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Effect of Lateral Stiffness on Bridge Deck Performance

Andrea del Pilar Toro
University of North Florida, n00929385@ospreys.unf.edu

EFFECT OF LATERAL STIFFNESS

ON BRIDGE DECK PERFORMANCE

by

Andrea P. Toro

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Certificate of Approval

The thesis of Andrea P. Toro is approved: (Date)

Dr. Adel ElSafty, Ph. D., PE (Committee Chairperson)

Dr. Thobias Sando, Ph.D. PE

Dr. James Fletcher, Ph.D,PE

Accepted for the School of Engineering:

Dr. Murat M. Tiryakioglu, Ph.D, CQE
Director of the School of Engineering

Accepted for the College of Computing, Engineering & Construction:

Dr. Mark A. Tumeo, Ph.D, JD, PE
Dean of the College of Computing, Engineering and Construction

Accepted for the University:

Dr. John Kantner, Ph.D.
Dean of Graduate School

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ABSTRACT

The use of the empirical deck design method has increased its acceptance due to the economic advantages that it presents when compared to its counterpart, the traditional method. This can be attributed to the fact that the empirical method provides an appropriate design where the deck withstands stress not only due to the steel reinforcement, but to an implicit arching membrane stress set-up as an effect of the lateral restraint surrounding the deck slab known as Compressive Membrane Action (CMA).

It has been proved through research that most design codes underestimate the strength of laterally restrained slabs. However, there is still a lack of acceptance in practical bridge design codes. This thesis presents an analysis addressing the influence that the lateral stiffness of the support beams has on the overall bridge deck performance. The lateral stiffness behavior was assessed through a programed electronic spreadsheet where a comparison with different current code requirements and an additional approach was made.

Through this analysis it was determined that not only does the support beam lateral stiffness play an important role in the overall bridge deck slab ultimate capacity, but mapping out this influence is a priority that may also be useful in setting the basis for a future design criteria.

1 INTRODCUTION

According to the Association of American State Highway and Transportation (AASHTO) and Load-Resistance Factor Design (LRFD) specifications, research has shown that the structural behavior under which a concrete deck resists wheel loads is not flexure, but a membrane stress state in which the deck behaves as a continuous membrane referred to as arching action or compressive membrane action (AASHTO, 2004, C.9.7.2.1). Compressive membrane action is the foundation for the empirical bridge deck design method. This method allows less flexural steel than that usually required by the traditional method by AASHTO Specifications (Standard Specifications for Highway Bridges, 1983).

Considering that the empirical deck design method is relatively new, numerous studies have been performed analyzing the behavior of existing bridges built according to this method and contrasting the results to finite element models of the empirical method versus the traditional method. The empirical method is believed to be highly beneficial since less steel reinforcement has an effect by decreasing the costs.

1.1 Problem Statement and Objectives

1.1.1. Problem Statement

This research is mainly focused on understanding and evaluating the effect of varying support beam lateral stiffness on empirical deck performance. The degree of lateral stiffness will be taken into account considering the studies performed by Queen's University of Belfast. The analysis will consider a contrast four types of beams of two different types of material (concrete and steel) and show the ultimate load capacity for the lateral stiffness that each supporting beam provides.

1.1.2. Objectives

The objectives of this research study are to determine how the influence of lateral stiffness affects the compressive membrane action and the empirical deck design. This was achieved through the TRC approach on computerized spreadsheets that predict the ultimate strength of laterally restrained slabs designed with the empirical method.

1.1.3. Tasks

The research plan consists of the following tasks:

- Investigate different values of slab thickness, bridge span length, and support beam spacing.
- Create an analytical spreadsheet tool for the American Concrete Institute ACI 318-05 method, the British Standard BS5400 method, the UK Highways Agency BD81/02 method, and the Taylor, Rankin, and Cleland's approach.
- Choose the appropriate reinforced concrete Florida I-Beam (FIB) girder and AASHTO girder.
- Identify two W-shape steel girders for the bridge conditions to be evaluated.

- Compare the effect of the girders’ stiffness on the deck slab ultimate strength when changing the thickness of the slab, the spacing between the structural elements, deck concrete compressive strength, beam span, reinforcement ratio, and the type of beams of either reinforced concrete or steel girders.
- Compare and plot the ultimate capacity estimated in each method.
- Compare and plot the results of the ultimate capacity in terms of the varying girders’ stiffness using Taylor, Rankin, and Cleland’s approach.
- Draw conclusions from analysis of results.

2 BACKGROUND ON BRIDGE DECK DESIGN

Among several types of bridge decks, cast-in-place reinforced concrete is the most commonly used for several reasons including cost, satisfactory resistance to displacement and the accessibility of materials needed to do the work. There are two methods considered for deck design; the traditional method and the empirical method.

2.1 Traditional Method

History shows that the most common reinforced concrete bridge deck has been designed using the Equivalent Strip method, typically known as the traditional method (American Association of State Highway and Transportation officials, 2002). This method basically can be applied to any situation since there is an analysis involved in order to meet the deck geometry (Standard Specifications for Highway Bridges, 1983, Section 1.3.2.). This method has shown to be rigorous and requires an extensive analysis of the bridge deck design (Meadway, 2008). It is believed that the traditionally designed slabs resist concentrating wheel loads on flexure in the traverse direction and that cracking in the concrete occurs in the positive moment region (Shoukry, William, McBride, Riad, & Wriston, 2010, p. 139). Additional reinforcement is placed when designing with this method in order to transfer the load to the principal longitudinal and transversal steel reinforcement, according to the serviceability, maximum deflection and to minimal shrinkage.

Generally, the traditional method designs the deck as a series of strips transverse to girders better known as the approximate strip design Method (Chen & Duan, 1999). This method

assumes that the bridge deck is a continuous beam across unyielding support girders. For analysis simplification, it is also assumed that the girders do not deflect; this way the maximum moments can be determined for design. The results of this traditional design method produces a design that is reliable and safe, but overly conservative (Muniz, 2013).

2.2 Empirical Method

The Ontario Ministry of Transportation introduced the empirical design method in 1979 for the design of reinforced concrete bridge decks (Bakht, 1993). The empirical design method is an approach that determines the amount of steel reinforcement that the deck is going to need, it is not a method typically used to analyze the bridge deck (Muniz, 2013). The primary attribute of this method is that it reduces the amount of reinforcing steel in the deck when compared to the traditional method (Shoukry et al., 2010).

Recently, federal and state agencies have been adopting the empirical deck design approach, as it has been proven through experimental tests to be a much simpler method that provides a satisfactory strength (Veen, 2008). These experimental tests were developed in order to have a better understanding on the internal load, and takes into account plane membrane forces that where previously ignored. These plane membrane forces create an arching action significantly enhancing the total strength capacity of the deck (Batchelor & Hewitt, 1976).

If a greater strength capacity were achieved in the deck due to this arching effect, less steel reinforcement would be necessary and would have a positive impact in the reduction of costs of the bridge deck as well. Less steel reinforcement reduces the probability of corrosion and improves longevity (Shoukry et al., 2010).

The Ontario Ministry of Transportation adopted the empirical method based on the investigations conducted by Queen’s University in Ontario, which discovered that reinforced

concrete bridge decks could sustain greater loads than what the Association of America State Highway and Transportation Official's stated (Bakht, 1993). Research concluded that the dominant failure mode for reinforced concrete bridge decks was punching shear (Fang, 1985). Gongkang Fu stated in 1992, that the compressive membrane force considered in the deck would improve its flexural strength by developing arching action when laterally restrained.

Since the empirical method requires no analysis, it is a much simpler method to adapt, particularly considering the additional economic benefit. The method consists of prescribing 0.3% of steel reinforcement in both transverse and longitudinal directions. Nonetheless, in order to adopt this method, some minimum conditions must be satisfied, such as the effective length between girders, the depth of the deck, the length of the deck overhang, and the concrete strength (Chen & Duan, 1999). Note that the deck overhang is not designed using the empirical method, rather these are designed using the traditional method mentioned before. According to the AASHTO LRFD Bridge Design Specifications, 2012, the deck conditions are as follows:

- The supporting elements must be designed in steel or concrete.
- The deck must be completely cast-in-place and water cured on sight.
- The deck has a uniform thickness, with the exception of the haunches at the girder flanges and other increases in thickness found.

$$6.0 \leq \frac{S_E}{e_L} \leq 18.0 \qquad \text{Eqn. 2.1.}$$

Where:

S_E=Effective Length e_L= deck thickness

$$S_E = S - \frac{(B_{1S} - B_A)}{2} \qquad \text{Eqn. 2.2.}$$

Where:

S= Space between girders (mm)

B_{1S}= Width of the superior flange of the girder (mm)

B_A= Width of the web (mm)

- Core depth of the deck not less than 100 mm (4in).
- Effective Length (S_e) must be equal or less than 4100 mm (13.5ft).
- Deck thickness should be greater than or equal to 174 mm (7 in).
- The overhang length should be greater than five (5) times the deck thickness, this condition is met if the overhang is at least three (3) times depth of the deck and a continuous concrete barrier is built.
- Concrete compressive stress (F_c) should be greater than 28 MPa (4 ksi).
- The deck works in conjunction with the supporting structural components.
- The least total of steel reinforcement for every bottom layer will be of 0.57 mm^2/mm (0.27 in^2/ft).
- The least total of steel reinforcement for every top layer will be of 0.38 mm^2/mm (0.18 in^2/ft).
- Steel reinforcement spacing shall not exceed 450 mm (18 in)
- Reinforcing steel shall be not less than 420 MPa (Grade 60 or better).

AASHTO Bridge Design Specifications 2007 recommends the deck be 7 in. for its minimum depth, with a 2 in. cover on top and 1.0 in cover on the bottom, and for a reinforced core to be 4 inches in width. Figure 2.1 indicates the deck details.

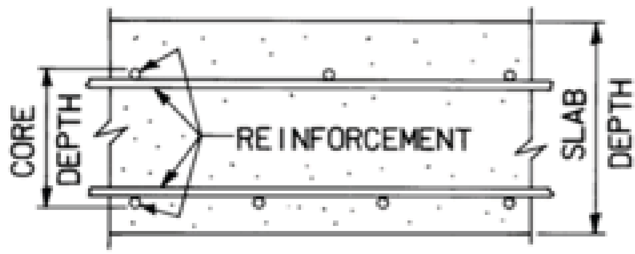

Figure 2.1 Core of a Concrete Slab
Adapted from "*AASHTO LRFD Bridge Design Specifications*", *by* the American Association of State highway and Transportation Officials, 2007, p. 9-11.

Taking into consideration that Canada plays an important role in the history of using the empirical method, the conditions that must be satisfied in their bridge design code are as follows: (Canadian Highway Bridge Design Code, 2006)

- Full-depth cast in place deck, the deck is composite with supporting beams and lines of the supports that are parallel to each other.
- The ratio of spacing of the supporting beams to the thickness of the slab must be less than 18.0.
- Spacing between supporting beams shall not exceed 4m.
- The deck extends sufficiently beyond the external beams to provide full development length for the bottom transverse reinforcement.
- The longitudinal reinforcement in the deck shall be provided for the negative moment region for continuous spans.
- Deck slab must contain two orthogonal assemblies of reinforcement, with a reinforcement ratio (ρ) in each direction near top and bottom of the slab in each assembly of at least 0.003,

unless otherwise specified.

- Reinforcement bars located closer to the top and bottom of the slab must be laid perpendicular to the axes of the supporting beams or laid on a skew parallel to the lines of the beam supports, but only if the slab is supported by parallel beams.
- A decrease in the reinforcement ratio (ρ) may apply from 0.003 to 0.002 if it satisfies and the reinforcement ratio of 0.003 is approved.
- If the transverse reinforcement is placed on a skew, the reinforcement ration should not be less than,

$$\frac{\rho}{cos^2\theta} \qquad \text{Eqn. 2.3.}$$

Where,

θ = is the skew angle

- If the unsupported length of the edge-stiffening beam surpasses 5m, the reinforcement ratio located in the exterior regions of the deck slab will be increased from 0.003 to 0.006.

When the conditions of the empirical method are not met, the ultimate capacity should be determined by the traditional method.

3 LITERATURE REVIEW

This chapter briefly discusses past research on compressive membrane action in reinforced concrete decks.

3.1 Introduction to compressive membrane action (CMA)

The compressive membrane action will occur whenever the wheel load acts upon the reinforced concrete deck, developing cracks in the positive moment region and displacing the neutral axis downward. When the empirical method is used, it is assumed that this compressive action will take place and will be resisted by the deck components acting together as required to resist the in-plane forces in the membrane coming from the stiff lateral boundaries in the deck as shown in Figure 3.1. (Fang, Worley, Burns, & Klinger, 1986).

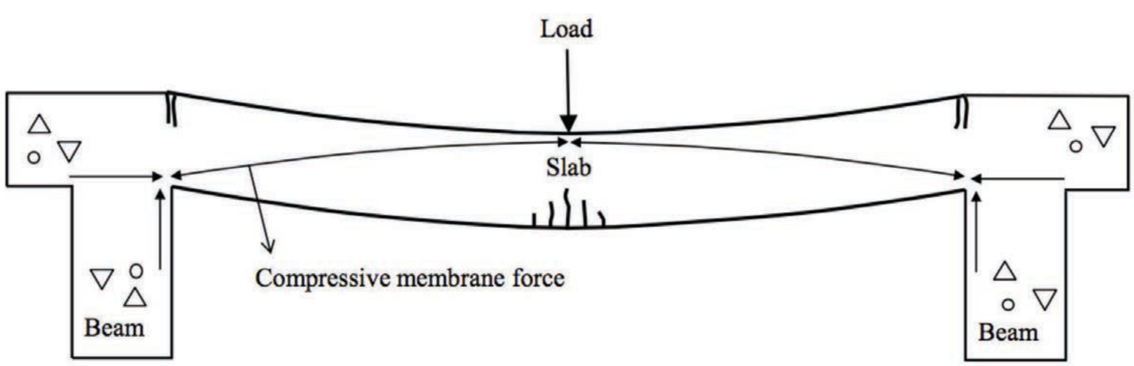

Figure 3.1 Compressive membrane action in reinforced concrete bridge deck slab
Adapted from "*Strength of reinforced concrete bridge decks under compressive membrane action*," by Hon et al., 2005

Research has shown that compressive membrane action increases the bearing capacity and leads to an ability to resist much higher loads than those predicted by the bridge standards (Bakht

& Jaeger, 1992, Batchelor, 1990, Fang, Lee, & Chen, 1994, Kirkpatrick, Rankin, & Long, 1984, Mufti, Jaeger, Bakht, & Wegner, 1993).

The degree of compressive membrane action developed in the slab will depend upon the level of lateral restraint proportioned as shown in Figure 3.2 (Hon, Taplin, & Al-Mahaidi, 2005).

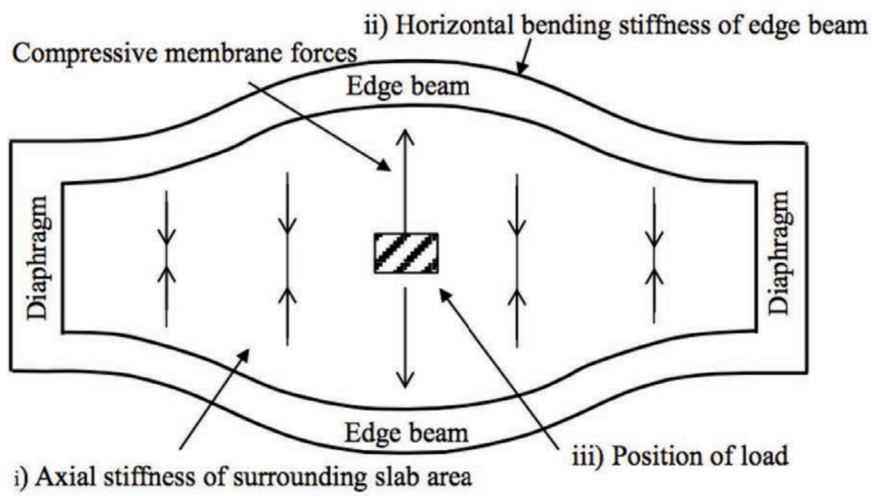

Figure 3.2 Contributions to horizontal translational restraint stiffness
Adapted from “*Strength of reinforced concrete bridge decks under compressive membrane action*,” by Hon et al., 2005

Compressive membrane action was first recognized in 1909 (Turner, 1909), but it was not brought to prominence until 1955 when Ockleston performed an experimental test on an old dental hospital in Johannesburg where he took into account the compressive membrane action and concluded that the failure loads obtained were higher than those predicted by the bridge standards (Ockleston, 1955). In 1956, McDowell et al. studied the arching action on masonry panels under transverse loading and presented a theory for arching action very different from what was believed at the time (McDowell, McKee, & Sevin, 1956).

