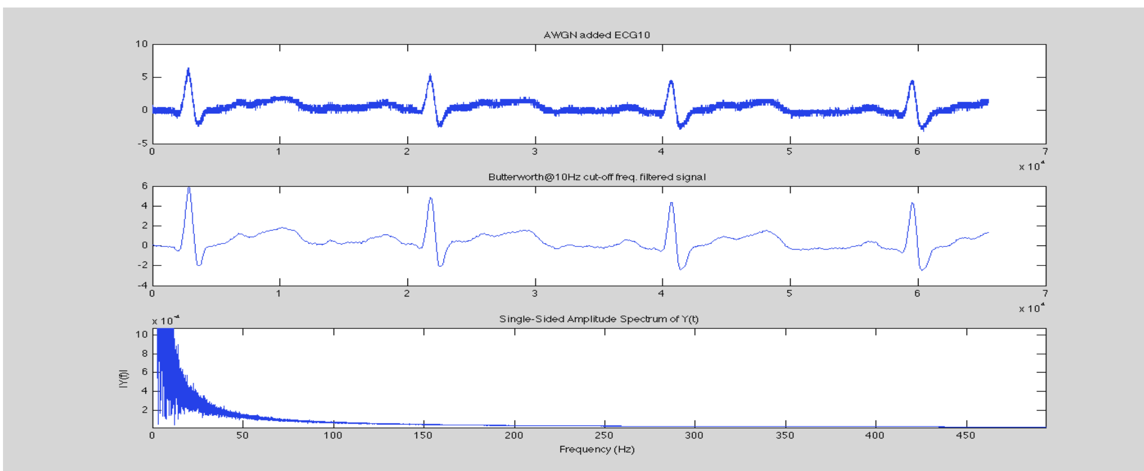

Figure 38. Butterworth @ 10 Hz cut-off freq. filtered AWGN added ECG signal (no. 10) and freq. spec.

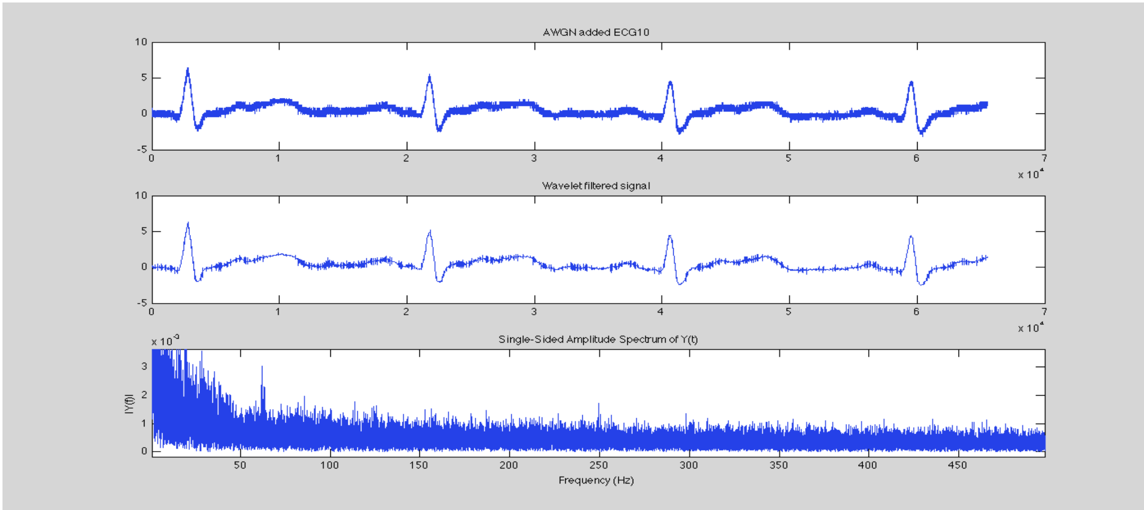

Figure 39. Wavelet with thresholding filtered AWGN added ECG signal (no. 10) and the freq. spectrum

The same test had also been done on ABP signals. As seen in figures 40 and 41, high frequency information could be kept after the noise filtering by using wavelet analysis but removed beyond the cut-off frequency when using the Butterworth filter. The experiments on both ECG and ABP signals showed that the AWGN did not have any effect on the de-noising performances on Fourier Analysis and Wavelet Analysis.

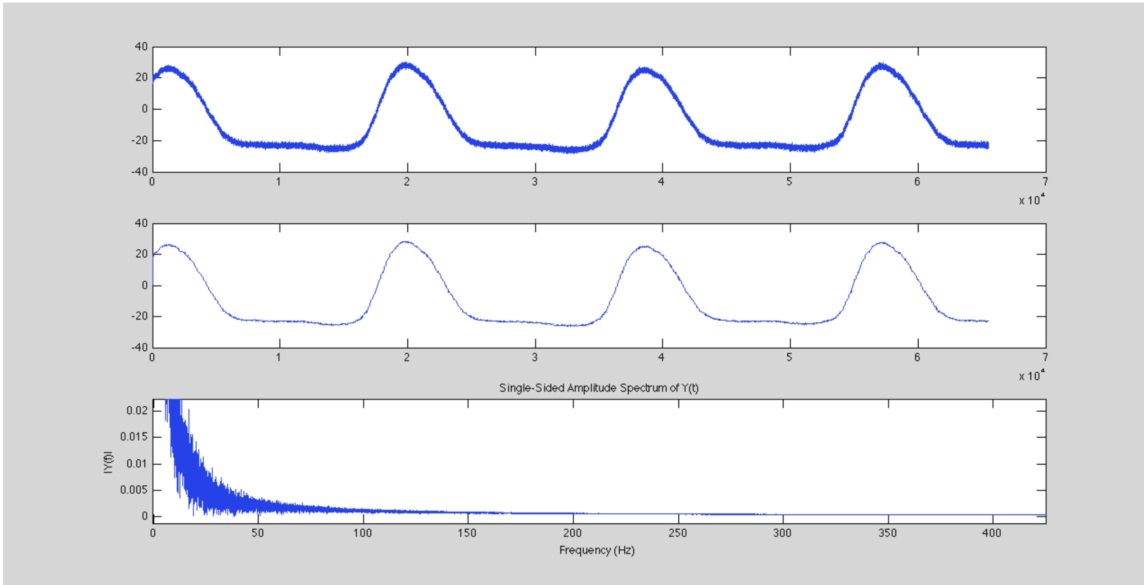

Figure 40. Butterworth @ 40 Hz cut-off freq. filtered AWGN added ABP signal (no. 13) and frequency spectrum

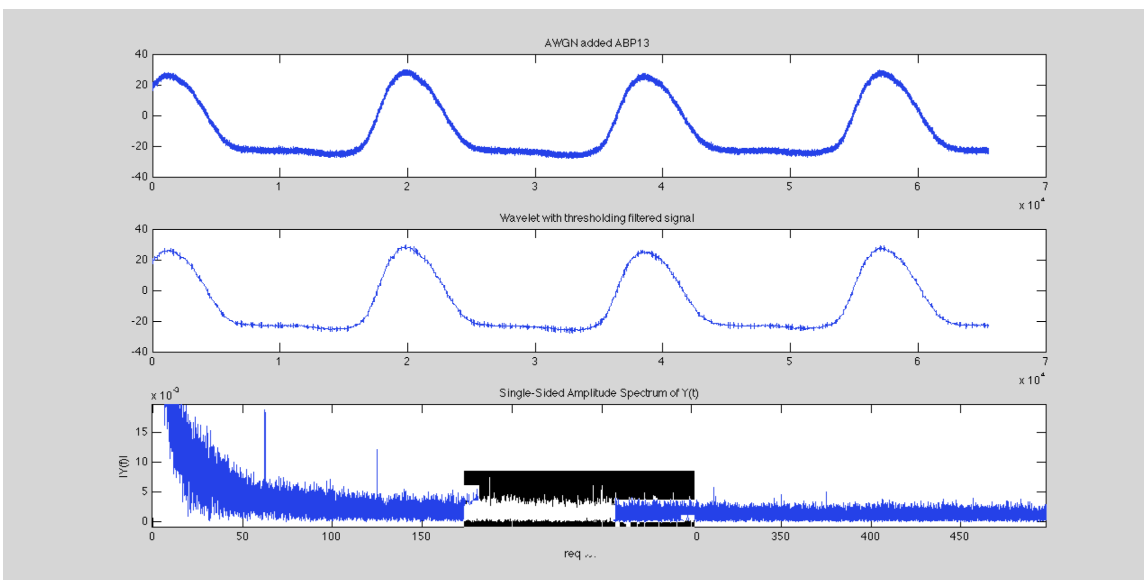

Figure 41. Wavelet with thresholding filtered AWGN added ABP signal (no. 13) and the frequency spectrum

4.3 Level of Decomposition

As stated, a signal could be de-composited into numerous approximations and details. In some applications and cases, only the approximation would be sufficient to represent the de-noised signal. That was because the approximation only showed the low frequency portions of a signal; all the high frequency contents including noises were removed. Although a higher level of approximation could give a cleaner signal, over decomposition could bring a serious drawback – distortion. To avoid this, the physical meaning of the approximations and details was studied and a method for determining the optimal level of decomposition was also created.

In DWT, the length of the approximation and the detail was divided into half of the previous level of approximation. Figure 42 showed an example of decomposition on an ECG signal (no. 14); for instance, the length of cA1 was only half of the ECG14. Not only was the length of the signal divided into half of the previous level, the frequency range was also divided into half of the previous level. If a signal had an original frequency range from 0 Hz to 500 Hz, the frequency ranges of each level of approximation and detail were as follows:

Level of decomposition	Frequency range of approximation	Frequency range of detail
1^{st}	0 Hz – 250 Hz	250 Hz – 500 Hz
2^{nd}	0 Hz – 125 Hz	125 Hz – 250 Hz
3^{rd}	0 Hz – 62.5 Hz	62.5 Hz – 125 Hz
4^{th}	0 Hz – 31.25 Hz	31.25 Hz – 62.5 Hz

5th	0 Hz – 15.625 Hz	15.625 Hz – 31.25 Hz
6th	0 Hz – 7.8125 Hz	7.8125 Hz – 15.625 Hz
7th	0 Hz – 3.90625 Hz	3.90625 Hz – 7.8125 Hz

Table 2. Level of decomposition

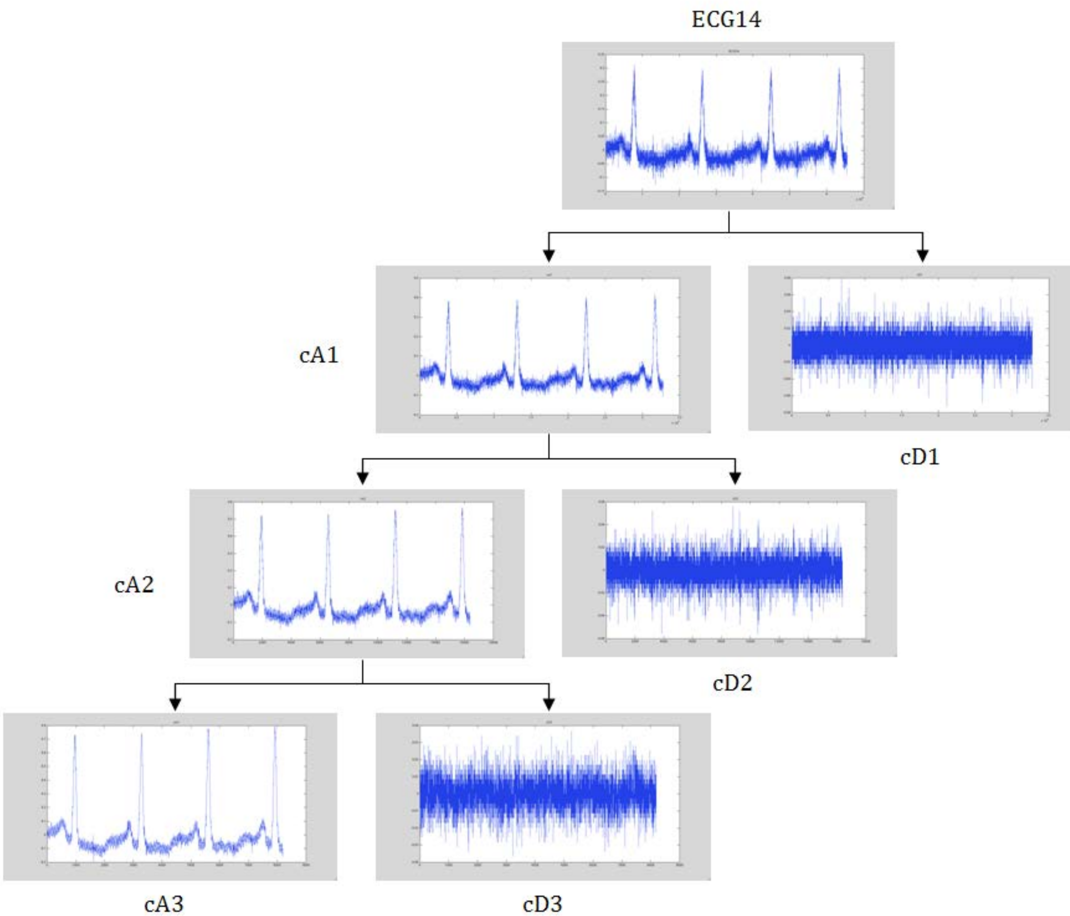

Figure 42. Wavelet decomposition tree

Figures 43, 44, 45, 46, 47 and 48were different physiological signals with their frequency spectrum. Most signals' low-frequency content were located from 0 Hz – 10 Hz. For example, the low-frequency content of ABP (no. 3) in figure 47 had a relatively large

frequency range; it ranges from 0 Hz – 50 Hz. In Fourier Analysis, 50 Hz was considered the cut-off frequency. According to the table 2, the 3rd level of approximation was the highest level that the ABP (no. 3) could be used without distortion; this was proved in figure 49. As shown, signal a4 in figure 49 was crooked with small square-shaped waves. The square-shaped wave was actually the mother wavelet; this phenomenon was due to signal distortion caused by over decomposition.

Another example was shown in figure 43, the low-frequency content of ECG (no. 7) had a range from 0 Hz to around 10 Hz. Therefore, it had roughly a 10 Hz cut-off frequency. According to the table 2, the 6th level of approximation was the highest level that the ECG (no. 7) could be used without distortion; this was proved in figure 50. The distortion of signal a7 was not obvious, because most of the low frequency contents were located from 0 Hz to 4 Hz. Once the level of approximation was moved from 7th to 8th, the distortion could be clearly seen at the signal a8 in figure 50.