3.2 Summary of previous research concerning compressive membrane action

3.2.1 Research in North America

Research into the load carrying capacity of composite structures considering the deck slab supported by concrete or steel girders, first started in 1960 at Queen's University, Ontario (Taylor, Rankin, & Cleland, 2002). Some years later, Hewitt in 1972, conducted research on concrete deck slabs of scaled models of composite steel/concrete bridges where the minimum steel reinforcement was reduced to a minimum of 0.2% and considered satisfactory for ultimate load limits (Hewitt, 1972). Thanks to the contribution of the Ontario Ministry of Transportation, Queen's University was able to conduct further research like the one done by Tong and Bachelor in 1975, where some experimental tests were performed by varying the steel reinforcement ratio in the slab. Even though the failure mode prevailing was punching shear, as expected for low steel reinforcement ratios, it was seen that the bearing capacities where higher than those predicted by yield line theory (Hewitt & Batchelor, 1975).

In 1978 the Ontario Ministry of Transportation sponsored Csagoly to perform tests on a full-scale model bridge (Csagoly, Holowka, & Dorton, 1978). Further research was performed on 28 existing bridges where each had different characteristics (some were decks composite with steel girders, others with pre-stressed concrete girders and others with simple reinforced concrete girders). The existing bridges had different restraint factors varying from 0.43 to 0.93 and each showed significant development of compressive membrane action in the slab (Bakht & Csagoly, 1972).

Based on the compressive membrane action benefits from the results of extensive studies, the empirical design specifications were introduced in the Ontario Design Code (OHBDC, 1979). The empirical method resulted in lighter and thinner bridge deck slabs, with a minimum steel

reinforcement of 0.3% for temperature crack control and shrinkage (He, 1992, p. 228).

The University of Texas at Austin performed a large number of experimental, numerical and analytical studies regarding the empirical method, known in Canada as Ontario-type reinforced concrete bridge decks (Fang, Worley, Burns, & Klinger, 1990, Graddy, Burns, & Klingner, 1995). Results from these studies showed that the flexural and punching shear modes presented significant compressive membrane action.

3.2.2 Research in United Kingdom

Just like Canada, Northern Ireland made a great contribution to the study of the compressive membrane action in the UK. Researchers tried to study the factors that would enhance the development of the compressive membrane action for its benefit to the slab (Masterson & Long, 1974). However, one of the greatest contributions was first presented by Rankin when he developed a method to predict the strength of laterally restrained reinforced concrete decks slabs (Rankin, 1982, p. 334). Rankin's method accounted for the compressive membrane action and was based on the theory presented by McDowell et al. (McDowell et al., 1956) on the arching deformation and considered an elastic plastic stress-strain criterion. Rankin estimated the ultimate flexural capacity by considering the bending and the arching capacity separately and later added together for the total resistance (Rankin, 1982).

At the same time, Kirkpatrick performed field tests at Queen's University Belfast where a slab was designed to have various levels of reinforcement in order to asses the influence on the serviceability of the slab. The conclusions mentioned that even at relatively low levels of load, compressive membrane forces played an important part in the control of cracking in the slab, improving the serviceability characteristics of the slab, and making inappropriate the calculation of crack widths based on the normal flexural method (Kirkpatrick, Rankin, & Long, 1986).

Further research on compressive membrane action was done between the years 1985 and 2000. An extensive number of reviews were published assessing the compressive membrane action (Rankin & Long, 1997, Long & Rankin, 1989).

Methods were developed to estimate restrained slabs subjected to uniformly distributed loads (Niblock, 1986). Taylor presented a procedure that assessed the degree of lateral restraint where a restraint model was used considering an effective width concept. This procedure accounted for the restraint provided by the diaphragms, the edge beams and the area of the slab adjacent the loaded area (Taylor, 2000). To evaluate the bridge deck ultimate capacity, the procedure integrated Rankin and Long’s theory for the flexural capacity (Rankin & Long, 1997) and Kirkpatrick et al. for punching shear capacity (Kirkpatrick et al., 1984). The ultimate capacity was determined by the lesser value between the flexural and the punching shear capacities (Taylor, 2000).

Numerous experimental tests have been conducted on scaled bridge models with changing parameters. Taylor et al. performed tests considering the use of high performance concrete in 2003 (Taylor, Rankin, & Cleland, 2003) and in 2007 further research was carried to assess shrinkage and enhance the durability of the concrete (Taylor, Rankin, Cleland, & Kirkpatrick, 2007). The development of compressive membrane action has been significant through all these experimental, analytical, and numerical studies (Zheng, Li, & Yu, 2011). The compressive membrane action has now been recognized by Northern Ireland through a code design (UK Highway Agencies [UKBD 81/02], 2002).

4 FINITE ELEMENT ANALYSIS

This chapter describes a nonlinear finite element analysis of two 3D bridge models and introduces the software used for the analysis.

4.1 STAAD.Pro V8i

According to the Bentley 2012 Technical Reference Manual, STAAD.Pro is a software package for structural analysis founded on the theory of Finite Elements (Bentley Systems, 2012) by which models can perform analysis, design, visualization and verification. It is used extensively throughout the field of civil engineering design for its versatility and high computing power.

Among key features is that the model allows one to design the structures according to various building codes that apply worldwide, including the regulations of countries like the United States (AISC and ACI), Spain, Britain, Canada, France Germany, China, Japan, etc (Bentley Systems, 2012).

The versatility that STAAD.Pro provides stems from its flexibility to work with any type of building material. Within the range of materials that STAAD.Pro uses, it is possible to find from the most common (such as steel or concrete) or any other desired or customized materials. Therefore, it is possible to define specific physical properties and assign them to the geometry already created at any point of the modeling. Another characteristic provided by the software is that includes a comprehensive database of commercial profiles commonly used worldwide.

Structural systems such as slabs, plates, and footings with tie beams that transmit loads in

two directions must be discretized as a finite element of three or four nodes interconnected by end nodes. The loads may be applied as distributed over the surfaces of the elements or concentrated loads in the joints.

4.2 Finite Element Modeling

A bridge section was analyzed using STAAD.Pro that allows a three-dimensional finite element modeling of the whole structure and its components, such as the piles, the girders and the deck. The deck was modeled using 4-noded plates with a thickness within the Empirical method specification in the AASHTO LRFD manual. The plates for the deck were designed in STAAD.Pro V8i with a parametric mesh generator tool with plate divisions of 2’x2’ as shown in Figure 4.1.

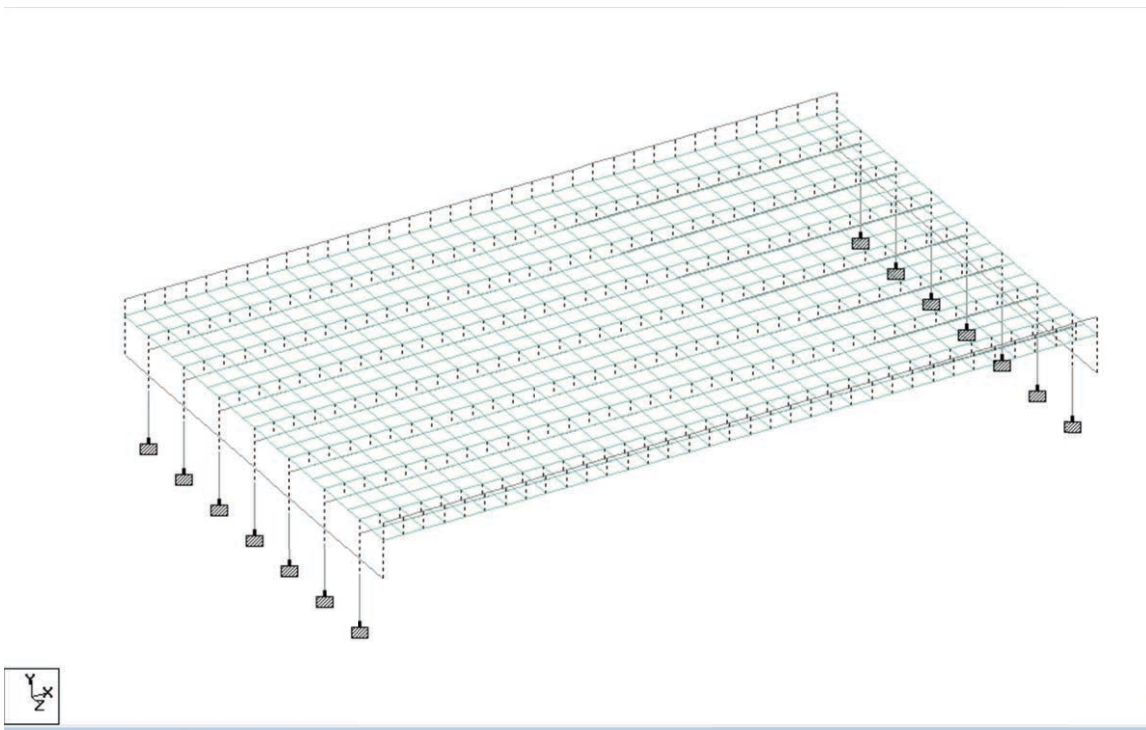

Figure 4.1 Finite element modeling showing the parametric mesh

Because the results obtained from this modeling involves principal stresses and moments in the plate, it is important to understand the sign convention for plate stresses and moments that STAAD.Pro V8i uses as shown in Figure 4.2 below.

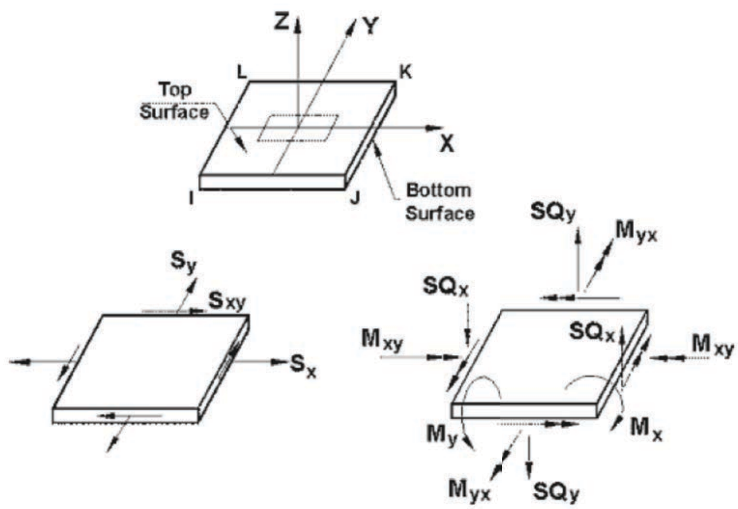

Figure 4.2 Sign conventions for plate stresses and moments
Adapted from "*Technical Reference Manual*", *by* Bentley, 2012.

The model properties, such as the type of beam and barrier, were created using the built-in feature that allows for the user to define a customized section and properties. Within this feature, the user can establish the coordinates of the shape in order to match the geometry of the FIB-36 and the FDOT F-Shape barrier. The two sections of a bridge were modeled with the same deck slab, but the supporting beams were changed from FIB-36 to a W-Shape (W44x335) beam appropriate for the bridge specifications as shown in Figures 4.3 and 4.4.

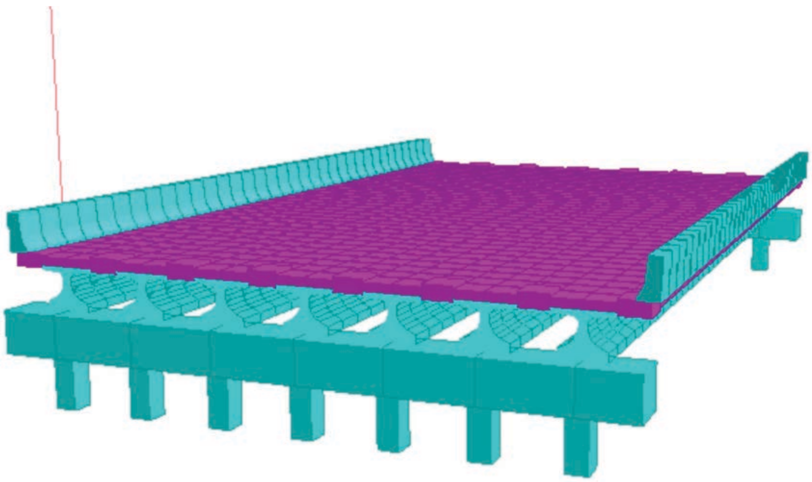

Figure 4.3 Bridge section model supported by FIB-36

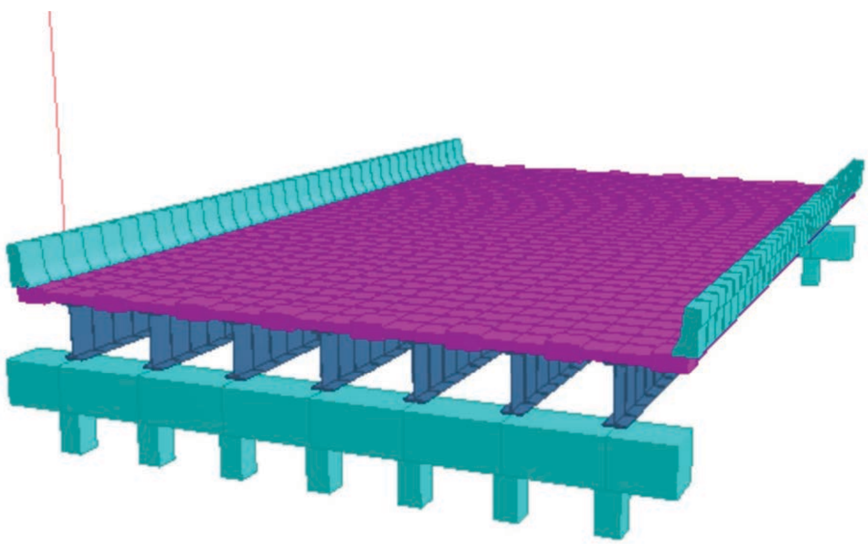

Figure 4.4 Bridge section model supported by W44x335

To understand the analysis of the two sections of the bridge, STAAD.Pro V8i has some geometry considerations that helped understand the interpretation of the results. There are 4 principal considerations:

1. In order to measure the strength in the 4-noded plate, the program automatically generates a fictitious 5^{th} node at the element center that was an average of the stresses present in all of the other nodes, giving a better representation of what happened in the plate.
2. Whenever assigning the nodes to an element it is important to keep into account the direction used (clockwise or counter-clockwise). This defined the orientation of the plate and it was convenient to keep an order in the definition of these orientations so as to effectively interpret the stress distributions.
3. The elements should maintain a relatively uniform aspect ratio.
4. It was preferable to maintain a parallelogram shape, and the angles between two adjacent elements should not exceed 90 degrees and never larger than 180. This resulted in more reliable results.

For this model, the following loads were considered:

- It was necessary to define a vehicle in order to design accordingly to the traffic in Florida; this is specified in the LRFD 3.6.1.3.3. The sections where evaluated for a HL-93 Design Truck and design Lane Loads with a dynamic load allowance of 33%.
- Self-weight for every element.
- Future wearing surface of 15 psf (FDOT SDG Table 2.2-1)
- Stay in place forms of 20 psf (FDOT SDG Table 2.2-1)

An analysis was done on the effect that changing the support beam lateral stiffness had on stresses in the bridge deck slab. The interpretation and results of the analysis follow.

4.2.1 Interpretation of Results

Upon completion of the design, results were obtained for the stress distribution within the deck slab. Based on the results, a comparison was drawn between the stress values obtained for the deck supported on FIB-36 girders and those obtained from the deck on steel W-shape girders. Initially, the results were very different.

The stresses that were developed in the steel W-shape girder were much higher than those developed by the FIB-36 counterpart. Originally, these differences were believed to have been due to calculations or modeling errors. Subsequently the models were extensively revised, eventually arriving at the assumption that the beam’s stiffness was possibly playing a much larger role in the stress distribution than originally thought. To further test the possibility of this assumption, different steel W-shape girders were considered and analyzed accordingly, each girder with a larger stiffness value than the previous one. Eventually arriving at the steel W44X335 girder, it was possible to observe that at this point the stress distribution was quite similar to the stress distribution in the FIB-36 as shown in Figures 4.5 and 4.6.

The STAAD analysis of the influence of the beam stiffness on bridge deck behavior did not give conclusive and definite answers to support the assumption of the correlation between girder stiffness and deck stress distributions. Given the nature of the finite element modeling limitations of STAAD, it was determined that STAAD might not be an adequate tool of analysis to achieve the specific purposes of this study.

For this reason, it was decided to go to the fundamental mechanic equations to further investigate the compressive membrane action and the beam stiffness.

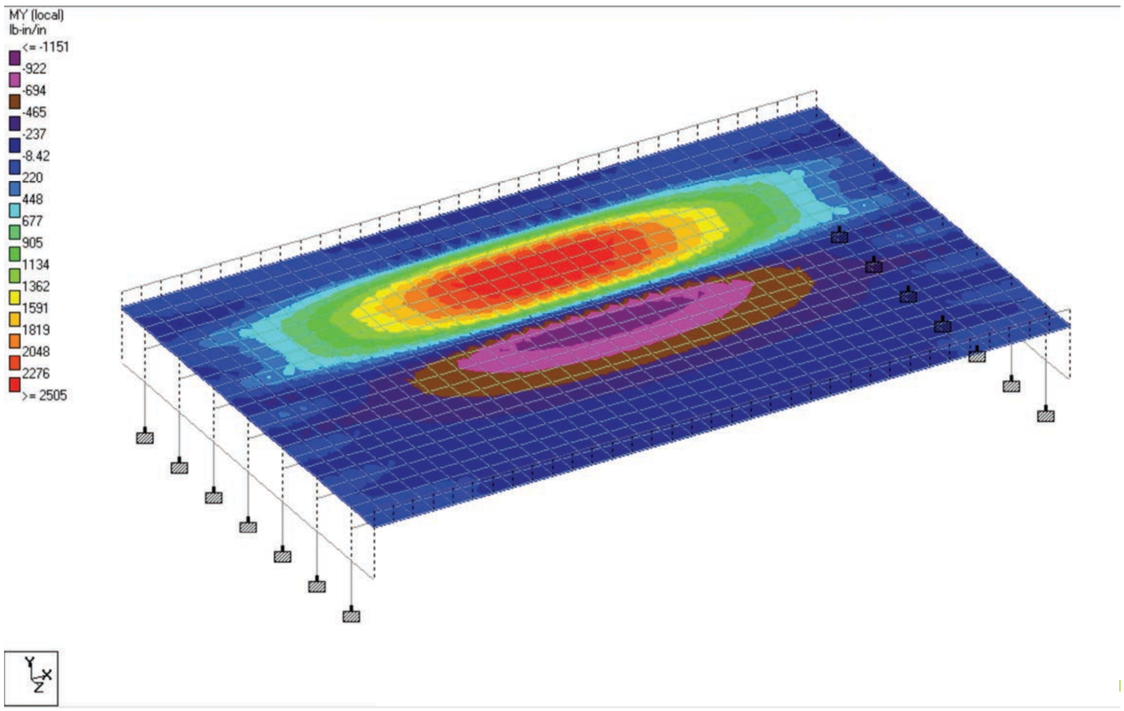

Figure 4.5 Transverse direction moment stress-contour supported on W44x335

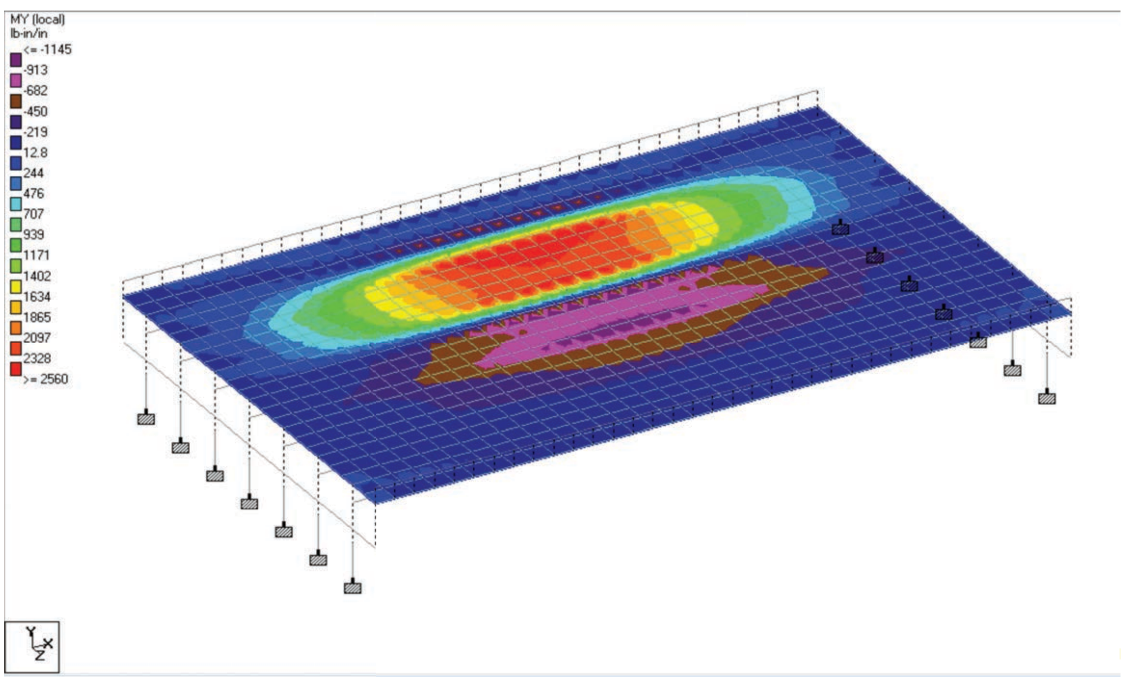

Figure 4.6 Transverse direction moment stress-contour supported on FIB-36

5 METHODS OF ANALYSIS

The main objective of this thesis was to evaluate the effect of lateral stiffness on bridge deck performance. This chapter includes the description and verification of the methods to be utilized in order to estimate the bridge deck ultimate capacity and evaluate the influence that lateral restraint has on the deck slab. The methods are defined in the following sections.

5.1 BS5400 method

In the design of the bridge deck slab the predominant criteria are the bending capacity and the local effect of the concentrated wheel load represented as such (with the partial safety factors removed)

$$M = A_s f_y d \left(1 - \frac{0.746 A_s f_y}{f_{cu} b d}\right) \qquad \text{Eqn.5.1.}$$

The code recommends using the Pucher Charts in order to establish the predicted flexural failure load from the maximum allowable internal moment (BS5400, 1978 to 1990). In this case, since it is a concentrated load analyzed in the center of the plate with two restrained edges, Pucher Chart 12 shown in Figure 5.1 was used and the relationship between the strain and the applied load is described by the following equation

$$M = 0.08P\ kN \cdot m/m \qquad \text{Eqn.5.2.}$$

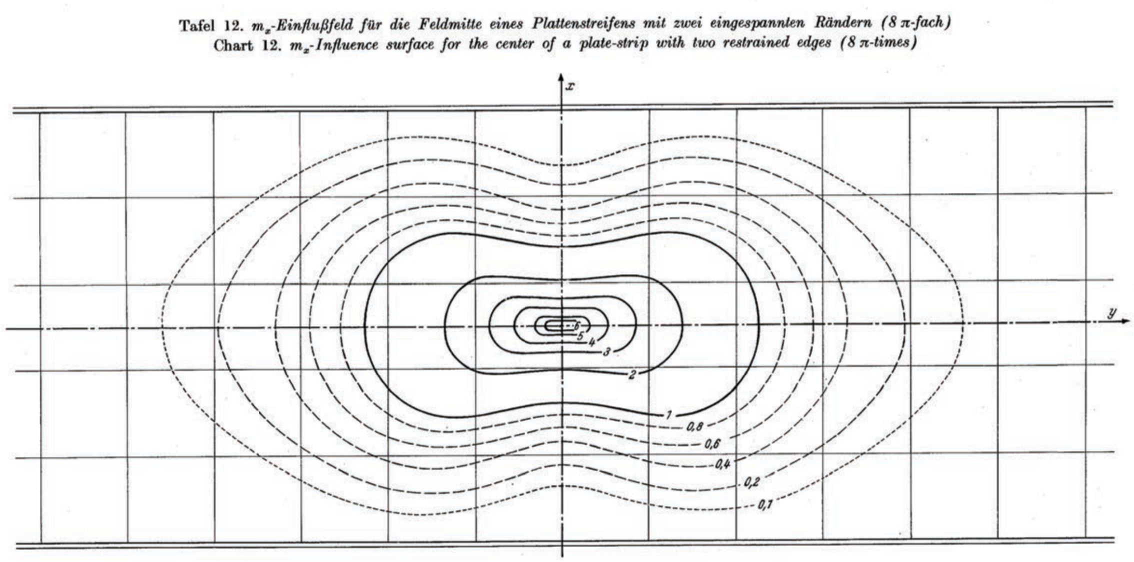

Figure 5.1 Chart 12. Influence surfaces for the center of the plate strip with two restraint edges.
Adapted from “*Influence surfaces of elastic plates*”, by Pucher Charts, 1964

The punching shear strength is given by the following equation

$$P_{vs} = 0.79 \cdot \sqrt[3]{100 \cdot \frac{A_s}{bd}} \cdot \sqrt[3]{\frac{f_{cu}}{25}} \cdot \sqrt[4]{\frac{500}{d}} \cdot b_o \cdot d \qquad \text{Eqn.5.3.}$$

5.2 ACI 318-05 method

The bending capacity and the local effect of a concentrated load can be represented by the following equations (all factors of safety removed):

$$M = \rho \cdot f_y \cdot d^2 \left(1 - \frac{0.5\rho f_y}{\beta \cdot f'c}\right) \qquad \text{Eqn.5.4.}$$

The same Pucher Chart was used to find the flexural capacity, since the ACI 318-05 punching shear capacity formula assumes the slab has been already correctly designed for flexure. The ACI formula for punching strength is the following

$$P_{vs} = 4 \cdot \sqrt{f'c} \cdot b_o d \qquad \text{Eqn.5.5.}$$

5.3 BD81/02 method

This method takes into account the development of the compressive membrane action in the slab. It is based on the different studies previously performed. It assumes that the slab's type of failure is punching shear and that it has an effective rigid restraint system (UKBD, 2002).

The method first accounts for an ideal elastic-plastic concrete stress block derived as

$$\varepsilon_c = (-400 + 60f'c - 0.33f'c^2)\times 10^{-6} \qquad \text{Eqn.5.6.}$$

This enables the estimation of McDowell's non-dimensional parameter *R*.

$$R = \frac{\varepsilon_c \cdot {L_r}^2}{h^2} \qquad \text{Eqn.5.7.}$$

Considering the moment ratio M_r and the deformation u, the maximum value for the arching moment ratio was derived as follows

$$M_r = 4.3 - 16.1\sqrt{3.3\times10^{-4} + 0.1243R} \qquad \text{Eqn.5.8.}$$

$$u = -0.15 + 0.36\sqrt{0.18 + 5.6R} \qquad \text{Eqn.5.9.}$$

This leads to the calculation of the maximum arching moment coefficient *k* used to find the equivalent area of flexural reinforcement ρ_e, given by

$$k = 0.0525\left(4.3 - 16.1\sqrt{3.3}\times10^{-4} + 0.1243R\right) \qquad \text{Eqn.5.10.}$$

$$\rho_e = \frac{k \cdot f'c \cdot h^2}{240d^2} \qquad \text{Eqn.5.11.}$$

Finally the equivalent area is substituted into Long's equation for the shear punching strength

$$P_{pv} = 1.52 \cdot (c_x + d) \cdot d \cdot \sqrt{f'c} \cdot (100\rho_e)^{0.25} \qquad \text{Eqn.5.12.}$$

Subsequent research done by Queen's University led to adjustments of the plastic strain value to incorporate high-performance concrete, this was later done to the drafting of BD81/02 described in the TRC approach.

5.4 Taylor, Rankin, and Cleland's approach (TRC)

In the process of assessing the degree of lateral stiffness that would enhance the strength of the bridge deck slab, Taylor et al. developed a simplified method predicting the ultimate load carrying capacity of bridge deck type slabs with a range of boundary conditions considering not only flexural, but also shear punching mode capacity on one-way spanning slab strips. The proposed method by Taylor et al. was found to more accurately predict the strength of the slabs compared to other methods when considering the compressive membrane action capacity acting in the bridge deck slab (Taylor et al., 2002).

The procedure considers a restraint system where the supporting edge beams, end diaphragm and surrounding area of unloaded slab were equated to a spring of an equivalent stiffness. A typical bridge deck restrained model is illustrated in Figure 5.2.

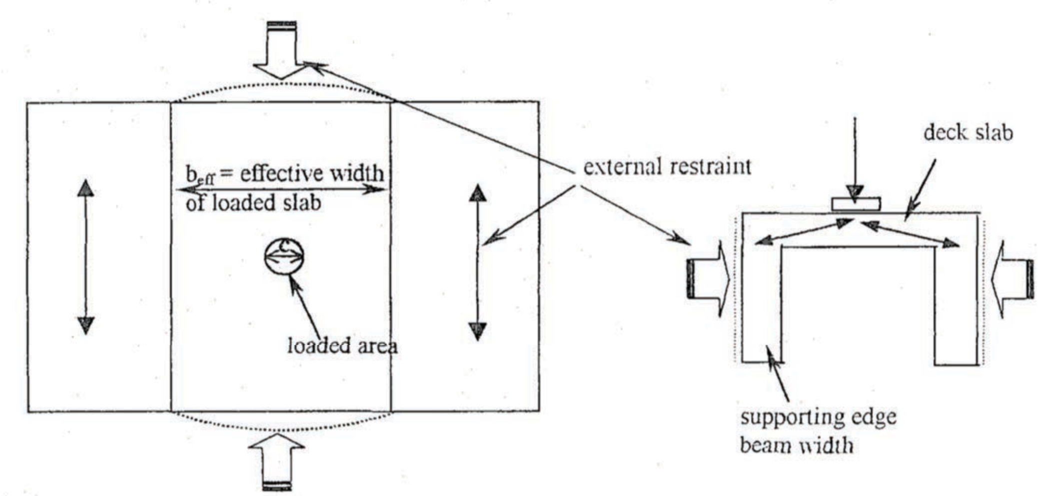

Figure 5.2 Restraint model proposed
Adapted from "*A guide to compressive membrane action*", by Taylor et al., 2002

The preceding analytical approach consists of twelve steps as illustrated in Figure 5.3.

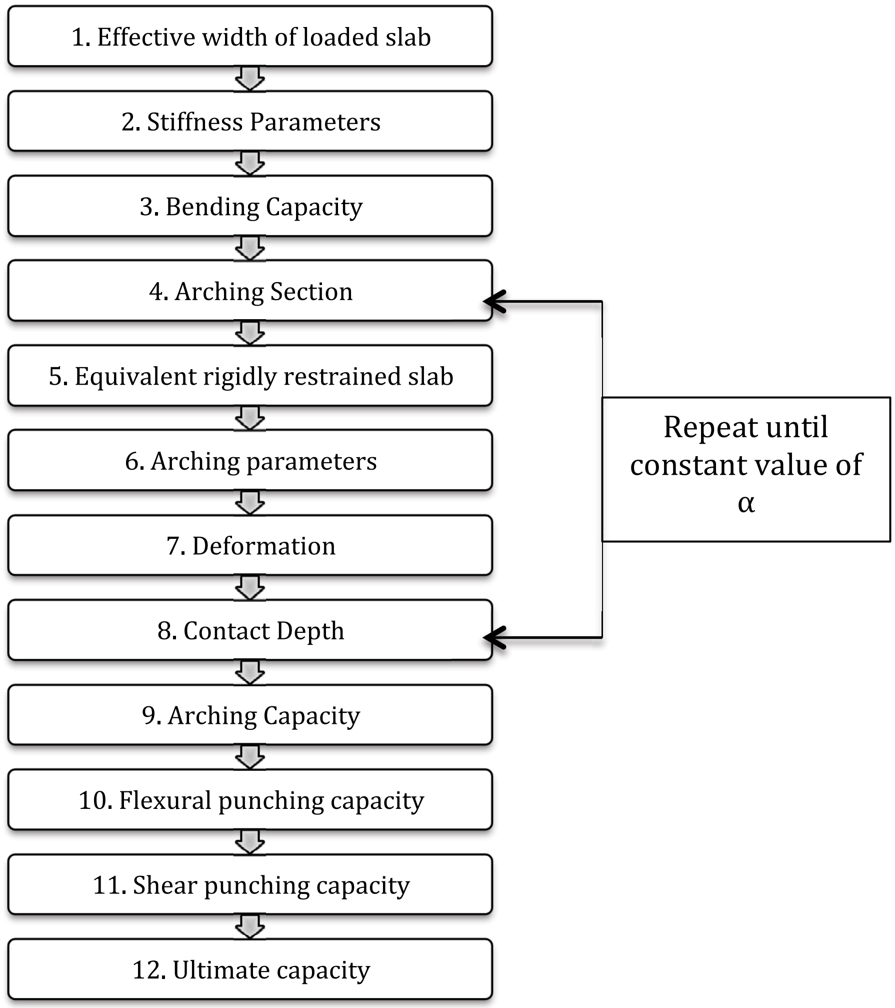

Figure 5.3 Simplified method procedure
Adapted from "*A guide to compressive membrane action*", by Taylor et al., 2002

Below, the subsequent equations present the detail of the proposed procedure.