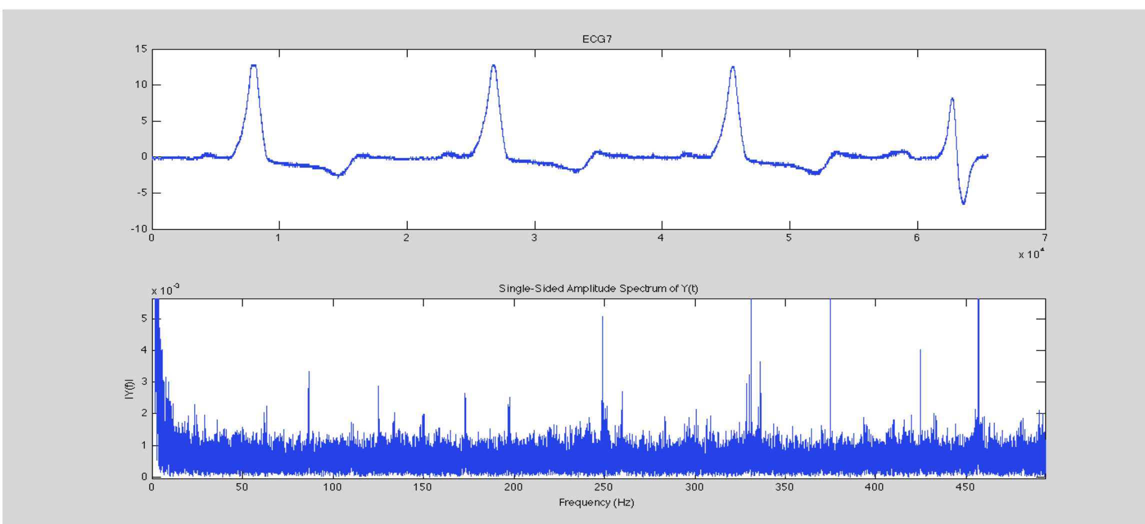

Figure 43. ECG signal (no. 7) with its frequency spectrum

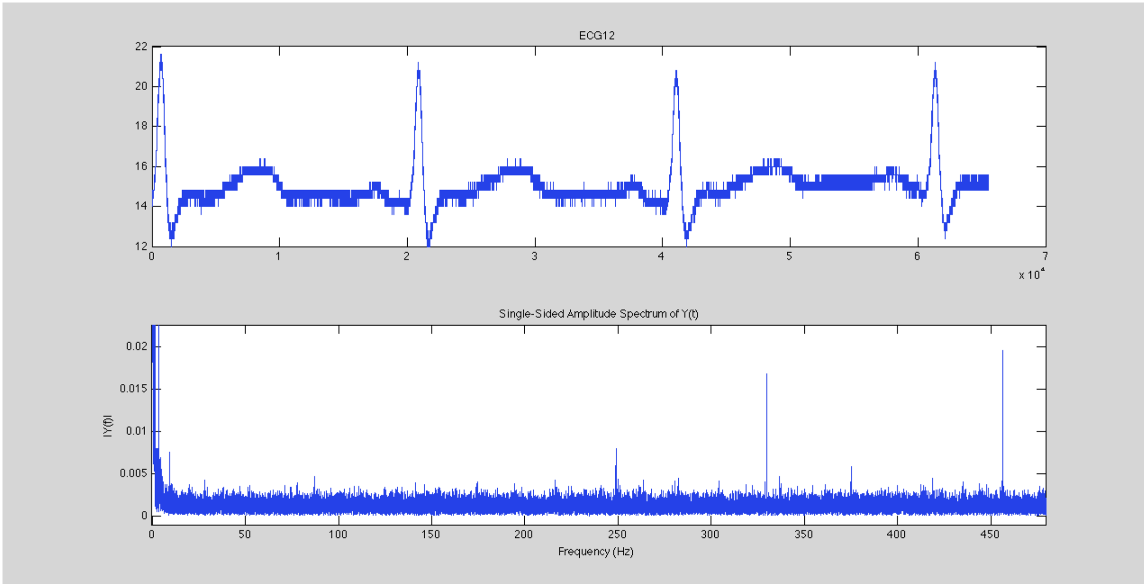

Figure 44. ECG signal (no. 12) with its frequency spectrum

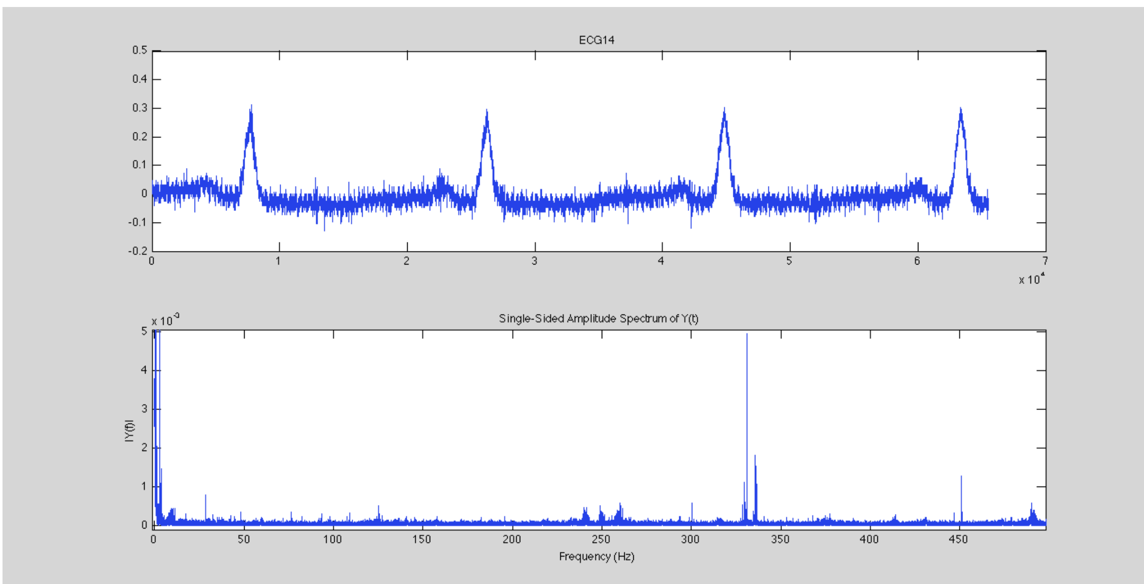

Figure 45. ECG signal (no. 14) with its frequency spectrum

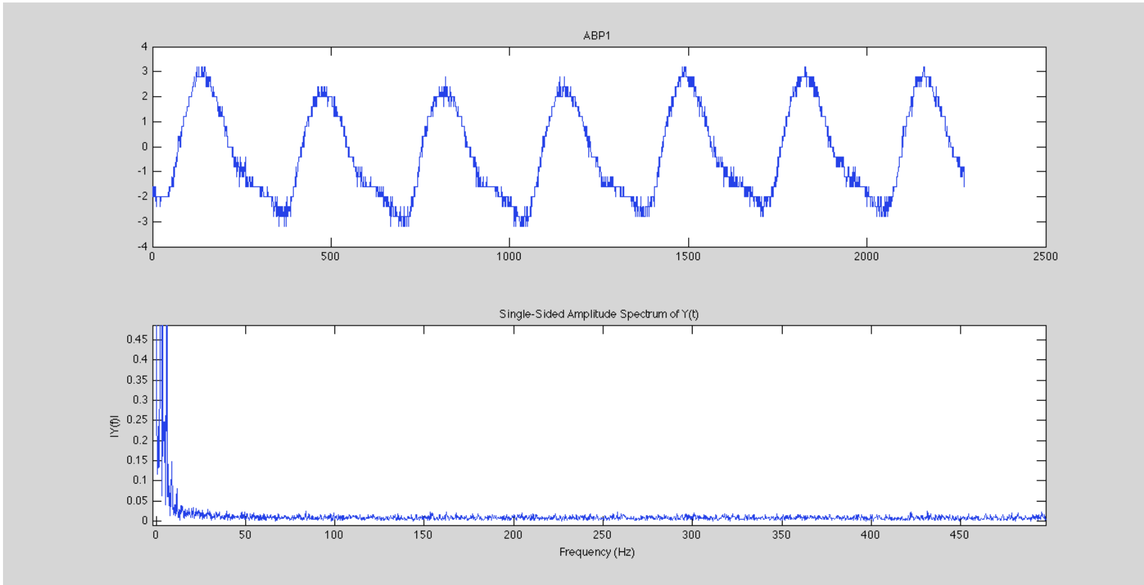

Figure 46. ABP signal (no. 1) with its frequency spectrum

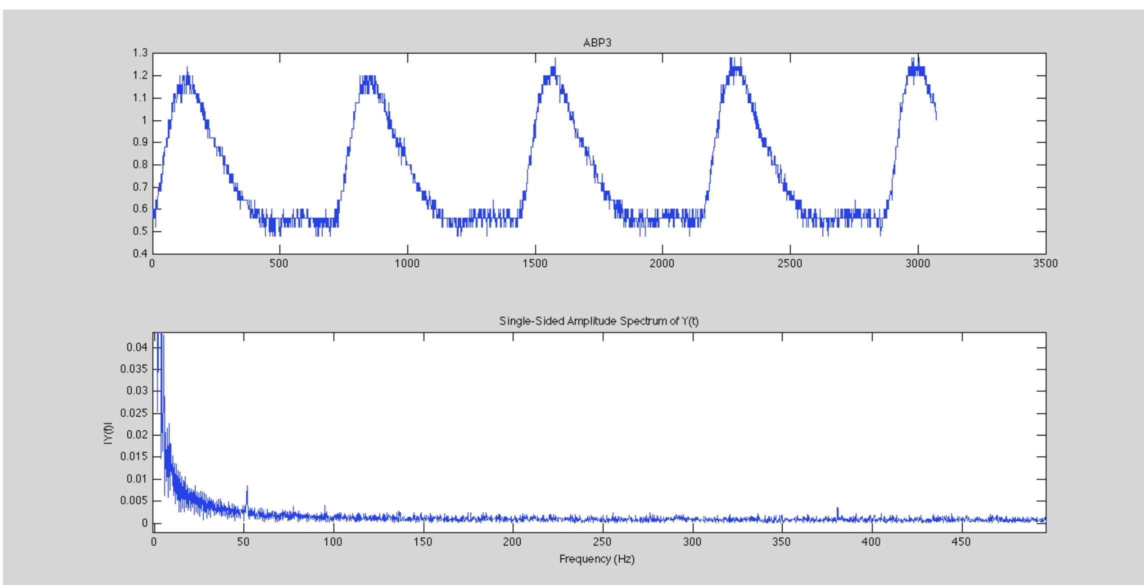

Figure 47. ABP signal (no. 3) with its frequency spectrum

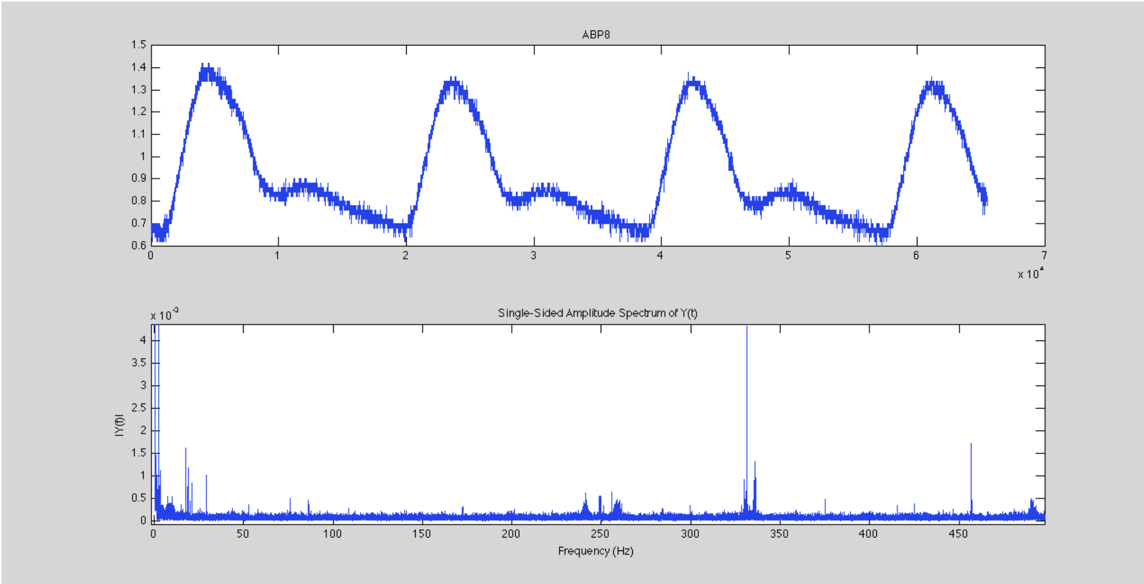

Figure 48. ABP signal (no. 8) with its frequency spectrum

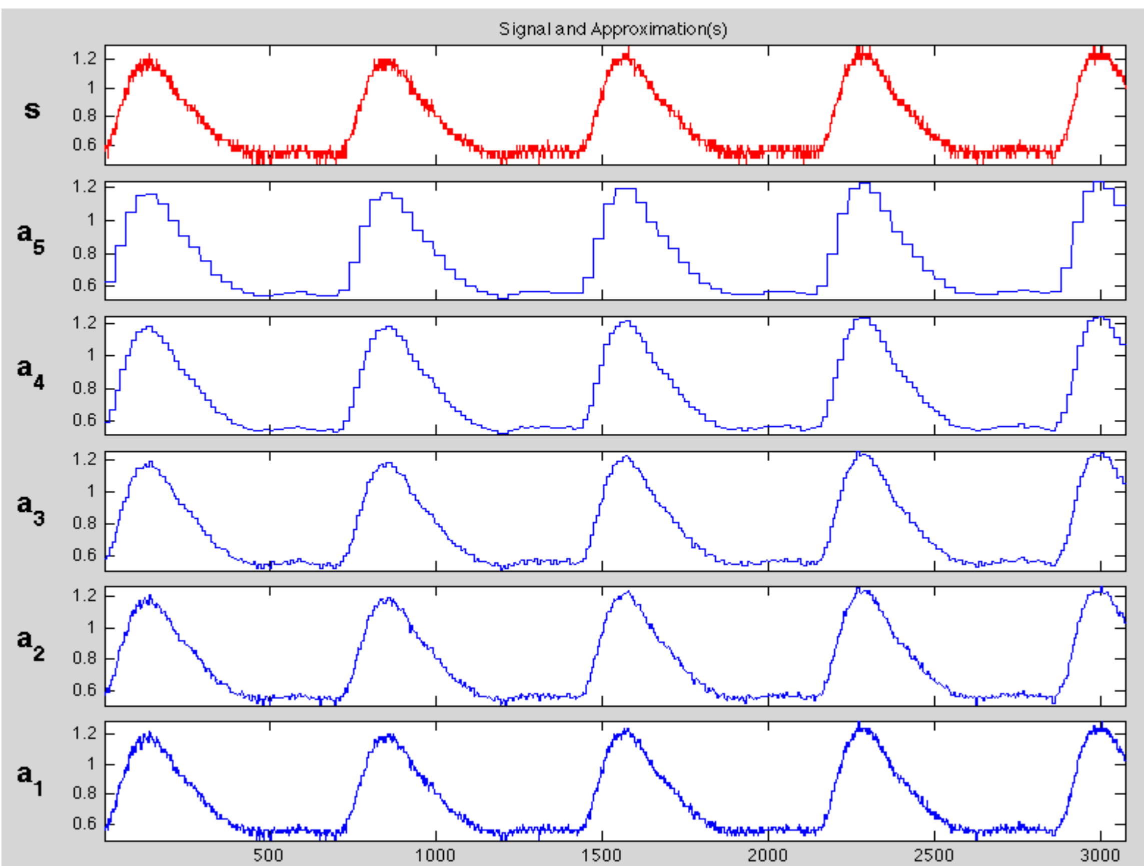

Figure 49. Original signal and approximations of ABP signal (no. 3)

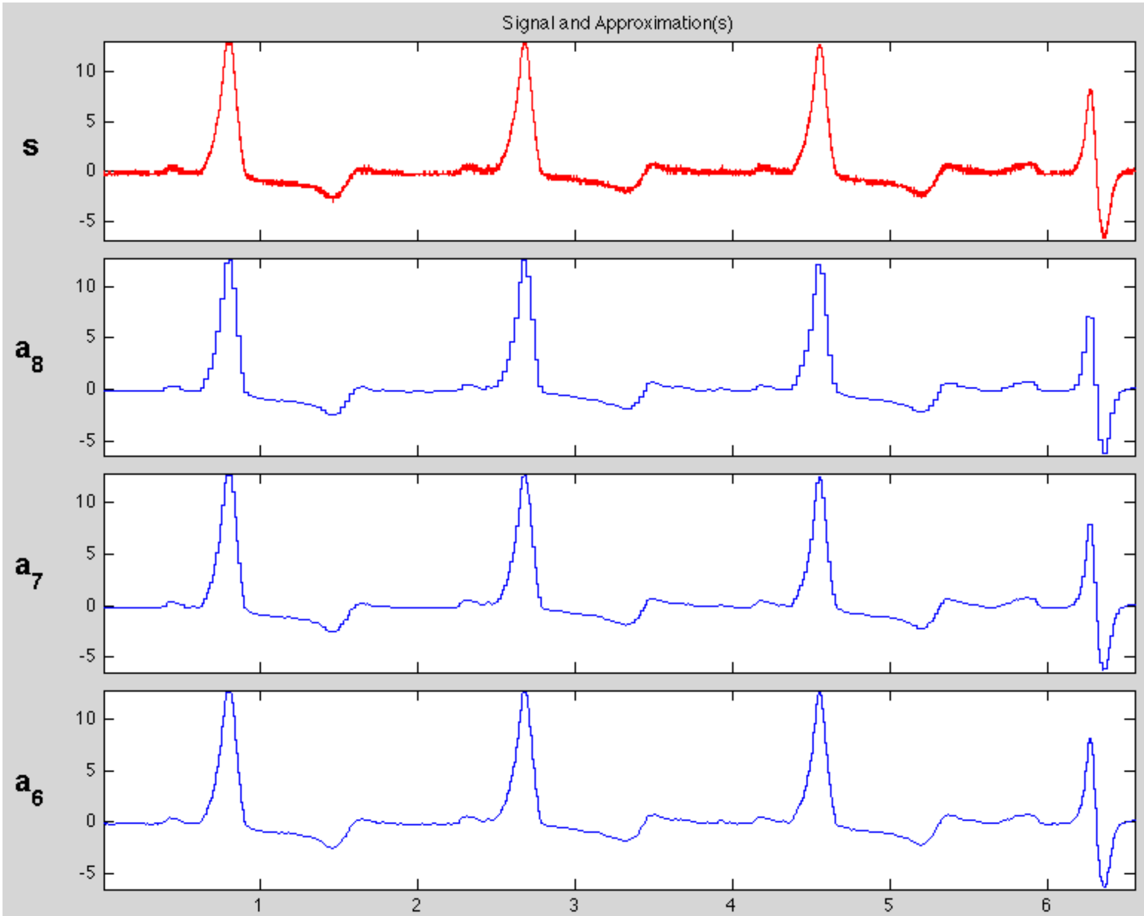

Figure 50. Original signal and approximations of ECG signal (no. 7)

Based on the experimental results, the suitable level of decomposition could be determined by examining the frequency spectrum of the signal. Using the algorithm shown in table 2, the frequency range of each level of approximations and details could be calculated. To avoid distortion, the frequency range of the selected approximation should always be smaller than the cut-off frequency of the signal. This method can help finding the optimal level of approximation and detail for maximizing the de-noising performance in Wavelet Analysis.

4.4 Adding Back the Details

The details could be a mixture of noise and important information on heart conditions; therefore, it is wise to selectively preserve the valuable portions of a signal. The selectivity features of both the magnitude and frequency band were studied.

4.4.1 Adjustable Threshold Values

In Wavelet Analysis, the adjustable threshold value could allow users to remove certain unwanted magnitudes from different levels of detail. This feature could let users have full selectivity on preserving the interested high frequency contents during the de-noising process. In physiological signals, high frequency content, such as humps and spikes, could be important for diagnosing the heart conditions. Therefore, the thresholding on physiological signals was studied.