1. Effective width of loaded slab

An effective width of slab subjected to arching forces is described by

$$b_{eff} = c_y + 2 \cdot L_e + 2h \qquad (where\ L_e = \frac{L}{2} - \frac{c_x}{2}) \qquad \text{Eqn. 5.13.}$$

Where,

b_{eff} = effective width of loaded slab

L_e= half the span of the arch length

c_y= width of patch load perpendicular to slab span

c_x= width of patch load parallel to slab span

L= spacing between supporting beams

h= depth of slab

2. Stiffness parameters

Research has shown that the widths of the supporting beams have a significant influence in the strength of the deck slab (Taylor et al., 2002). Considering that the supporting beams are related to a spring of an equivalent stiffness then the 'equivalent area' of lateral stiffness, A_b, gives an external stiffness of ${}^{EA_b}/{}_{L_e}$.

$$E_c = 4.23\sqrt{f'c} \qquad \text{Eqn.5.14.}$$

$$K_s = \frac{E_c h b_{beff}}{L_e} \qquad \text{Eqn.5.15.}$$

$$Calculate\ the\ second\ moment\ of\ area\ of\ support\ beam$$

$$about\ the\ vetical\ axis\ (I_{yb})$$

$$A_b = \frac{\zeta L_e I_{yb}}{{b_{eff}}^3} \qquad \text{Eqn.5.16.}$$

where ζ= constant support condition (114.5 if simply supported or 985 for fixed ends)

$$K_b = \frac{A_b E_C}{L_e} \qquad \text{Eqn.5.17.}$$

A similar approach is made in assessing the restraint inherent in a bridge deck slab.

$$A_d = area\ of\ diaphragm + area\ of\ slab\ outside\ the\ effective\ width$$

$$K_d = \frac{\Sigma A_d E_c}{L_e} \qquad \text{Eqn.5.18.}$$

Nevertheless, since the supporting beams don’t act parallel to the end diaphragms and unloaded slab like the deck slab does, the combined flexibility of the total restraint is expressed by

$$K_r = \frac{1}{\frac{1}{K_b}+\frac{1}{K_d}} \qquad \text{Eqn.5.19.}$$

Where,

E_c=concrete elastic modulus

K_s= stiffness of slab within effective width

A_b= equivalent area of support beam

K_b= equivalent stiffness of support beam

K_d= stiffness of diaphragm and slab

K_r= combined stiffness of restraint

3. Bending capacity

The bending capacity is estimated by taking into account the equivalent rectangular stress block as can be observed in the following procedure.

Depth of stress block, $\beta = 1 - 0.003\ f'c$ but <0.9 Eqn.5.20.

Depth of neutral axis, $x = \frac{f_y A_s}{0.67\ f'c\beta\ b}$ Eqn.5.21.

Lever arm, z=d – 0.5 β x Eqn.5.22.

$$M_b = f_y A_s\ z \qquad \text{Eqn.5.23.}$$

$$P_b = k_b M_b \qquad \text{Eqn.5.24.}$$

Where,

β = proportional depth of stress block (=0.9 in BS)

x = depth of concrete compression zone

f_y = reinforcement yield strength

A_s = area of steel reinforcement

b = width of section

M_b= flexural moment of resistance at principal section

P_b= predicted ultimate flexural capacity

k_b= static moment coefficient for a strip under uniform loading

4. Arching Section

The arching section may be estimated by using the following considerations.

$$2d_1 = h - 2x\beta \qquad \text{Eqn.5.25.}$$

new d_1 from previous iterations

Whered_1 is the half the arching depth.

5. Affine Strip

The following equations are used in determining the affine strip.

$$A = \alpha b d_1 \qquad \text{Eqn.5.26.}$$

$$L_r = L_e \sqrt[3]{\left(\frac{EA}{KL_e} + 1\right)} \qquad \text{Eqn.5.27.}$$

Where,

A= cross section area

L_r= half the span of the rigidly restrained arch

6. Arching parameters

The arching parameters are estimated considering the plastic strained formula. This is determined through the non-dimensional parameter for the arching moment of resistance R from previous research by McDowell et al.

$$\varepsilon_u = 0.0043 - [(f'c - 60)2.5\times10^{-5}] \quad \text{but} < 0.0043 \qquad \text{Eqn.5.28.}$$

$$\text{and } R = \frac{\varepsilon_u {L_r}^2}{4{d_1}^2} \qquad \text{Eqn.5.29.}$$

$$\varepsilon_c = 2\varepsilon_u(1 - \beta) \qquad \text{Eqn.5.30.}$$

Where,

ε_u= concrete maximum compressive strain

ε_c= concrete compressive plastic strain value

R: McDowell’s non-dimensional parameter (elastic deformation)

7. Deformation

R>0.26 ➔ u=0.31 (constant)

0<R<0.26 ➔ $u = -0.15 + 0.36\sqrt{0.18 + 5.6R}$ Eqn.5.31.

where, u= McDowell’s non-dimensional parameter (deflection)

8. Contact depth

$$\alpha = 1 - \frac{u}{2} \qquad \text{Eqn.5.32.}$$

$\boldsymbol{\alpha}\mathbf{d_1}$ use for refined arching action section above until value remains constant.

Where α is the proportion of d_1 in contact with the support.

9. Arching capacity

The arching capacity for the section is determined by the maximum value for the arching moment M_r

$$R>0.26 \Rightarrow M_r = \frac{0.3615}{R} \qquad \text{Eqn.5.33.}$$

$$0<R<0.26 \Rightarrow M_r = 4.3 - 16.1\sqrt{3.3\times10^{-4} + 0.1243R} \qquad \text{Eqn.5.34.}$$

$$M_a = 0.168bf'cd_1{}^2M_r\left({}^{L_e}/_{L_r}\right) \qquad \text{Eqn.5.35.}$$

$$[for\ maximum\ arching\ L_e = L_r \Rightarrow M_{ar} = 0.168f'cd_1{}^2M_r] \qquad \text{Eqn.5.36.}$$

$$P_a = k_aM_a \qquad \text{Eqn.5.37.}$$

Where,

M_r= moment ratio (non-dimensional)

M_{ar}= arching moment of resistance of rigidly restrained slab strip

M_a= arching moment of resistance

P_a= predicted ultimate arching capacity

k_a= static moment coefficient under concentrated mid-span loading

10. Flexural punching capacity

The flexural punching capacity is established by taking into account the bending and the arching capacity:

$$P_{pf} = P_a + P_b \qquad \text{Eqn.5.38.}$$

11. Shear punching capacity

An equivalent area of reinforcement is estimated in order to determine the shear punching capacity as follows

$$\rho_e = (\rho_e + \rho)\left(\frac{f_y}{320}\right) = \left(\frac{M_a + M_b}{M_b}\right)\left(\frac{f_y}{320}\right)\rho \qquad \text{Eqn.5.39.}$$

Where,

ρ_e= effective reinforcement ratio at principal section

ρ = reinforcement ratio at principal section

$$P_{pv} = \frac{0.43}{r_f}\sqrt{f'c}(critical\ permeter)d(100\rho_e)^{0.25} \qquad \text{Eqn.5.40.}$$

Critical perimeter at 0.5d from face of loaded area

12. Ultimate capacity

The ultimate capacity for the bridge deck slab was determined according to the following relationships

If $P_{pf} < P_{pv}$ ➔ $P_p = P_{pf}$

If $P_{pf} > P_{pv}$ ➔ $P_p = P_{pv}$

Where P_p is the ultimate capacity

A spreadsheet tool was developed based on each method using programing software called MathCAD in order to study the effect of different parameters on the bridge deck bearing capacity. The tool was validated against results from papers in the literature review (Taylor et al., 2002,2007). The validation is presented in the following section.

5.5 Code work validation

This section describes previous studies performed by Taylor et al. (Taylor et al., 2007) to verify the analyzing tool developed for the purpose of this study. An experimental investigation

was carried in 2007 in order to assess the benefits of the arching action. The results were compared with the current code requirements.

The test consisted of evaluating the influence of the steel reinforcement, the concrete compressive strengths, and position on the service behavior of the bridge deck slab. The test results in the study were compared to the predicted strengths from the bridge design codes, such as the British Standard (BS5400), the American Concrete Institute (ACI 318-05), and the UK Highways Agency (BD81/02). For the purpose of this study, the validated analyzing tool developed based on the TRC approach was also incorporated in this example.

Table 5.1 presents the results obtained using the developed tool that matches the predicted values estimated in the paper (Taylor et al., 2007).

Table 5.1
Summary of predicted capacities under concentrated load

Test Panels	BS5400	ACI 318	BD81/02	TRC	TEST LOAD
C1	66.6	66.2	588	418	333
C2	92.2	91.7	588	435	428
D1	127.9	126.7	553	394	368
D2	177.3	175.5	568	420	428
E1	202.1	200.7	632.8	465	392
E2	280	274.5	648.7	470.9	428
F1	199.5	198.9	566.5	410.564	371
F2	275.2	260.4	601.2	425.215	428

The results obtained matches the study performed by Taylor et al., but most importantly state that the TRC approach predicts an ultimate capacity closer to the real test loads than the standard codes. This is better illustrated in Figure 5.4 below.

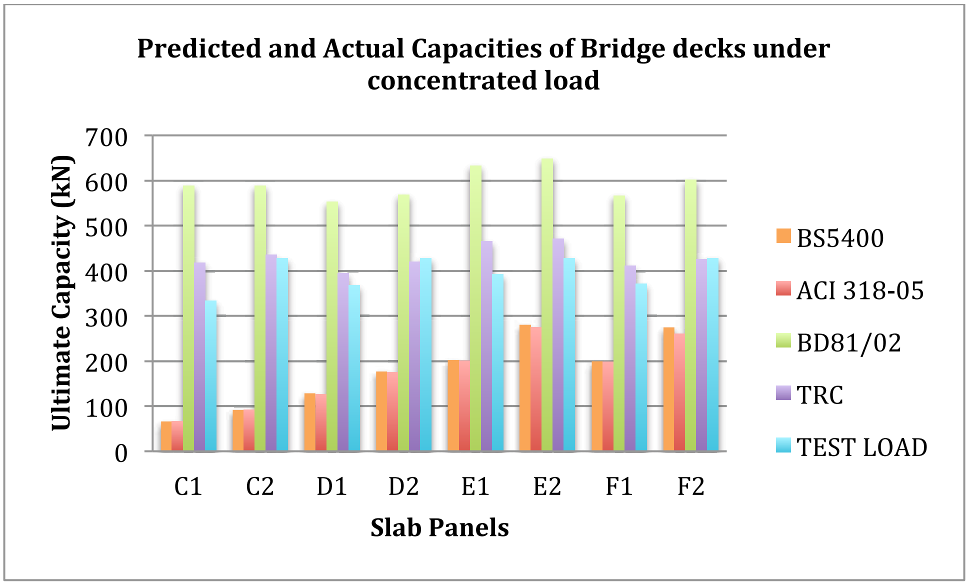

Figure 5.4 Comparison between predicted and actual test bridge deck capacities under concentrated loads

The methods that account for arching action (BD81/02, TRC approach) gave more accurate predictions when compared to the current codes. It has been proven through research (Taylor et al., 2002, p. 26) that the TRC approach gives more consistent predictions when compared to actual test results.

In order to evaluate the consistency of the predictions represented by each method (BS5400, ACI 318-05, BD81/02, and TRC), a comparative analysis was performed considering different parameters.

6 COMPARATIVE ANALYSIS BETWEEN METHODS

This chapter details the comparison done between the four methods (BS5400, ACI 318-05, BD81/02, TRC) to study the effect of different parameters for the bridge lateral stiffness.

6.1 Parameters for comparative analysis

This section introduces the parameters that were varied to analyze their effect on the predicted ultimate capacity of the bridge deck slab. The following aspects were analyzed and results are presented:

- 5 different deck slab thickness (7.5, 8, 8.5, 9, 9.5 inches)
- 5 different support beam spacing (6, 8, 10, 12, 14 feet)
- Steel reinforcement ratio of 0.454%
- 80 foot bridge span length
- Florida I- Beam (FIB-36)
- Compressive concrete strength of 5ksi
- Reinforcement yield strength of 60ksi

A comparison analysis was conducted considering the following methods:

- British Standard (BS5400)
- American Concrete Institute (ACI 318-05)
- UK Highways Agency (BD81/02)
- Taylor, Rankin, and Cleland’s approach (TRC)

6.2 Results of analysis

Tables 6.1 to 6.5 summarize the results obtained using the analyzing tool developed for this study. An evaluation was performed to determine the influence that different support beam spacing and deck thickness have on the ultimate capacity of the bridge deck slab.

Table 6.1

Summary of predicted capacities using a 7.5-inch slab thickness for different methods

SPACING	THICKNESS	Method	Flexural Capacity kip	Shear Capacity kip	Ultimate Capacity kip	Type of Failure
6'	7.5"	BS5400	35.144	93.457	35.144	Flexural
		ACI-318	35.088	118.48	35.088	Flexural
		BD81	-	255.101	255.101	Shear
		TRC	180.064	200.951	180.064	Flexural
8'		BS5400	28.425	93.457	28.425	Flexural
		ACI-318	28.38	118.48	28.38	Flexural
		BD81	-	248.771	248.771	Shear
		TRC	154.188	192.314	154.188	Flexural
10'		BS5400	24.514	93.457	24.514	Flexural
		ACI-318	24.475	118.48	24.475	Flexural
		BD81	-	241.824	241.824	Shear
		TRC	128.731	182.22	128.731	Flexural
12'		BS5400	24.514	93.457	24.514	Flexural
		ACI-318	24.475	118.48	24.475	Flexural
		BD81	-	234.169	234.169	Shear
		TRC	105.344	170.799	105.344	Flexural
14'		BS5400	21.543	93.457	21.543	Flexural
		ACI-318	21.509	118.48	21.509	Flexural
		BD81	-	225.646	225.646	Shear
		TRC	84.484	157.969	84.484	Flexural

Table 6.2

Summary of predicted capacities using an 8-inch slab thickness for different methods

SPACING	THICKNESS	Method	Flexural Capacity kip	Shear Capacity kip	Ultimate Capacity kip	Type of Failure
6'	8"	BS5400	42.245	105.05	42.245	Flexural
		ACI-318	42.178	133.117	42.178	Flexural
		BD81	-	282.585	282.585	Shear
		TRC	211.876	222.876	211.876	Flexural
8'		BS5400	34.155	105.05	34.155	Flexural
		ACI-318	34.1	133.117	34.1	Flexural
		BD81	-	276.174	276.174	Shear
		TRC	180.215	213.541	180.215	Flexural
10'		BS5400	29.457	105.05	29.457	Flexural
		ACI-318	29.411	133.117	29.411	Flexural
		BD81	-	269.178	269.178	Shear
		TRC	151.451	202.78	151.451	Flexural
12'		BS5400	29.457	105.05	29.457	Flexural
		ACI-318	29.411	133.117	29.411	Flexural
		BD81	-	261.53	261.53	Shear
		TRC	125.385	190.801	125.385	Flexural
14'		BS5400	25.896	105.05	25.896	Flexural
		ACI-318	25.855	133.117	25.855	Flexural
		BD81	-	253.108	253.108	Shear
		TRC	101.963	177.366	101.963	Flexural

Table 6.3

Summary of predicted capacities using an 8.5-inch slab thickness for different methods

SPACING	THICKNESS	Method	Flexural Capacity	Shear Capacity	Ultimate Capacity	Type of Failure
			kip	kip	kip	
6'	8.5"	BS5400	49.999	117.138	49.999	Flexural
		ACI-318	49.919	148.32	49.919	Flexural
		BD81	-	310.904	310.904	Shear
		TRC	243.824	245.506	243.824	Flexural
8'		BS5400	40.424	117.138	40.424	Flexural
		ACI-318	40.36	148.32	40.36	Flexural
		BD81	-	304.413	304.413	Shear
		TRC	208.312	235.451	208.312	Flexural
10'		BS5400	34.864	117.138	34.864	Flexural
		ACI-318	34.809	148.32	34.809	Flexural
		BD81	-	297.36	297.36	Shear
		TRC	176.005	224.007	176.005	Flexural
12'		BS5400	34.864	117.138	34.864	Flexural
		ACI-318	34.809	148.32	34.809	Flexural
		BD81	-	289.699	289.699	Shear
		TRC	147.111	211.456	147.111	Flexural
14'		BS5400	30.649	117.138	30.649	Flexural
		ACI-318	30.601	148.32	30.601	Flexural
		BD81	-	281.336	281.336	Shear
		TRC	121.282	197.282	121.282	Flexural

Table 6.4

Summary of predicted capacities using a 9-inch slab thickness for different methods

SPACING	THICKNESS	Method	Flexural Capacity kip	Shear Capacity kip	Ultimate Capacity kip	Type of Failure
6'	9"	BS5400	58.406	129.717	58.406	Flexural
		ACI-318	58.314	164.089	58.314	Flexural
		BD81	-	340.062	340.062	Shear
		TRC	278.921	268.835	268.835	Shear
8'		BS5400	47.221	129.717	47.221	Flexural
		ACI-318	47.146	164.089	47.146	Flexural
		BD81	-	333.491	333.491	Shear
		TRC	238.485	258.039	238.485	Flexural
10'		BS5400	40.727	129.717	40.727	Flexural
		ACI-318	40.662	164.089	40.662	Flexural
		BD81	-	326.376	326.376	Shear
		TRC	202.398	245.902	202.398	Flexural
12'		BS5400	40.727	129.717	40.727	Flexural
		ACI-318	40.662	164.089	40.662	Flexural
		BD81	-	318.688	318.688	Shear
		TRC	170.527	232.765	170.527	Flexural
14'		BS5400	35.803	129.717	35.803	Flexural
		ACI-318	35.746	164.089	35.746	Flexural
		BD81	-	310.354	310.354	Shear
		TRC	142.336	218.484	142.336	Flexural

Table 6.5

Summary of predicted capacities using a 9.5-inch slab thickness for different methods

SPACING	THICKNESS	Method	Flexural Capacity	Shear Capacity	Ultimate Capacity	Type of Failure
			kip	kip	kip	
6'	9.5"	BS5400	67.479	142.793	67.479	Flexural
		ACI-318	67.372	180.423	67.372	Flexural
		BD81	-	370.062	370.062	Shear
		TRC	316.512	292.857	292.857	Shear
8'		BS5400	54.557	142.793	54.557	Flexural
		ACI-318	54.47	180.423	54.47	Flexural
		BD81	-	363.413	363.413	Shear
		TRC	270.741	281.303	270.741	Flexural
10'		BS5400	47.054	142.793	47.054	Flexural
		ACI-318	46.979	180.423	46.979	Flexural
		BD81	-	356.231	356.231	Shear
		TRC	230.638	268.464	230.638	Flexural
12'		BS5400	47.054	142.793	47.054	Flexural
		ACI-318	46.979	180.423	46.979	Flexural
		BD81	-	348.505	348.505	Shear
		TRC	195.642	254.731	195.642	Flexural
14'		BS5400	41.365	142.793	41.365	Flexural
		ACI-318	41.299	180.423	41.299	Flexural
		BD81	-	340.177	340.177	Shear
		TRC	164.916	239.992	164.916	Flexural

6.3 Discussion of results

From the results, certain interpretations can be drawn when the ultimate capacity values for different methods are compared with the respective spacing and the corresponding thicknesses. Figures 6.1 to 6.5 represent graphical interpretations for the comparison between methods.

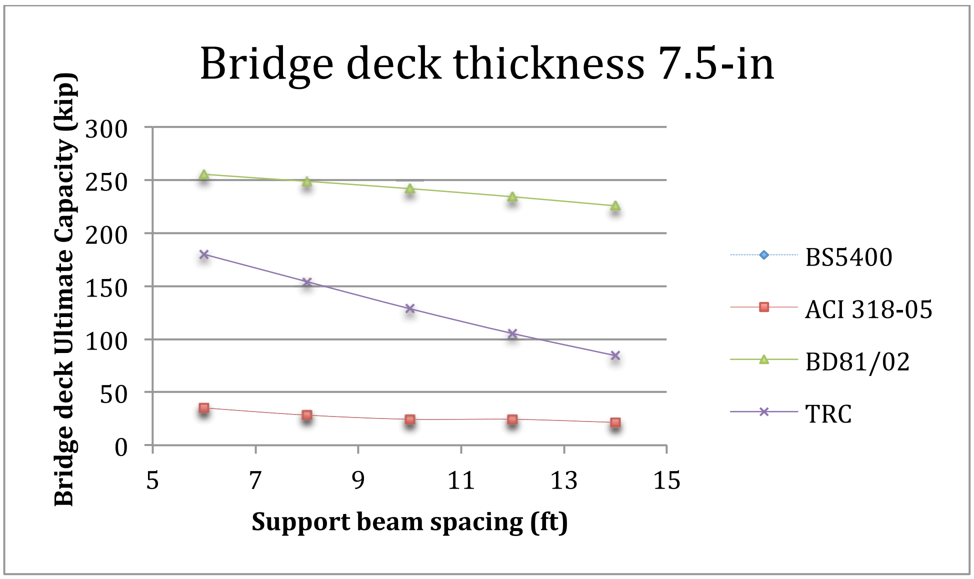

Figure 6.1 Varying Capacity due to support beam spacing using different methods with a 7.5-inch slab thickness.

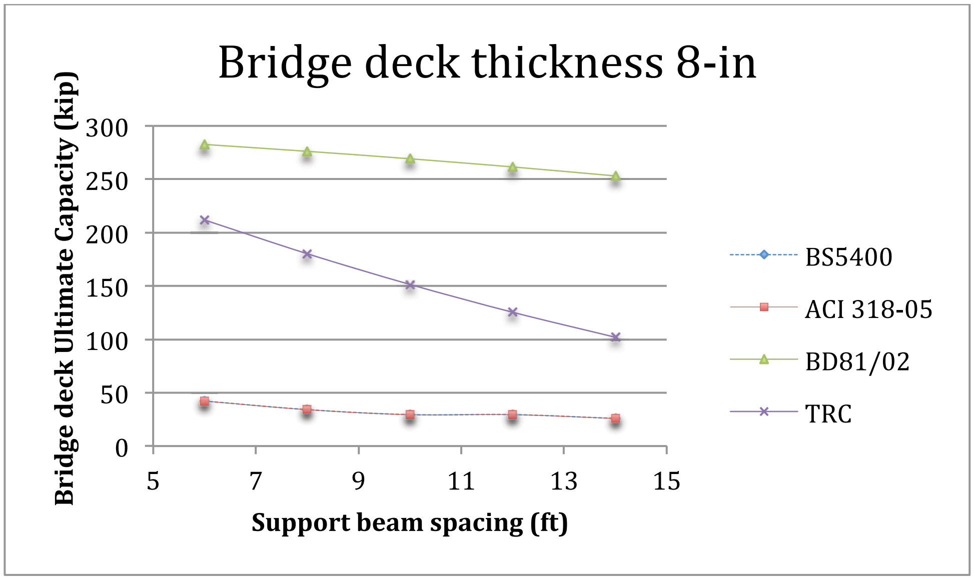

Figure 6.2 Varying Capacity due to support beam spacing using different methods with an 8-inch slab thickness.

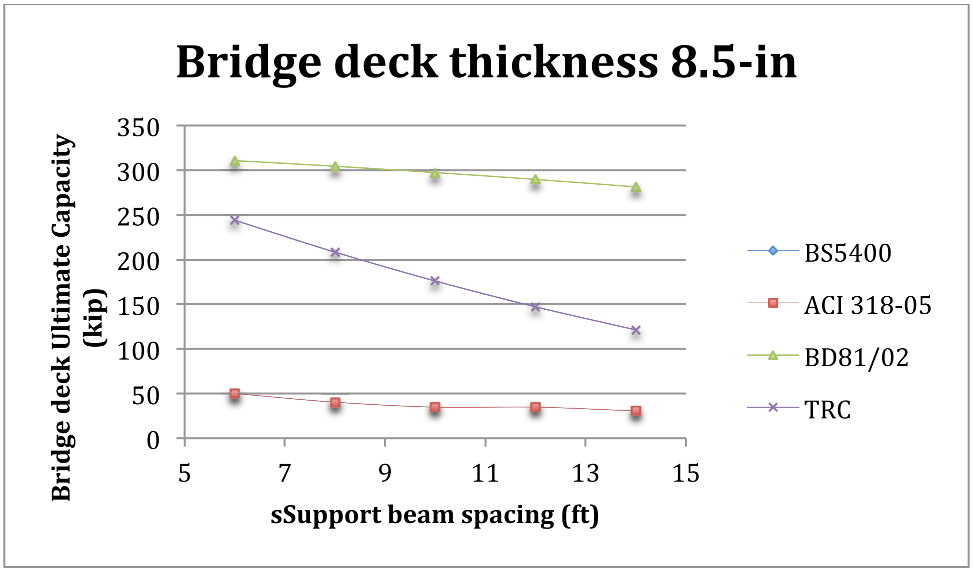

Figure 6.3 Varying Capacity due to support beam spacing using different methods with an 8.5-inch slab thickness.

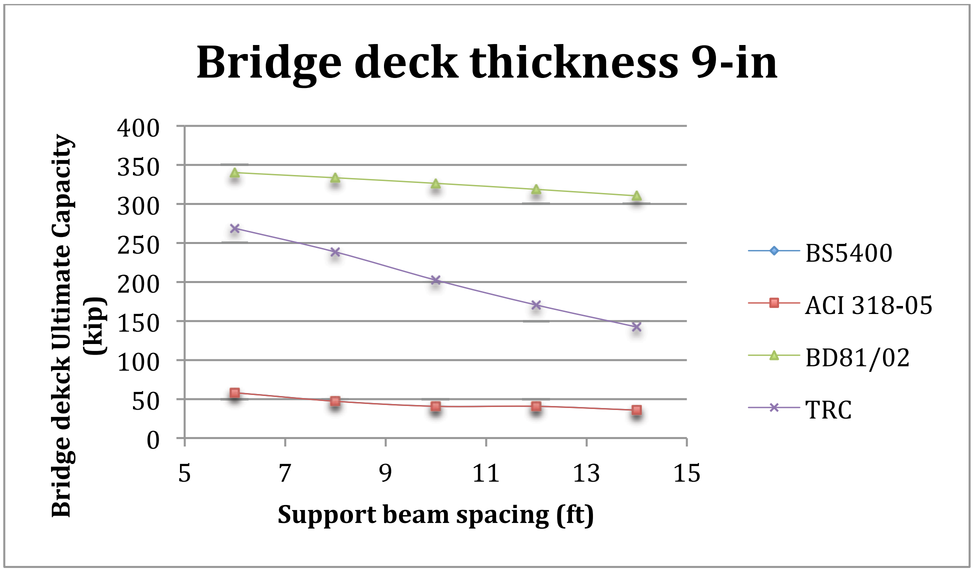

Figure 6.4 Varying Capacity due to support beam spacing using different methods with a 9-inch slab thickness.

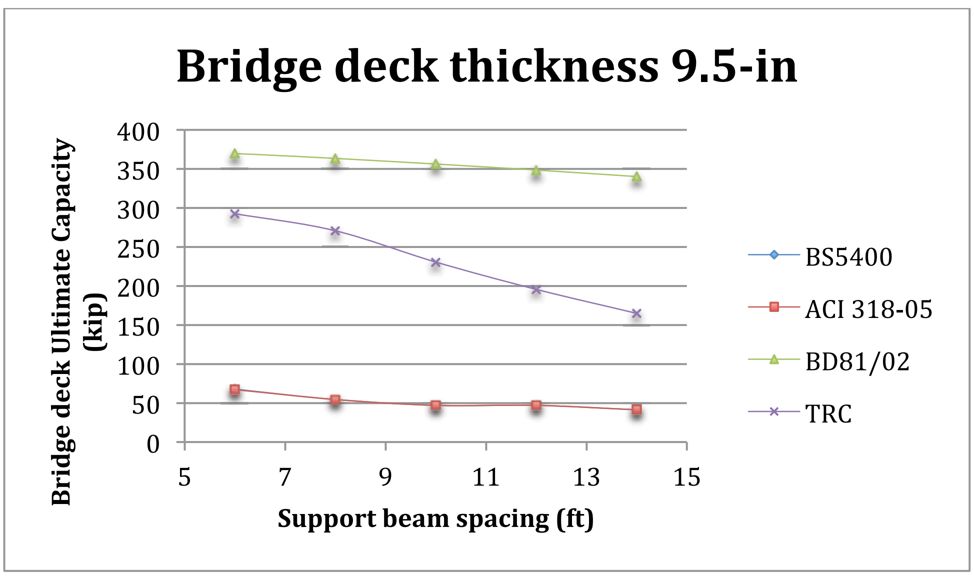

Figure 6.5 Varying Capacity due to support beam spacing using different methods with a 9.5-inch slab thickness.

When varying the slab thickness, it was observed that the TRC approach confirmed the behavior found in the research, where the TRC is more conservative than the UK BD81/02 method. However, the ACI 318-05 and the BS5400 standard codes are significantly more conservative than the BD81/02 and the TRC approach. This discrepancy can be attributed to the different factors that each method includes. For instance, even though ACI 318-05 and BS5400 methods take into account both the flexural and the shear punching capacity, the BD81/02 only takes into account the latter. However, the ACI 318-05 and BS5400 methods do not consider the spacing as the BD81/02 method does. Nevertheless, the TRC approach not only takes into consideration the flexural and shear punching, and the spacing between girders, but it also considers a series of different stiffness parameters that contribute to the development of the compressive membrane action. While the BD81/02 method also accounts for the compressive membrane action, it does not take into consideration the lateral restraint provided by the supporting beams, end diaphragms, and surrounding area, as does the TRC approach.

It was observed that when the support beam spacing increased, the predicted ultimate capacity decreased. This was more drastically observed for the TRC approach then for the other three methods. This can also be attributed to the factors considered in each method, since the TRC approach does take into account many more parameters than the other three methods.

When The American Concrete Institute (ACI 318-05) method is compared to the other methods, it was observed to yield the most conservative results. For the case of the BD81/02 method, the results overestimated the bridge deck slab ultimate capacity. This can be to the fact that this method has its own limitations such that it assumes that the bridge deck slab has an effectively rigid restraint system and that the mechanism of failure is always a punching shear mode (Taylor et al., 2007, p. 46).

Studies have shown that the TRC approach presents more precise predictions when compared to the experimental strengths obtained from practical testing (Taylor et al., 2003). This can be attributed to the fact that the method considers the variations of the external restraint stiffness.

In view of all things considered, it was determined that further analysis based on varying the external restraint factors using the TRC approach should be performed in this study. This analysis provided a more thorough characterization of the structural response to be expected of the bridge deck as the lateral stiffness of the supporting member is changed.

7 LATERAL STIFFNESS ANALYSIS

This chapter details the analysis performed in order to study the influence that different stiffness parameters have on the ultimate capacity of the bridge deck slab.

7.1 Analysis Parameters

This section introduces the parameters considered in order to analyze and evaluate the effect of lateral stiffness on bridge deck performance. The following parameters were analyzed using the developed tool based on the TRC approach only.

- 5 different deck slab thickness (7.5, 8, 8.5, 9, 9.5 inches)
- 5 different support beam spacing (6, 8, 10, 12, 14 feet)
- 5 different bridge span lengths (50, 60, 70, 80, 90 feet)
- 4 different types of girders (two reinforced concrete girders: FIB-36 and AASHTO Type III, and two steel W-shape girders: W44x335 and a Built-up steel girder)
- 2 steel reinforcement ratio (0.454% and 0.63%)
- 2 different compressive concrete strengths (4, 5 ksi)
- Reinforcement yield strength of 60ksi

Each beam accounted for different properties that were considered on the spreadsheet developed. Table 7.1 details the variables used to evaluate the bridge deck ultimate capacity.

Table 7.1
Support beam properties

	FIB-36	AASHTO TYPE III	W44X335	BUILT UP
CROSS SECTION AREA (in2)	806.58	560	98.5	106
Ix (in4)	127,564	125,390.35	31,100	99,734
Iy (in4)	81,131	12,216.56	1,200	2,884.55
Material	Concrete	Concrete	Steel	Steel
Modulus of Elasticity N/mm2 (ksi)	2.85E+04 (4.134E+03 ksi)	2.85E+04 (4.134E+03 ksi)	2.00E+05 (2.90E+04 ksi)	2.00E+05 (2.90E+04 ksi)
Rectangular load patch (in)	10x20	10x20	10x20	10x20

Considering the fixed and variable data, tables were created in order to evaluate the effect of lateral stiffness on bridge deck performance.

7.2 Analysis and discussion

The developed analysis tool was used to predict the mechanism of failure of the bridge deck slab. Due to the amount of data obtained, the results presented in Tables 7.2 to 7.5 describe only by the behavior developed on a 6-feet support beam spacing, 7.5-inch thickness on four different types of girders and two steel reinforcement ratio (0.454% and 0.63%). The additional work is shown in Appendix A.