On the left-hand side of figure 51, arbitrary threshold values were set at each level of detail. The unwanted portions (within the blue-dotted lines) were to be removed. Users, such as physicians, could adjust the number of maximum values or sharpest features from a de-noised signal. To improve the efficiency of adjusting the threshold value, an algorithm was created to generate the threshold value for each level of detail. Matlab computing software was used to program the algorithm; the threshold values were calculated based on the number of percentage of the maximum values. Some of the generated threshold values were listed on Table 3.

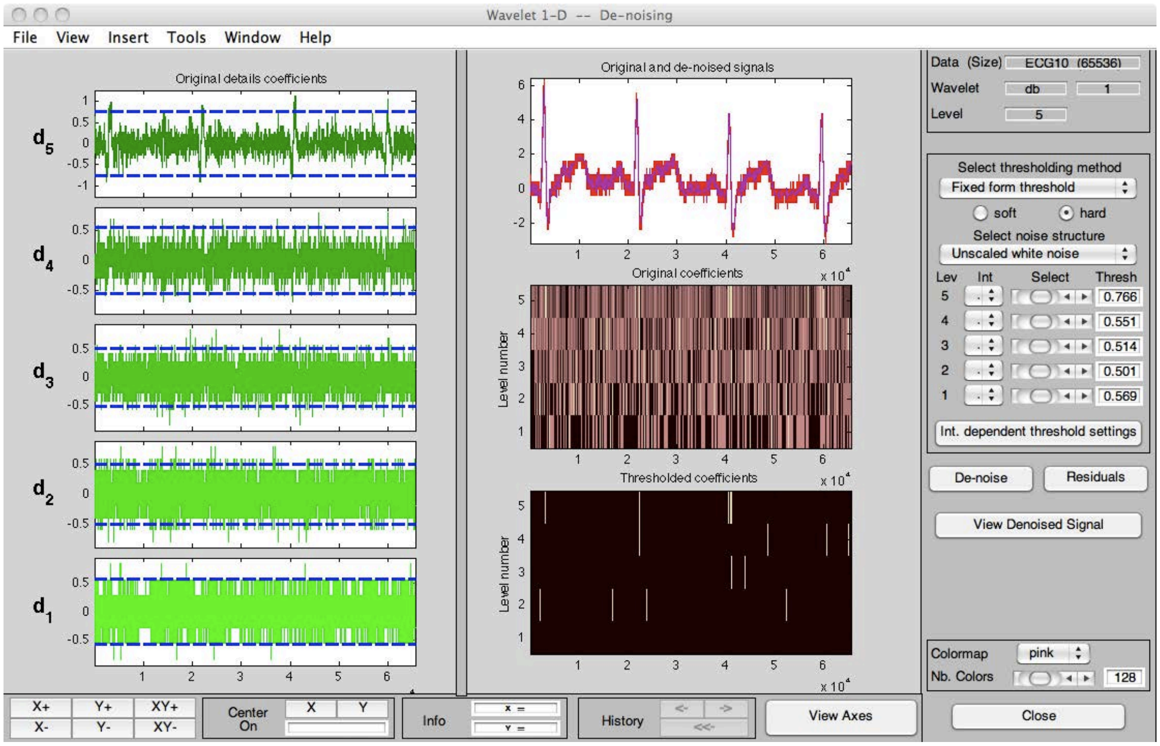

Figure 51. Threshold setting on ECG (no. 10)

Signal	Percentage	Lv. 5	Lv. 4	Lv. 3	Lv. 2	Lv. 1
ECG 10	5%	0.6745	0.4956	0.4781	0.4608	0.4579
ECG 10	4%	0.7149	0.5195	0.4915	0.4760	0.5016
ECG 10	3%	0.7662	0.5512	0.5139	0.5012	0.5691
ECG 14	5%	0.0322	0.0227	0.0218	0.0260	0.0279
ECG 14	4%	0.0340	0.0235	0.0227	0.0275	0.0292
ECG 14	3%	0.0362	0.0246	0.0237	0.0292	0.0314
ABP 7	5%	0.0346	0.0270	0.0237	0.0264	0.0306
ABP 7	4%	0.0361	0.0281	0.0243	0.0280	0.0312
ABP 7	3%	0.0376	0.0291	0.0254	0.0306	0.0322

ABP 9	5%	0.2562	0.2251	0.2085	0.2147	0.1972
ABP 9	4%	0.2669	0.2314	0.2244	0.2183	0.2111
ABP 9	3%	0.2799	0.2419	0.2285	0.2244	0.2344

Table 3. Generated threshold values

Figures52, 53, 54, and 55were the original physiological signals with their de-noised signals; the threshold values were set at top 5%, 4%, and 3% of each level of detail. These examples showed that the threshold values could be easily adjusted by changing the percentage.

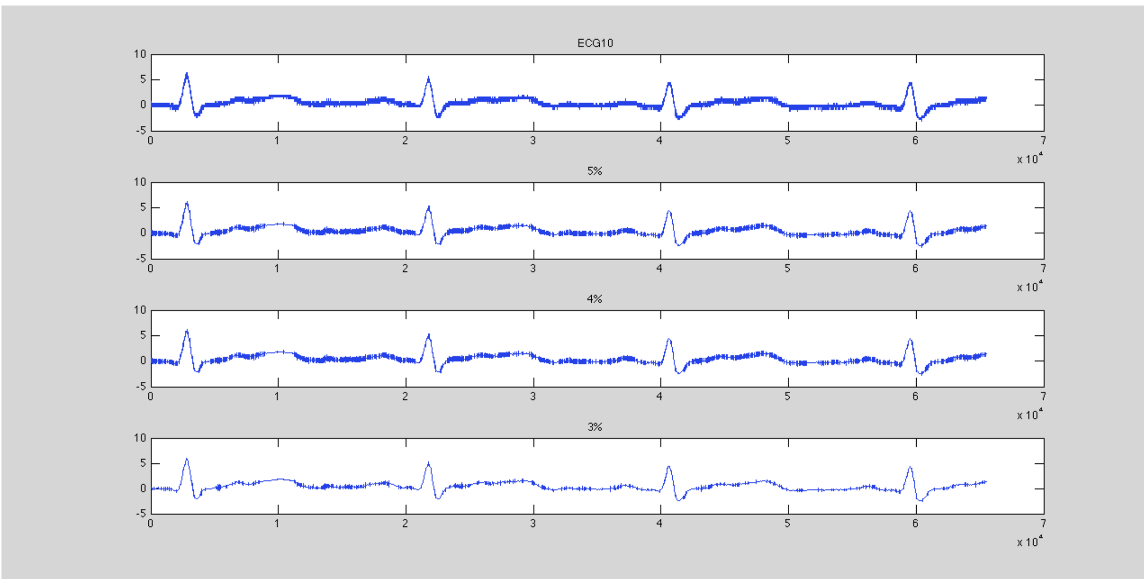

Figure 52. ECG (no. 10) signal and de-noised ECG signals with threshold at top 5%, 4%, and 3% of the details

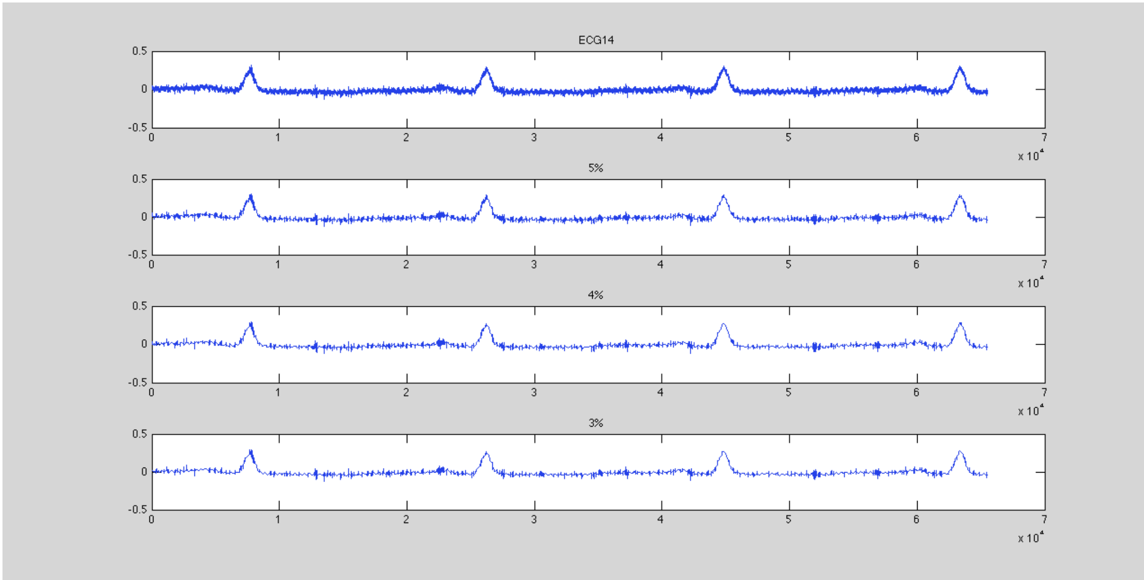

Figure 53. ECG (no. 14) signal and de-noised ECG signals with threshold at top 5%, 4%, and 3% of the details

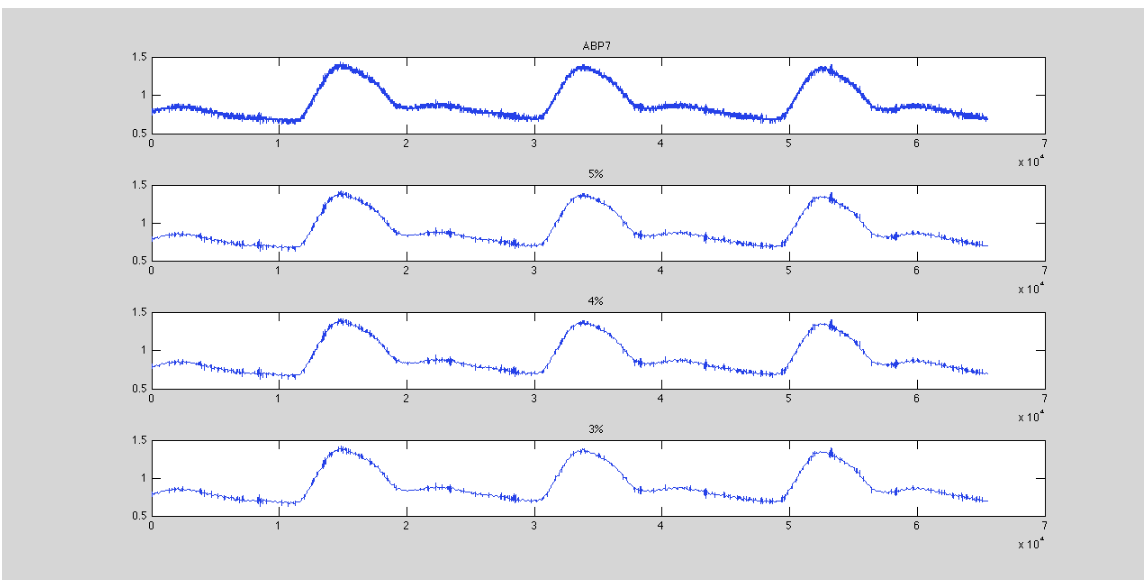

Figure 54. ABP (no. 7) signal and de-noised ABP signals with threshold at top 5%, 4%, and 3% of the details

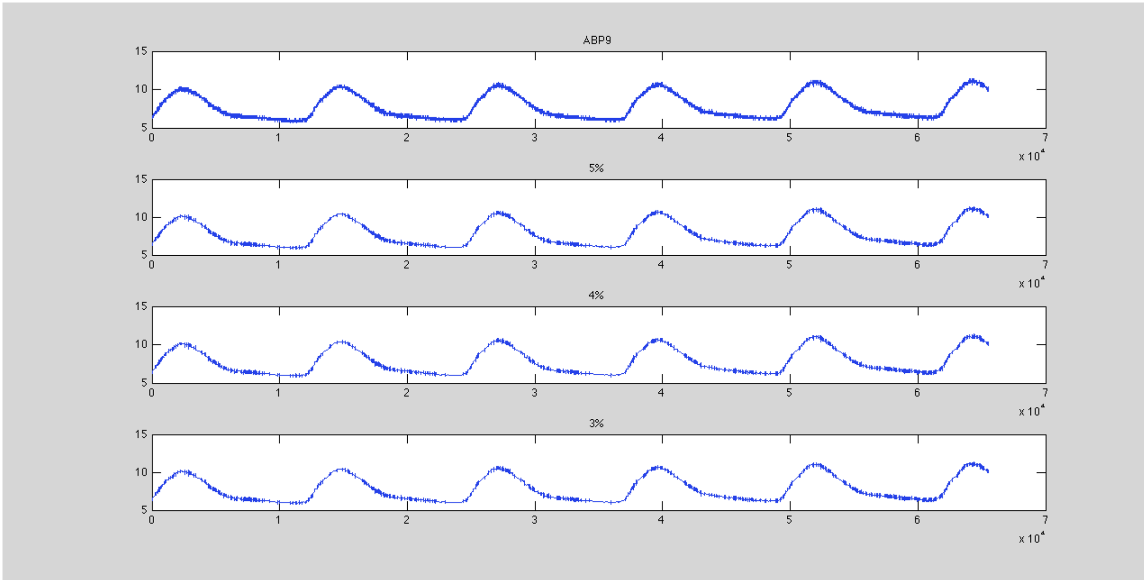

Figure 55. ABP (no. 9) signal and de-noised ABP signals with threshold at top 5%, 4%, and 3% of the details

Figures 56, 57, 58, and 59 showed a closer look of some examples. The signals on the lower half in these figures were generated by combining the 5th level of approximation and the threshold applied details.

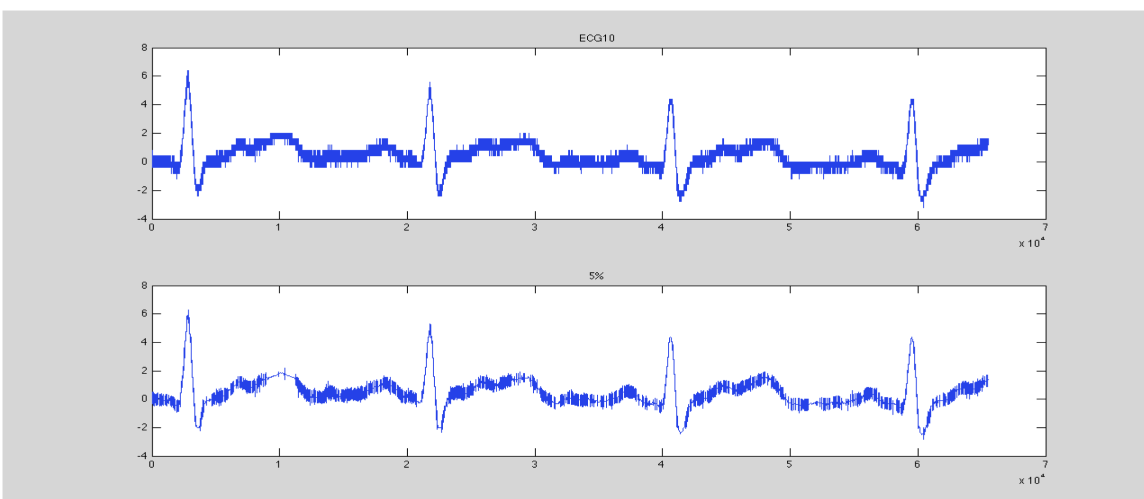

Figure 56. ECG signal (no. 10) with de-noised signal (5th approx. plus top 5% of details)

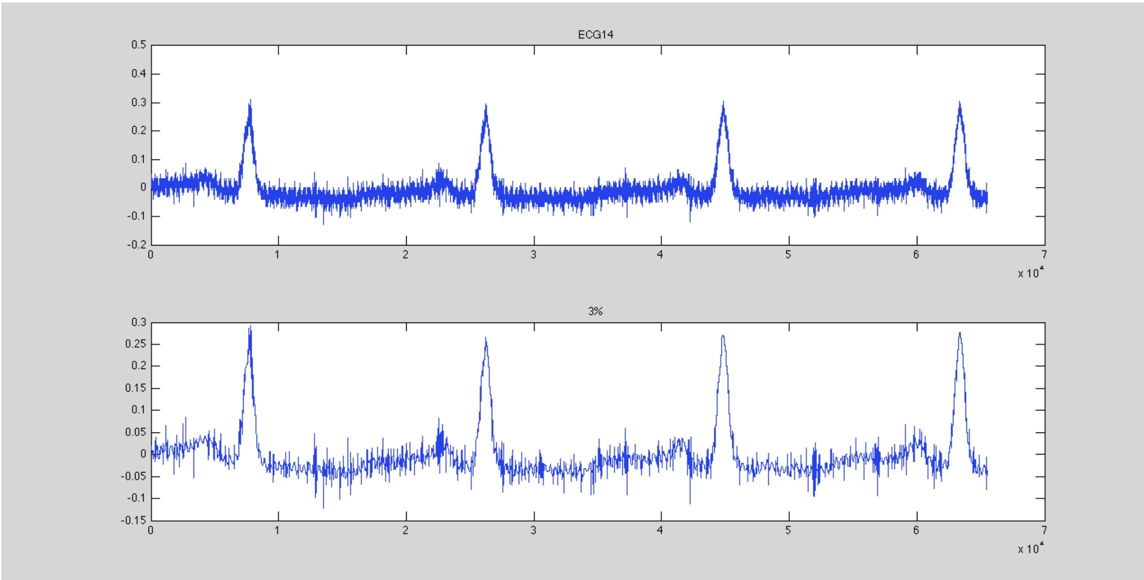

Figure 57. ECG signal (no. 14) with de-noised signal (5^{th} approx. plus top 3% of details)