Table 7.2

Ultimate capacity on a 7.5-inch slab thickness for a 6-foot support spacing on a FIB-36 – f'c= 5ksi

FIB- 36				Flexure	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	180.24	200.676	180.24	Flexural
60				60.165	180.615	200.801	180.615	Flexural
70				60.165	180.874	200.888	180.874	Flexural
80				60.165	181.064	200.951	181.064	Flexural
90				60.165	181.209	200.999	181.209	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	190.314	201.206	190.314	Flexural
60				82.118	190.644	201.317	190.644	Flexural
70				82.118	190.872	201.394	190.872	Flexural
80				82.118	191.039	201.45	191.039	Flexural
90				82.118	191.166	201.493	191.166	Flexural

Table 7.3

Ultimate capacity on an 7.5-inch slab thickness for a 6-foot support spacing on an AASHTO Type III – f'c= 5ksi

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	139.254	185.305	139.254	Flexural
60				60.165	139.351	185.346	139.351	Flexural
70				60.165	139.417	185.374	139.417	Flexural
80				60.165	139.466	185.395	139.466	Flexural
90				60.165	139.503	185.411	139.503	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	153.7	187.535	153.7	Flexural
60				82.118	153.788	187.572	153.788	Flexural
70				82.118	153.848	187.597	153.848	Flexural
80				82.118	153.892	187.615	153.892	Flexural
90				82.118	153.926	187.629	153.926	Flexural

Table 7.4

Ultimate capacity on an 7.5-inch slab thickness for a 6-foot support spacing on a W44x335– f’c= 5ksi

W44X335				Flexure	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	133.28	182.716	133.28	Flexural
60				60.165	133.57	182.75	133.57	Flexural
70				60.165	133.409	182.773	133.409	Flexural
80				60.165	133.447	182.79	133.447	Flexural
90				60.165	133.476	182.803	133.476	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	148.291	185.242	148.291	Flexural
60				82.118	148.36	185.272	148.36	Flexural
70				82.118	148.407	185.292	148.407	Flexural
80				82.118	148.442	185.307	148.442	Flexural
90				82.118	148.468	185.319	148.468	Flexural

Table 7.5

Ultimate capacity on an 7.5-inch slab thickness for a 6-foot support spacing on a steel Built-up section– f’c= 5ksi

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	6'	9.5"	0.454	121.501	268.668	278.08	268.668	Flexural
60				121.501	268.876	278.149	268.876	Flexural
70				121.501	269.018	278.197	269.018	Flexural
80				121.501	269.122	278.232	269.122	Flexural
90				121.501	269.201	278.258	269.201	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	165.833	297.975	281.601	281.601	Shear
60				165.833	298.16	281.662	281.662	Shear
70				165.833	298.287	281.703	281.703	Shear
80				165.833	298.38	281.734	281.734	Shear
90				165.833	298.45	281.757	281.757	Shear

7.2.1 Effect of bridge span length

Based on the results shown in Tables 7.2 to 7.5, it is possible to assess the effects of the different parameters on the bridge deck performance. The effect of varying the bridge span length on the slab's ultimate strength capacity was observed to be negligible. This is due to the fact that the span length of the bridge has very little influence on increasing stiffness in the transverse direction. This can also be observed on Figure 7.1 of the results using the FIB-36 as an example and a compressive concrete strength of 4ksi.

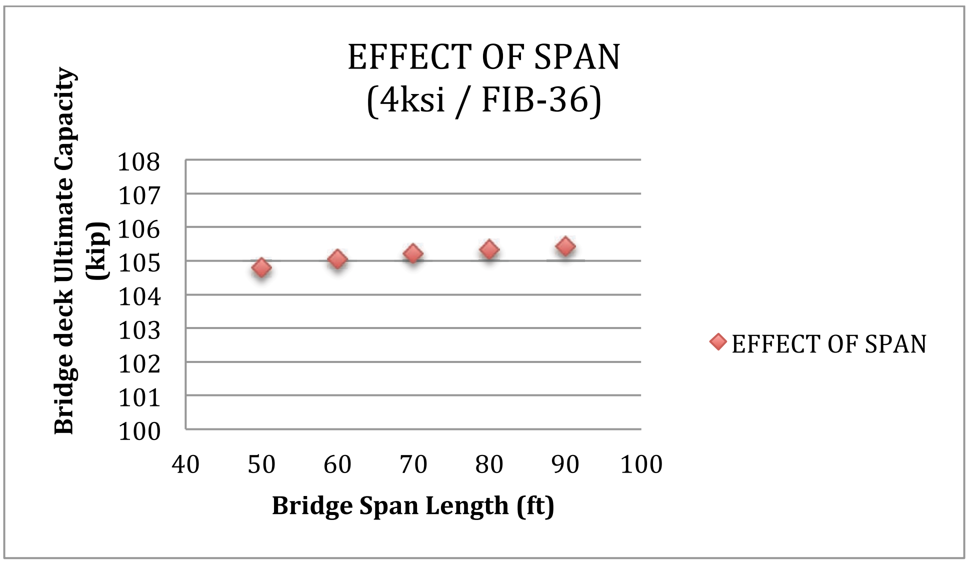

Figure 7.1 Effect of bridge span length on the bridge deck ultimate capacity

7.2.2 Effect of compressive concrete strength

A sample table (Table 7.6) was created to analyze the effect of the compressive concrete strength on the bridge decks slab supported by FIB-36 girder.

Table 7.6

Sample of the effect of compressive concrete strength on bridge deck ultimate capacity

EFFECT OF F_c					Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
f'c (ksi)	Length (ft)	Spacing (ft)	Thickness (in)	ρ empirical (%)	kip	kip	kip	
4	80	12'	7.5	0.45	92.898	146.676	92.898	Flexural
5					105.344	170.799	105.344	Flexural
8					119.593	224.327	119.593	Flexural

When varying the compressive concrete strength, the ultimate capacity increases approximately 10%. This can be attributed to the fact that increasing the compressive strength of the concrete results in a capacity increase on the portion of the slab that is subjected to compression. Hence, this results in an increase on the flexural punching capacity of the bridge deck slab. The shear punching capacity sees a similar increase.

7.2.3 Effect of support beam spacing and slab thickness

The spacing length had an inverse proportional relationship to the slab's ultimate capacity; the larger the spacing, the smaller the ultimate strength. However, the ultimate capacity was directly proportional to the slab thickness, i.e. the thicker the deck the higher the capacity. This can be attributed to the fact that increasing the spacing between support beams reduces the stiffness of the deck slab. Conversely, increasing the thickness on the deck slab augmented the stiffness, which in turn increased the ultimate strength capacity. This can be observed in Figure 7.2 shown below.

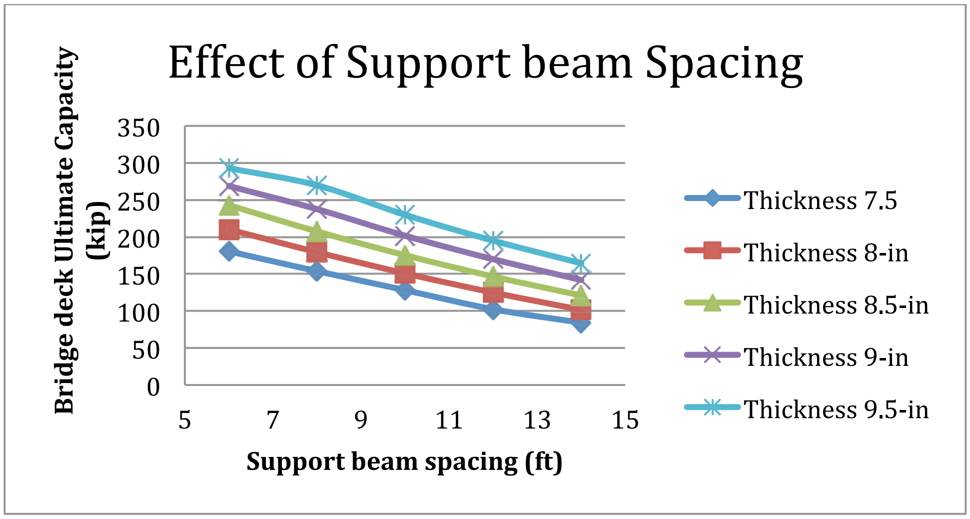

Figure 7.2 Effect of support beam spacing and slab thickness

7.2.4 Effect of steel reinforcement ratio

It was observed that increasing the steel reinforcement ratio did not result in a significant increase on the ultimate load capacity. This can be attributed to the development of arching action which enhanced the ultimate load capacity the bridge deck slab could carry on low steel reinforcement ratio as found in the literature review (Batchelor & Hewitt, 1976). Figures 7.3 to 7.10 are representative of the impact that the steel reinforcement ratio has on the bridge deck slab considering the varying support beam spacing and slab thickness when supported by different types of girder.