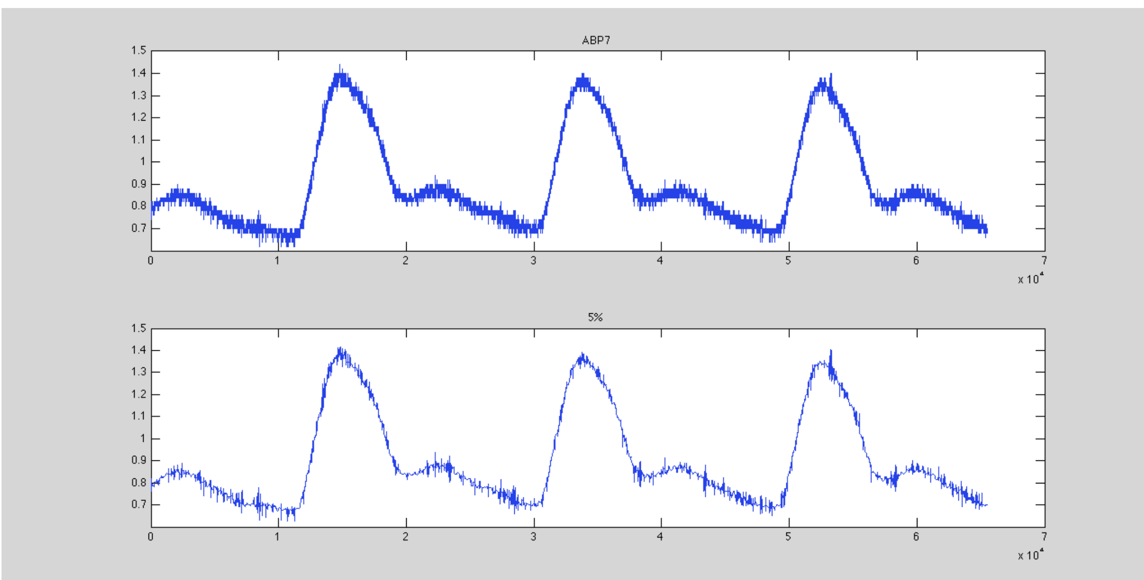

Figure 58. ABP signal (no. 7) with de-noised signal (5^{th} approx. plus top 5% of details)

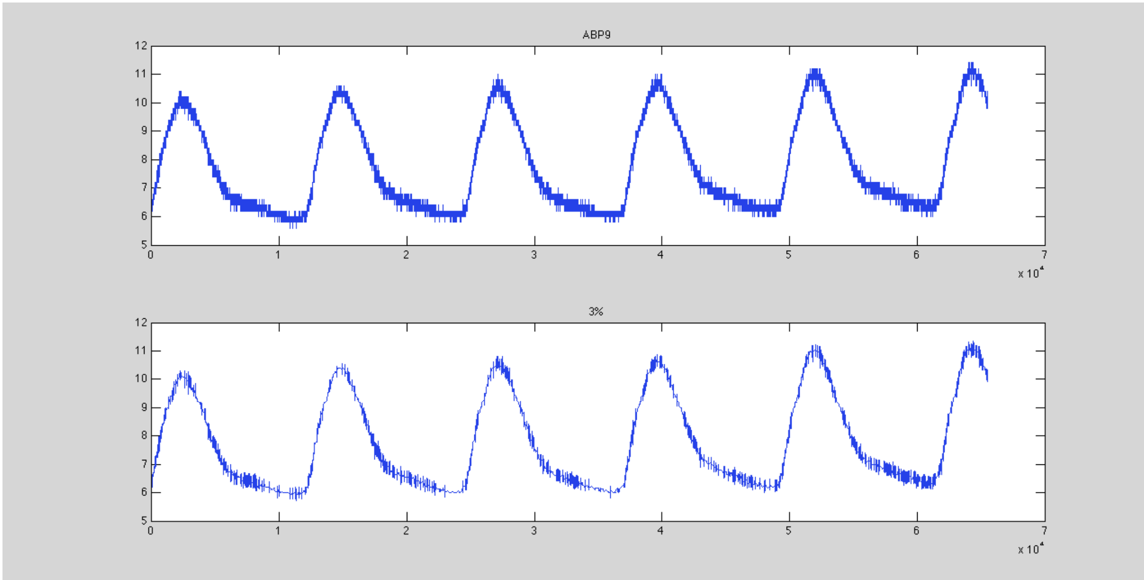

Figure 59. ABP signal (no. 9) with de-noised signal (5^{th} approx. plus top 5% of details)

The experimental results showed that the thresholding feature of wavelet analysis allowed adjustable magnitudes on each frequency band. The uniform portion (inside the blue-dotted line in figure 51) of the magnitude could be treated as noise or useless information since this portion of detail would only blur and give no sharp features to the fundamental signal. With the created algorithm, users were able to preserve the sharp features and remove unwanted magnitude at different frequency bands systemically and efficiently.

4.4.2 Different Frequency Bands

The threshold value could be used to control the number of maximum values; it also had control of the frequency band. In wavelet analysis, the signal was divided into different frequency bands; each level of detail represented an individual frequency range. Removing the frequency band or different frequency bands at once allowed physicians to study the effect and location of each frequency band on the original signal.

For example, the middle signal in figure 60 showed only two frequency bands: the 5th order approximation (0 Hz – 15.625 Hz) and 1st order detail (250 Hz – 500 Hz) with 5% threshold value. The 5% threshold value (0.4579) came from the table 3. The lower signal in figure 60 also showed only two frequency bands: the 5th order approximation (0 Hz – 15.625 Hz) and 2nd order detail (125 Hz – 250 Hz) with 5% threshold value. The 5% threshold value (0.4608) came from the table 3. By comparing the two de-noised signals in figure 60, the locations of the frequency bands could be clearly seen. As shown, the high frequency detail (250 Hz – 500 Hz) was located relatively evenly on the original signal; however, the high frequency detail (125 Hz – 250 Hz) was located mainly between the first and the second period.

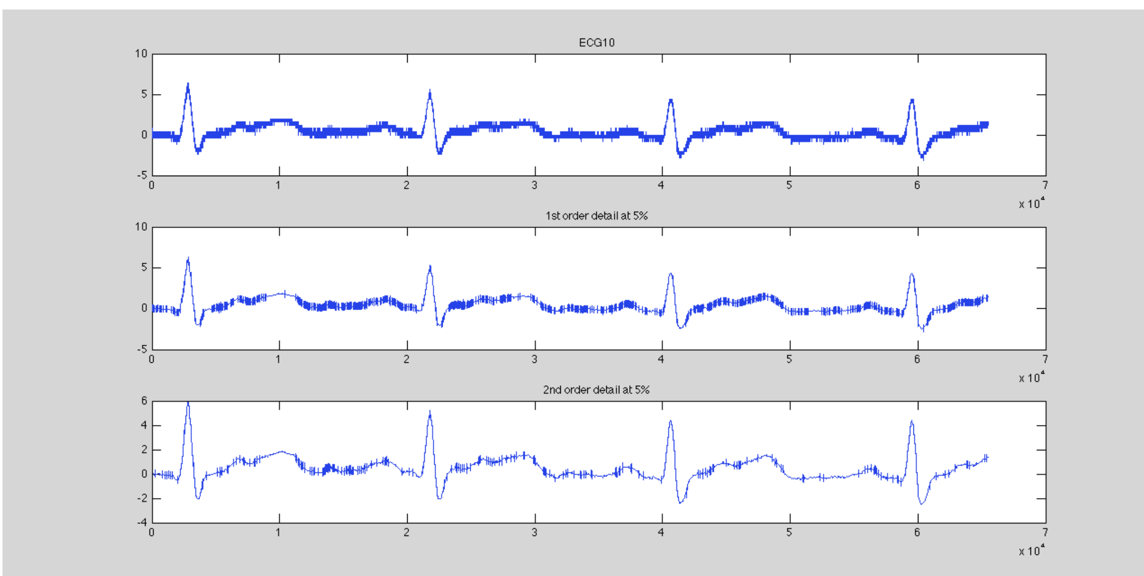

Figure 60. ECG signal (no. 10), 1st order detail at 5%, 2nd order detail at 5%

Similar to the previous example, the middle signal in figure 61 showed only two frequency bands: 5^{th} order approximation (0 Hz – 15.625 Hz) and 1^{st} order detail (250 Hz – 500 Hz) with 5% threshold value. The 5% threshold value (0.0306) came from the table 3. The lower signal in figure 61 showed only two frequency bands: 5^{th} order approximation (0 Hz – 15.625 Hz) and 2^{nd} order detail (125 Hz – 250 Hz) with 5% threshold value. The 5% threshold value (0.0264) came from the table 3. By comparing the two de-noised signals, the 1^{st} order detail and the 2^{nd} order detail both distributed evenly on the original signal.