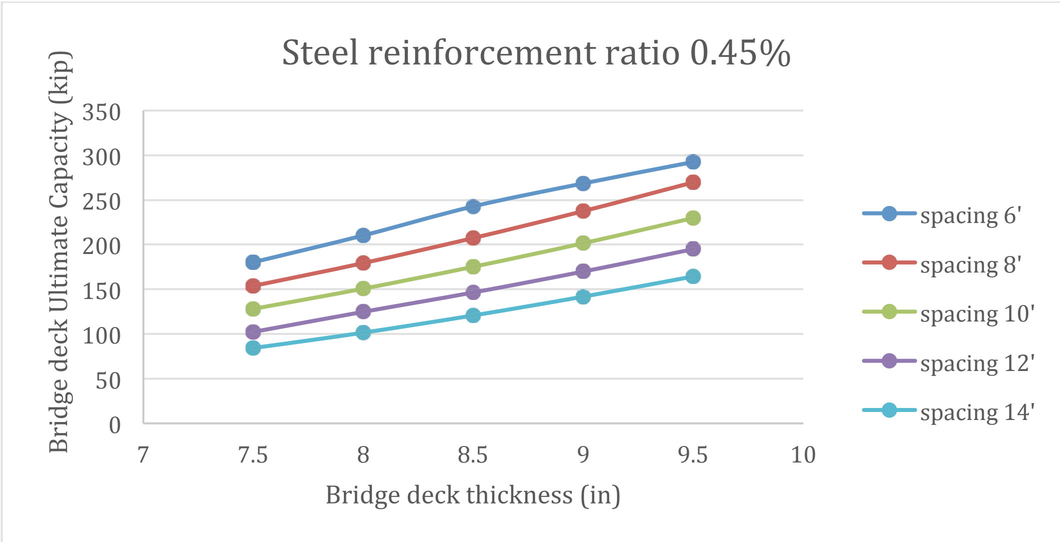

Figure 7.3 Effect of steel reinforcement ratio (ρ=0.45%) - FIB-36

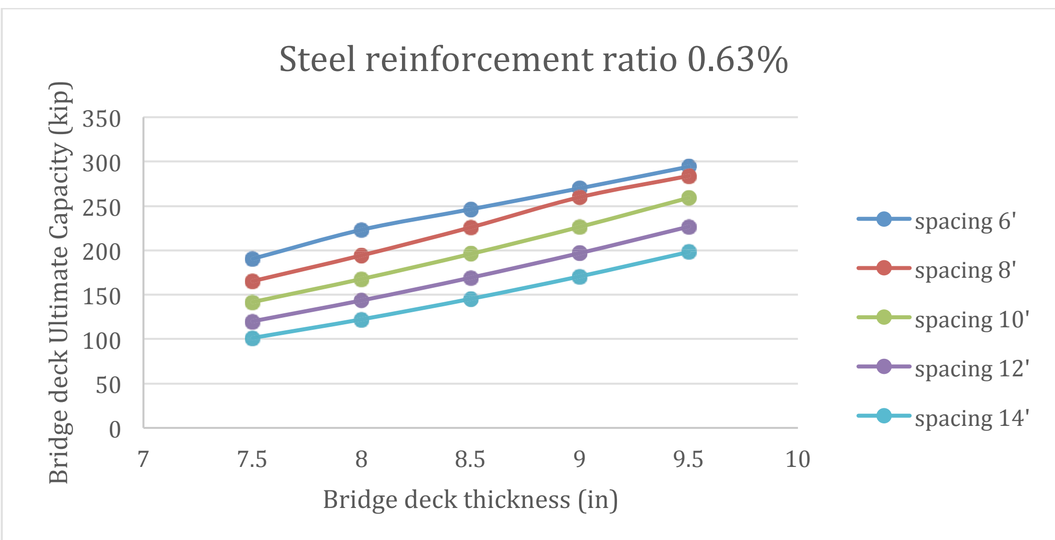

Figure 7.4 Effect of steel reinforcement ratio (ρ=0.63%) - FIB-36

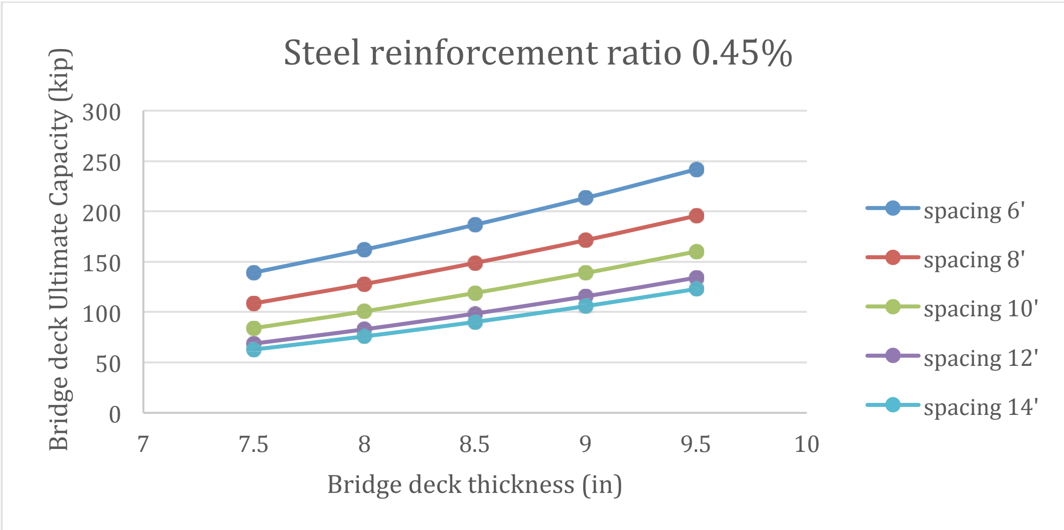

Figure 7.5 Effect of steel reinforcement ratio (ρ=0.45%) – AASHTO TYPE III

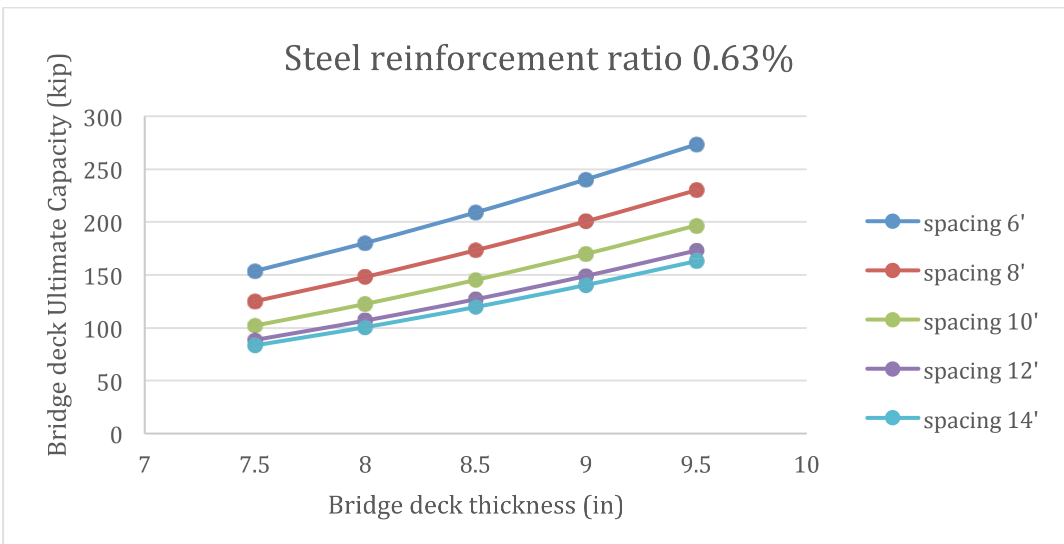

Figure 7.6 Effect of steel reinforcement ratio (ρ=0.63%) – AASHTO TYPE III

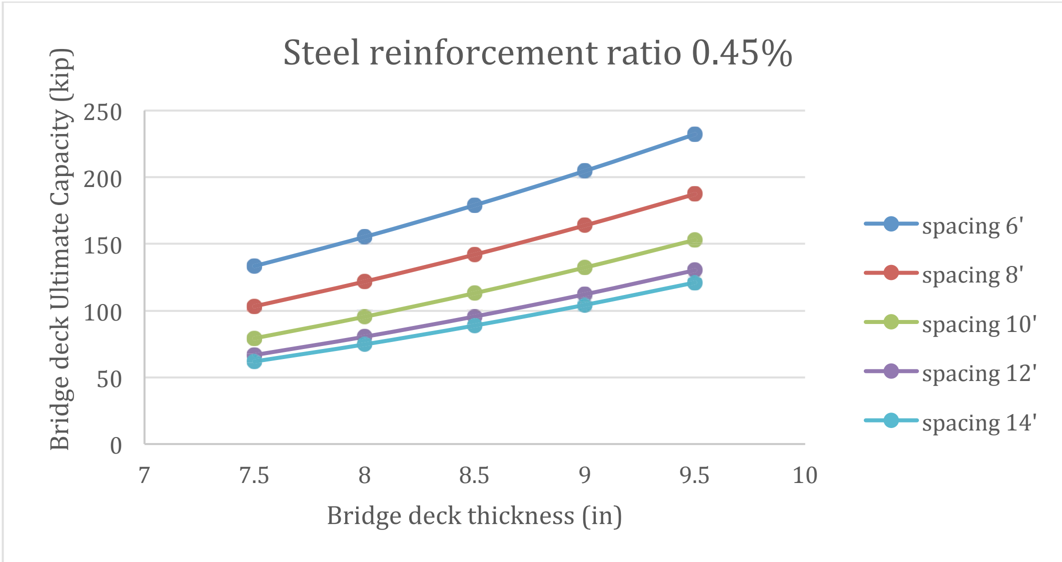

Figure 7.7 Effect of steel reinforcement ratio (ρ=0.45%) – W44x335

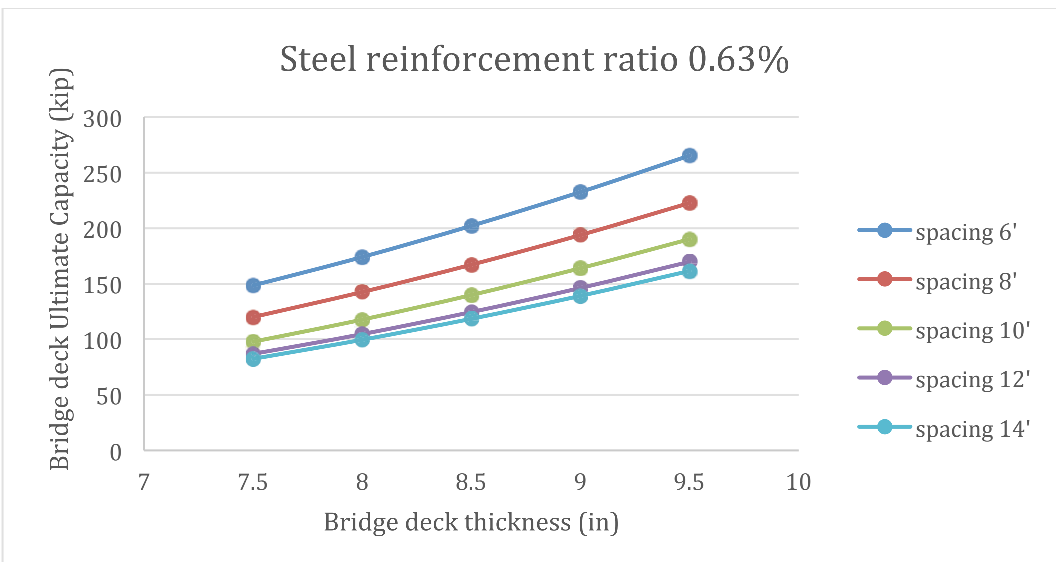

Figure 7.8 Effect of steel reinforcement ratio (ρ=0.63%) – W44x335

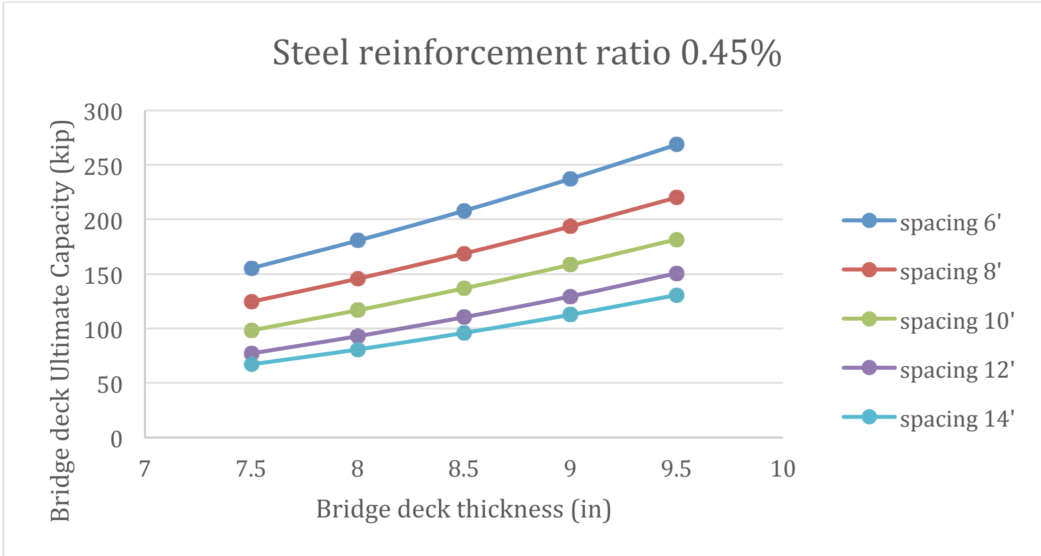

Figure 7.9 Effect of steel reinforcement ratio (ρ=0.45%) – Steel Built-up section

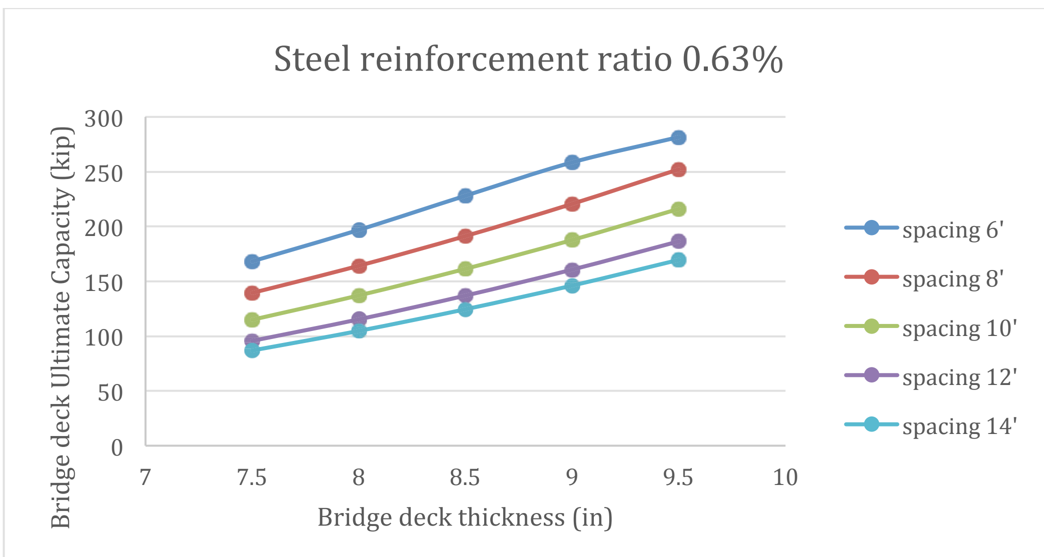

Figure 7.10 Effect of steel reinforcement ratio (ρ=0.63%) – Steel Built-up section

It can be observed that the bridge deck ultimate capacity does not vary significantly within the same type of girder when increasing the reinforcement ratio. When comparing the types of girder, the ultimate load capacity varies. This can be attributed to the stiffness each type of girder provides to the slab.

7.2.5 Effect of support beam stiffness

The total axial stiffness of the slab accounts for the area of the diaphragms and the surrounding area of the slab resisting the outward arching thrust given by K_d. On the other hand, the width and material of the support beams have significant influence on the deck slab by means of the beams lateral stiffness, quantified in this approach through $K_{b.}$ The combined flexibility of the total restraint considering the stiffness of the slab and the support beams is represented by K_r.

The relationship of each restraint stiffness can be seen as that of an equivalent spring to two springs set in series with one another. In this manner, equation 5.19 illustrates this relationship. Further analyses were done in order to represent how the bridge deck slab thickness in combination with the type and material of the girders affect the lateral stiffness of the deck. The following Tables 7.7 to 7.11 show this in detail.

Table 7.7
Support beam lateral stiffness Kb (kip/in) for bridge deck slab thickness 7.5-inches

THICKNESS 7.5"										
SPAN 50'										
	Kb6 (kip/in)	UC 6 (kip)	Kb8 (kip/in)	UC 8 (kip)	Kb10 (kip/in)	UC 10 (kip)	Kb12 (kip/in)	UC 12 (kip)	Kb14 (kip/in)	UC 14 (kip)
W44X335	8730	133.28	3868	103.23	2041	79.27	1205	66.7	769.77	61.88
AASHTO	11,040	139.254	4891	108.67	2580	84.06	1523	68.8	973.16	62.88
BUILT STEEL	20990	155.352	9299	124.35	4905	98.30	2896	77.2	1850	66.97
FIB 36	73300	180.24	32480	153.41	17130	128.07	10120	104.8	6463	84.10

Table 6.8
Support beam lateral stiffness Kb (kip/in) for bridge deck slab thickness 8-inches

THICKNESS 8"										
SPAN 50'										
	Kb6 (kip/in)	UC 6 (kip)	Kb8 (kip/in)	UC 8 (kip)	Kb10 (kip/in)	UC 10 (kip)	Kb12 (kip/in)	UC 12 (kip)	Kb14 (kip/in)	UC 14 (kip)
W44X335	8399	155.2	3756	121	1992	95.419	1181	80.44	756.57	74.72
AASHTO	10620	162.1	4748	127	2519	100.77	1493	82.93	956.47	75.90
BUILT STEEL	20190	180.7	9028	145	4789	116.73	2839	92.95	1819	80.78
FIB 36	70510	210.3	31530	179	16730	150.74	9915	124.8	6352	101.5

Table 7.9

Support beam lateral stiffness Kb (kip/in) for bridge deck slab thickness 8.5-inches

THICKNESS 8.5"										
SPAN 50'										
	Kb6 (kip/in)	UC 6 (kip)	Kb8 (kip/in)	UC 8 (kip)	Kb10 (kip/in)	UC 10 (kip)	Kb12 (kip/in)	UC 12 (kip)	Kb14 (kip/in)	UC 14 (kip)
W44X335	8084	179.1	3647	142.0	1946	113.1	1158	95.6	743.676	88.8
AASHTO	10220	186.8	4611	148.8	2460	119.0	1463	98.5	940.169	90.2
BUILT STEEL	19430	208.0	8768	168.7	4677	136.8	2783	110.4	1788	96.0
FIB 36	67870	242.8	30620	207.4	16340	175.3	9719	146.5	6244	120.8

Table 7.10

Support beam lateral stiffness Kb (kip/in) for bridge deck slab thickness 9-inches

THICKNESS 9"										
SPAN 50'										
	Kb6 (kip/in)	UC 6 (kip)	Kb8 (kip/in)	UC 8 (kip)	Kb10 (kip/in)	UC 10 (kip)	Kb12 (kip/in)	UC 12 (kip)	Kb14 (kip/in)	UC 14 (kip)
W44X335	7784	204.7	3543	163.9	1900	132.3	1135	112.2	731.071	104.30
AASHTO	9841	213.3	4479	171.4	2403	138.8	1435	115.6	924.235	105.91
BUILT STEEL	18710	237.3	8517	193.6	4668	158.5	2728	129.4	1757	112.62
FIB 36	65360	268.5	29750	237.5	15960	201.6	9529	169.9	6138	141.84

Table 7.11

Support beam lateral stiffness Kb (kip/in) for bridge deck slab thickness 9.5-inches

THICKNESS 9.5"										
SPAN 50'										
	Kb6 (kip/in)	UC 6 (kip)	Kb8 (kip/in)	UC 8 (kip)	Kb10 (kip/in)	UC 10 (kip)	Kb12 (kip/in)	UC 12 (kip)	Kb14 (kip/in)	UC 14 (kip)
W44X335	7500	232.2	3443	187.4	1857	153.1	1113	130.3	718.75	121.1
AASHTO	9481	241.8	4353	195.7	2347	160.2	1407	134.2	908.658	123.0
BUILT STEEL	18030	268.7	8276	220.2	4463	181.8	2675	150.2	1728	130.7
FIB 36	62970	292.5	28910	269.7	15590	229.8	9343	195.0	6034	164.4

Given the objectives of this study, it becomes important to present the relationship between the lateral stiffness of the supporting beam, K_b, and the bridge deck's ultimate capacity. For this purpose a graphic representation was conceived such that the deck thickness was a constant and the spacing between girders varied, effectively granting a means of comparison between the lateral stiffness of the supporting beam and the ultimate capacity. This can be observed in Figures 7.11 through 7.15 the support beam spacing values considered in this study ranged from 6 feet to 14 feet. The bridge span length was not taken into account as a varying parameter for this evaluation.

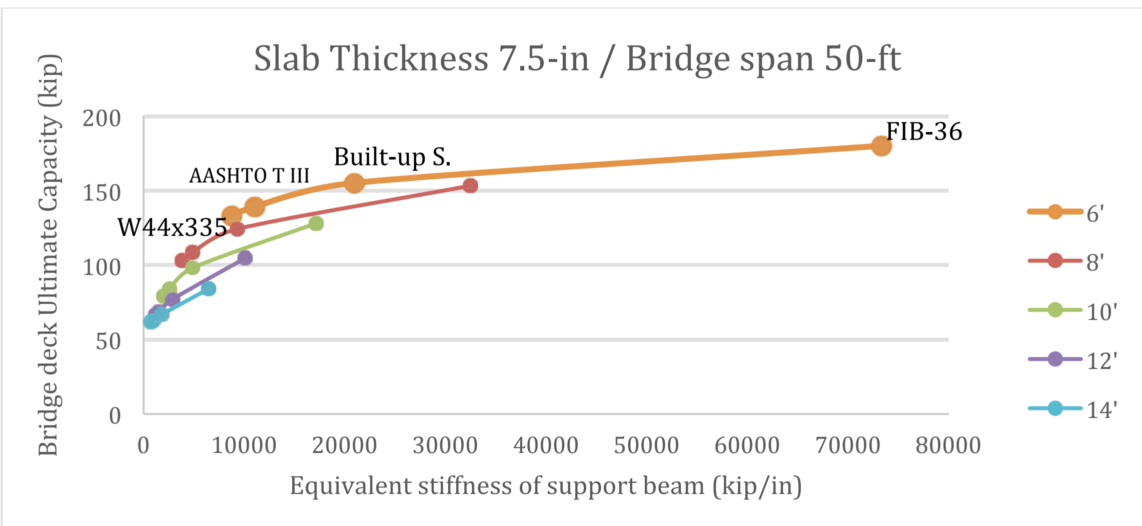

Figure 7.11 Relationship between Ultimate capacity and equivalent stiffness of support beam-Thickness 7.5-inch

The results from the data showed that the FIB-36 reached higher ultimate capacity and provided greater stiffness to the bridge deck slab. The thicker the slab, the ultimate capacity and the equivalent stiffness from the support beams increased as shown in Figures 7.12 to 7.15.