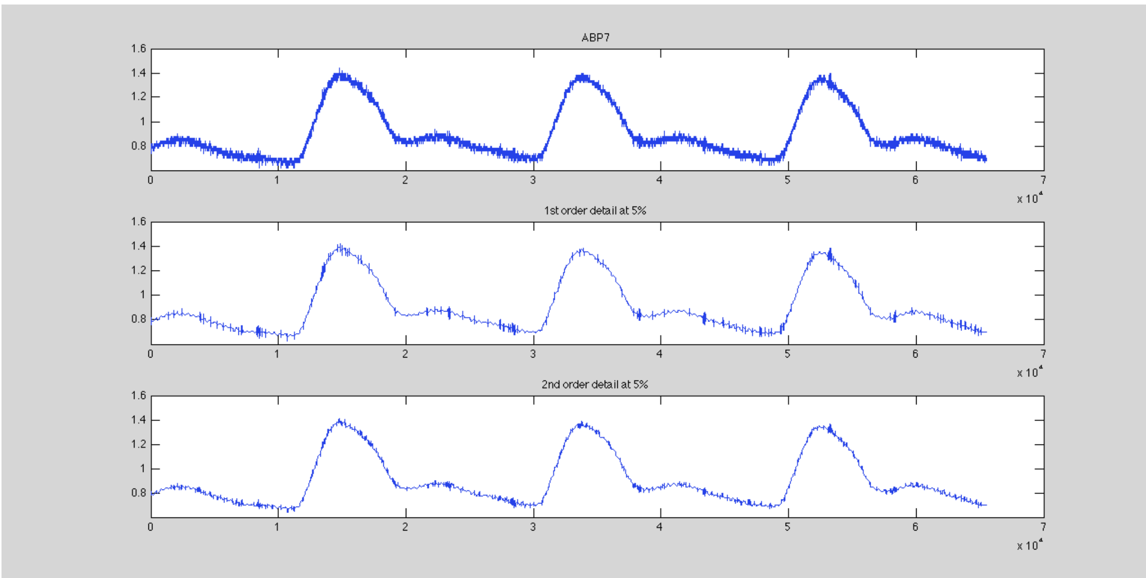

Figure 61. ABP signal (no. 7), 1^{st} order detail at 5%, 2^{nd} order detail at 5%

The experimental results showed that the wavelet analysis had the selectivity of different frequency bands. This feature could be used along with the thresholding to maximize the de-noising performances in physiological signals.

Chapter 5

CONCLUSION

The goal of this thesis is to study the de-noising performance of Wavelet Analysis on physiological signals. The literature study shows that Wavelet Analysis has the ability to overcome some drawbacks of Fourier Analysis and experimental results prove that Wavelet Analysis performs excellent noise filtering on the ECG & ABP signals.

Similar to Fourier Analysis, Wavelet Analysis is shown to be able easily to remove all high frequency contents from the physiological signals. The approximations can provide the same quality of noiseless signal as Butterworth filtered signals while taking only a fraction of the size. This saves tremendous storage space in places like hospitals. For example, if all the signals are de-noised and saved in a 2nd order of approximation, the storage size will be cut in half without further signal compression.

If signals have the same time-invariant characteristics, both analytical methods are excellent for filtering the noise. However, physiological signals are time-varying signals, so the characteristics are not only changing from patient to patient but also from time to time. In these cases, some important high frequency information is changing over time. Thus, designing a filter based on Fourier Analysis to find the suitable passband will require tremendous effort. Wavelet Analysis treats signals in a totally different way. It breaks down a signal into a number of small frequency bands. This can preserve the

timing transitory information of the signal. Therefore, the Wavelet Analysis is more suitable for de-noising the physiological signals, such as ECG and ABP signals.

Over filtering is one of the common problems in Fourier Analysis. Some signal characteristics, such as humps and peaks, are always suppressed or even removed by the filter. Similar problem can also happen in Wavelet Analysis if too high a level is selected for decomposition. In the thesis, an algorithm was created for finding the optimal level of decomposition; it can help prevent over decomposition and signal distortion. The optimal level of approximation can provide the best basic shape of a signal that requires the smallest storage space.

Wavelet Analysis is also shown to have the capability of selecting the magnitude and the frequency bands in filtering a signal. The unwanted magnitude can be easily removed from a special frequency band. The algorithm of magnitude selection provides a systematical way on selecting the threshold values. Certain high frequency bands can be deleted from the original signal as needed. Since the approximation shows no high frequency information but the basic shape of the signal in time domain, it can be treated as a track of time. When the detail is added back to the approximation, the location of that frequency band is revealed in time domain. If the frequency of each frequency band is known, users are able to study the effect of certain frequency band to original signal. The add-back feature of the Wavelet Analysis makes the frequency information more readable to inexperienced physicians, and also gives an innovative way to examine the physiological signals.

To conclude the thesis, the results of the experiment show that there are some advantages in using Wavelet Analysis over Fourier Analysis on preserving useful info while reducing noise on physiological signals. Wavelet decomposition can let users, such as physicians, to analyze a signal in different detail levels for different frequency bands. Unlike Fourier Analysis, these frequency bands will be shown in the time-frequency domain. The time information could greatly assist the analyzing of the physiological signals. Adding back details capability allows users to have the flexibility to add back certain amount of spikes and peaks at certain frequency ranges. The approximations portion plus the reduced details from different levels gives flexibility to the noise reduction and filtering effect. The same process also can provide signal compression effect for data storage. In the future, the experimental results in this thesis could help manufacturers to build a better monitoring device for physicians.

Chapter 6

FUTURE TOPICS

Wavelet Analysis is a new and powerful analytical method that can de-noise physiological signals with frequency and magnitude selectivity. It compensates for the losses of the time information when using the traditional de-noising method, filters based on Fourier Analysis. Different small frequency bands can be selected and added to the low frequency portion, the basic shape of the signal. Thus, the location of the frequency information can be seen and studied easily. This could greatly help scientists and physicians understand when heart conditions are changing or happening. Also, because some medical conditions can only appear at certain frequency bands, storing certain details of medical interest may be efficient and sufficient for the future comparison and diagnosis. This thesis will also provide a very good foundation for future research study on the following topics:

1. The physical meaning of the information at different frequency bands.
2. The relationship between the info at different frequency bands and the heart related diseases or cardiovascular system conditions.
3. The forms and patterns of the high frequency information on the de-noised physiological signals can also be a good research topic in the future.

REQUIRED HARDWARE AND SOFTWARE

Hardware: 2.16 GHz Intel Core 2 Duo, 2.5 GB 667 MHz DDR2 SDRAM

Software: Mac OS X Version 10.6.5, MATLAB (R2009b) 64-bit

APPENDIX

Matlab code for Frequency Spectrum:

```
Fs = 1000;                    % Sampling frequency
T = 1/Fs;                     % Sample time
L = length(ABP8)              % Length of signal
t = (0:L-1)*T;                % Time vector

x = ABP8;
y = x %+ awgn(x,30,'measured');     % Add noise
figure;
plot(Fs*t(1:length(ABP8)),y)

NFFT = 2^nextpow2(L);
Y = fft(y,NFFT)/L;
f = Fs/2*linspace(0,1,NFFT/2+1);

% Plot single-sided amplitude spectrum.
% subplot(3,1,1)
% plot(ECG12);title('ECG12');
% subplot(3,1,2)
% plot(x)
% subplot(3,1,3)

subplot(2,1,1)
plot(x);title('ABP8');
subplot(2,1,2)
plot(f,2*abs(Y(1:NFFT/2+1)))
title('Single-Sided Amplitude Spectrum of Y(t)')
xlabel('Frequency (Hz)')
ylabel('|Y(f)|')
```

Matlab code for generating threshold values:

```
[C,L]=wavedec(ABP9,5,'db1');    % Apply wavelet
cD5=detcoef(C,L,5);             % 5th level of detail
cD4=detcoef(C,L,4);             % 4th level of detail
cD3=detcoef(C,L,3);             % 3rd level of detail
cD2=detcoef(C,L,2);             % 2nd level of detail
cD1=detcoef(C,L,1);             % 1st level of detail

s=cD1;
len=length(s);                  % length of signal
Pe=round(len*0.03);             % Percentage of points

ab=abs(s);                      % Absolute values
so=sort(ab);
```

```
i=len-Pe; j=0;
whilei<len
su=so(i+1,:);
j=j+su;
i=i+1;
end

av=j/Pe;                                % Calculated threshold
value

m=1;
while so(m,:)<av
m=m+1;
end
```

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