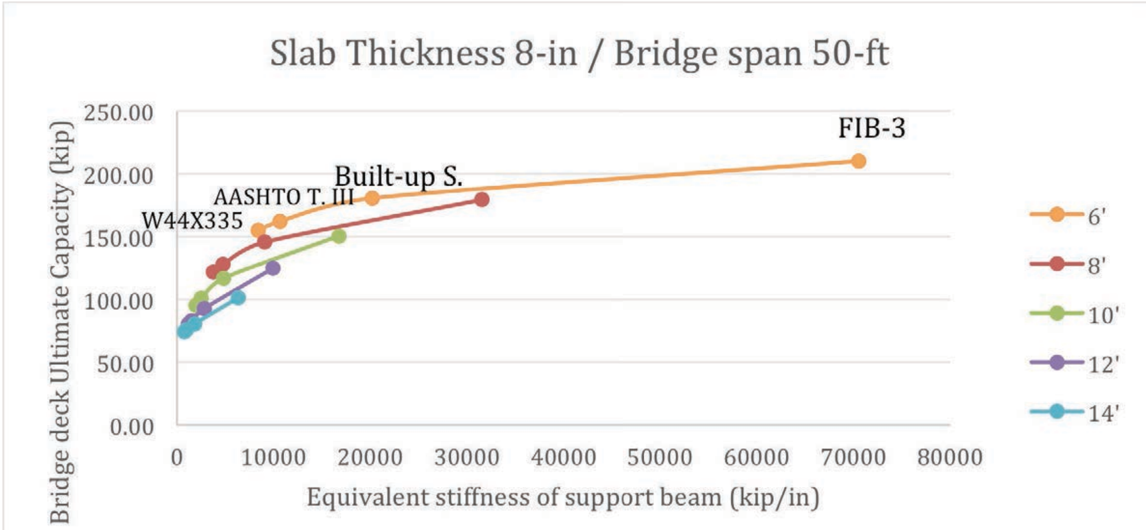

Figure 7.12 Relationship between Ultimate capacity and equivalent stiffness of support beam-Thickness 8-inch

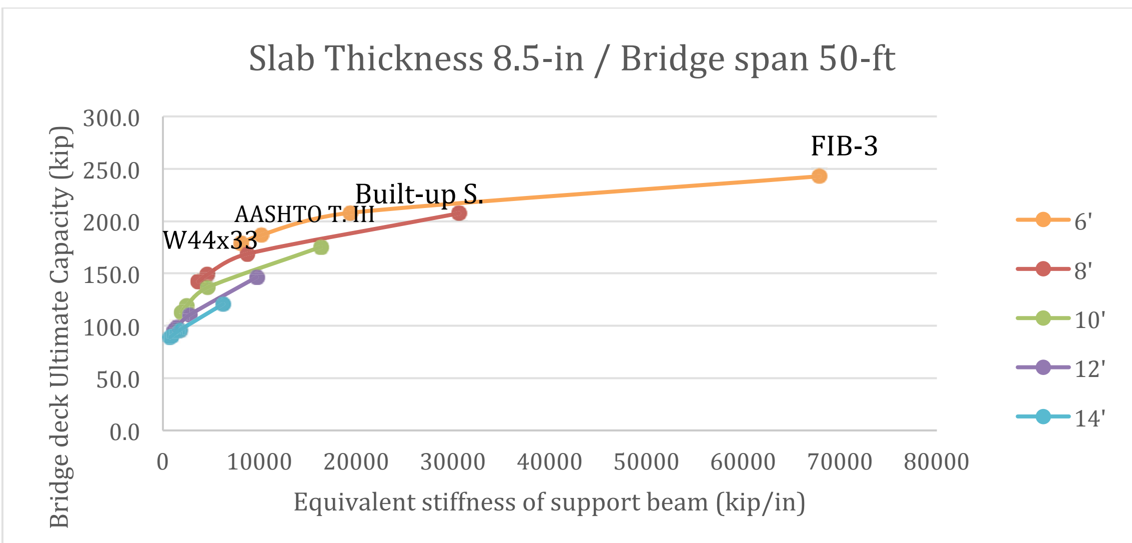

Figure 7.13 Relationship between Ultimate capacity and equivalent stiffness of support beam-Thickness 8.5-inch

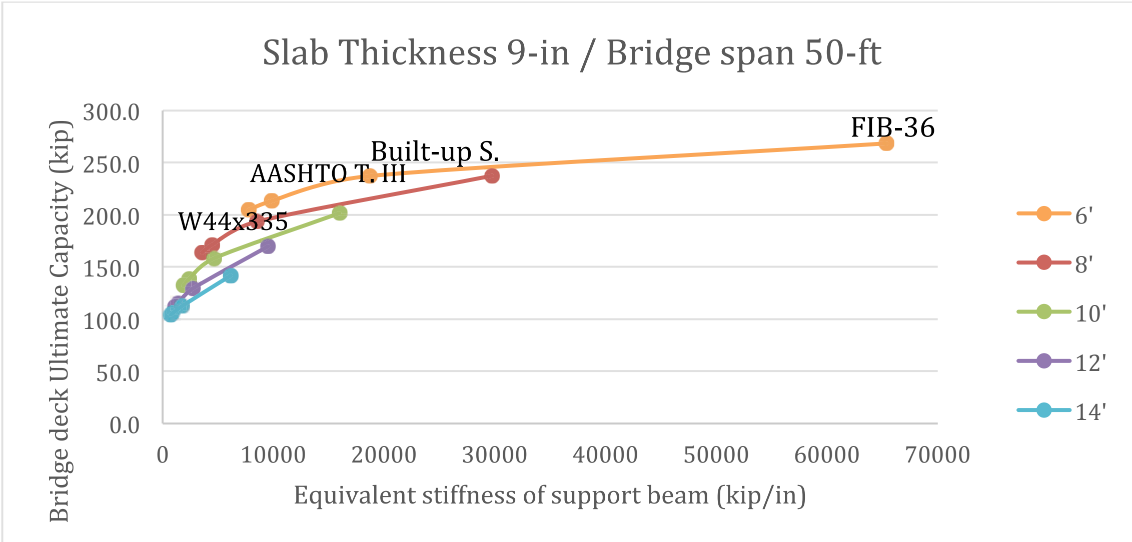

Figure 7.14 Relationship between Ultimate capacity and equivalent stiffness of support beam-Thickness 9-inch

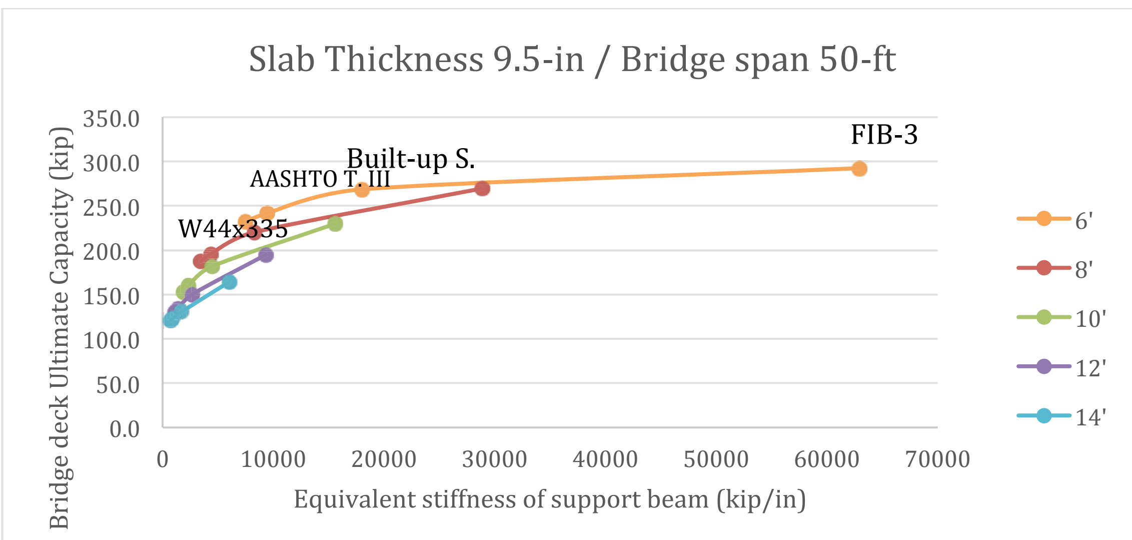

Figure 7.15 Relationship between Ultimate capacity and equivalent stiffness of support beam-Thickness 9.5-inch

After the evaluation, it was observed that Figures 7.11 to 7.15 above could serve as a design criterion based on the construction characteristics of the deck, the type of girder to be used, and the support beam spacing. The design assumes the support beam spacing in order to obtain the equivalent stiffness of the girder as shown in Eqn. 5.16. Knowing these two variables the chart can help estimate the bridge deck ultimate capacity. This is better explained in the

following example.

EXAMPLE:

Considering the following input for evaluation:

Bridge span length:	80ft
Spacing:	8ft
Deck thickness:	8.5
Fy:	60ksi
f'c:	5ksi
ρ	0.454%
d	0.5ft

Assuming a support beam spacing of 8ft and using an FIB-36, the equivalent stiffness of the support beam can be estimated as shown bellow.

$$I_{yb} := 81131\,\text{in}^4$$

- Second moment of area of support beam about the vertical axis

$$\zeta := \begin{cases} 114.5 & \text{if Support} = 1 \\ 985 & \text{otherwise} \end{cases}$$

- Constant for support condition

$$A_b := \zeta \cdot L_e \cdot \frac{I_{yb}}{{b_{eff}}^3}$$

- Equivalent area of support beam

$$A_b = 2.084 \times 10^5 \cdot \text{mm}^2$$

$$K_b = 3.062 \times 10^4 \cdot \frac{\text{kip}}{\text{in}}$$

- Equivalent stiffness of support beam

Figure 7.18 can be used by assuming the spacing and the equivalent stiffness of the support beam estimated and designing for a deck thickness of 8.5-inches as shown below in Figure 7.19.

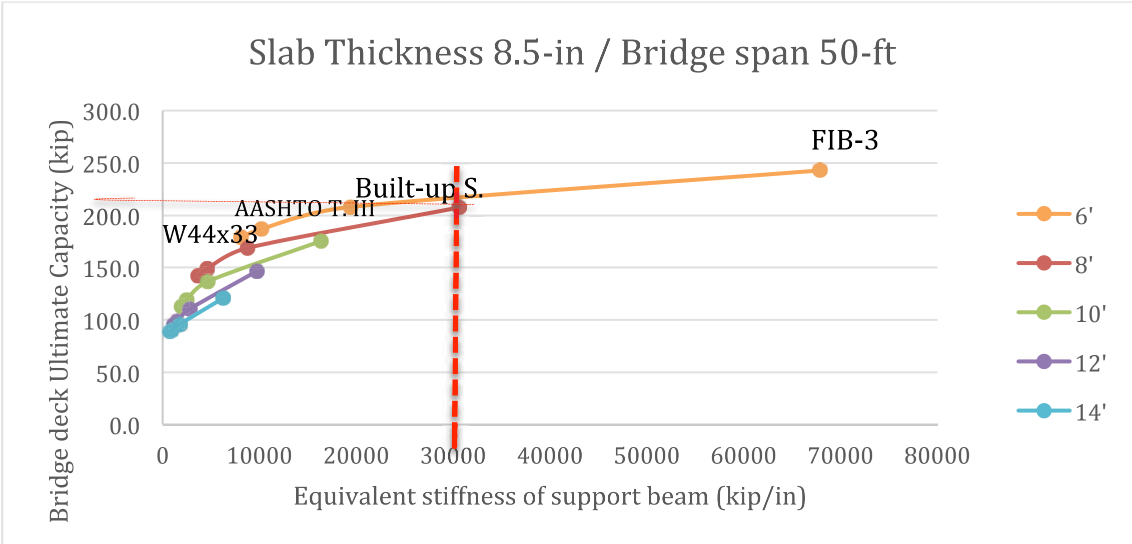

Figure 7.16 Example- Thickness 8.5-inch

Figure 7.16 show that the estimated bridge deck ultimate capacity is approximately 210 kip. The results from the developed spreadsheet show that bridge deck ultimate capacity is 208.3 kip. This can be further developed for better estimations, but it presents approximations close to the predicted results from the calculations.

8 CONCLUSIONS

This research study investigated the arching action developed in a bridge deck under loading. The effect of support beam lateral stiffness on empirical deck performance was also investigated along with several other parameters. The variables included deck thickness, deck concrete compressive strength, beam spacing, beam span length, steel reinforcement ratio, and support beam type and material. This chapter includes the conclusions drawn and the recommendations for future research.

8.1 Conclusions

- The support beam lateral stiffness has a direct relationship with the ultimate capacity of the bridge deck slab as shown in Figures 7.14 through 7.18. The resulting charts illustrate the relationship between support beam lateral stiffness and the bridge deck ultimate capacity, in separate series of varying spacing. These charts can be used for design. Knowing the support beam lateral stiffness, one can estimate the required spacing in order to achieve a given ultimate capacity for the bridge deck slab.
- It was observed that the FIB-36 girder contributed to a higher lateral stiffness when compared to the other girders (AASHTO type III, steel built-up section, and W44x335). This resulted in greater lateral restraint that allowed the compressive membrane action to further develop, which consequently increased the ultimate load capacity of the bridge deck slab.
- The predicted ultimate capacity estimated using ACI 318-05 and BS5400 methodologies, when predicting the ultimate capacity, remained highly conservative when varying the

support beam spacing. This can be attributed to the fact that they don't consider the compressive membrane action in the ultimate capacity prediction and calculation. Even though the BD81/02 and the TRC approaches take into account the compressive membrane action, only the TRC approach takes into consideration a greater amount number of properties in determining the ultimate capacity of the bridge deck slab that results in better predictions when compared to the real test loads from research.

- The concrete compressive strength (F_c) using the TRC approach had a significant effect on the ultimate capacity of the bridge deck slab. As the F_c increased, so did the ultimate capacity. Increasing the compressive strength of the concrete, results in a capacity increase on the portion of the slab that is subjected to compression. Hence, this results in an increase on the bridge deck ultimate capacity.
- The TRC approach used to determine ultimate strength capacity uses the lesser value between the flexural punching and shear punching capacity. Through the performed analysis, it was observed that when considering a small spacing, an increase to the steel reinforcement ratio would give a proportional increment to the flexural punching capacity. This relationship implies that the failure type mode for these cases could most likely be identified as the shear punching capacity. Increasing the steel reinforcement ratio increases the flexural punching capacity and would have no effect if the shear punching capacity which would the critical of the two failure modes.
- Varying the bridge span length under fixed supporting beam spacing had little to no impact on the ultimate bridge deck capacity. This is due to the fact that the span length of the bridge has very little influence on increasing stiffness in the transverse direction.
- Upon analyzing the results, it was observed that when increasing the support beam spacing

under a fixed deck slab thickness, the deck ultimate strength decreases. This can be attributed to the fact that increasing the spacing between support beams reduces the stiffness of the deck slab. Conversely, increasing the thickness on the deck slab augmented the stiffness, which in turn increased the ultimate strength capacity.

8.2 Recommendations

- Further research will be needed to properly assess the effect of the support beam torsional rigidity on the ultimate capacity.
- It is recommended to further develop the design criterion tool by considering more than the four girders studied in this research.
- For further verification, this research should be tested and compared to the predictions analyzed in this study.

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Appendix A: Effects of different variables in the Ultimate Capacity Using an FIB-36 girder.

A. Ultimate load capacity using FIB-36/Thickness 7.5-inch

f'c=5ksi	FIB-36							
FIB- 36				Flexure	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	180.24	200.676	180.24	Flexural
60				60.165	180.615	200.801	180.615	Flexural
70				60.165	180.874	200.888	180.874	Flexural
80				60.165	181.064	200.951	181.064	Flexural
90				60.165	181.209	200.999	181.209	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	190.314	201.206	190.314	Flexural
60				82.118	190.644	201.317	190.644	Flexural
70				82.118	190.872	201.394	190.872	Flexural
80				82.118	191.039	201.45	191.039	Flexural
90				82.118	191.166	201.493	191.166	Flexural

FIB- 36				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	7.5"	0.454	59.189	153.768	192.152	153.768	Flexural
60				59.189	154.011	192.246	154.011	Flexural
70				59.189	154.188	192.314	154.188	Flexural
80				59.189	154.188	192.314	154.188	Flexural
90				59.189	154.321	192.365	154.321	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	80.784	165.316	193.24	165.316	Flexural
60				80.784	165.633	193.633	165.633	Flexural
70				80.784	165.849	193.445	165.849	Flexural
80				80.784	166.005	193.506	166.005	Flexural
90				80.784	166.124	193.551	166.124	Flexural

FIB- 36				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	7.5"	0.454	58.602	128.069	181.916	128.069	Flexural
60				58.602	128.377	182.057	128.377	Flexural
70				58.602	128.583	182.152	128.583	Flexural
80				58.602	128.731	182.22	128.731	Flexural
90				58.602	128.842	182.271	128.842	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.985	141.76	184.042	141.76	Flexural
60				79.985	142.036	184.167	142.036	Flexural
70				79.985	142.22	184.25	142.22	Flexural
80				79.985	142.353	184.31	142.353	Flexural
90				79.985	142.452	184.355	142.452	Flexural

FIB- 36				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	7.5"	0.454	58.212	104..802	170.495	102.306	Flexural
60				58.212	105.057	170.638	102.51	Flexural
70				58.212	105.22	170.733	102.659	Flexural
80				58.212	105.344	170.799	102.763	Flexural
90				58.212	105.433	170.849	102.839	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.451	120.099	173.788	120.099	Flexural
60				79.451	120.329	173.912	120.329	Flexural
70				79.451	120.481	173.994	120.481	Flexural
80				79.451	120.588	174.052	120.588	Flexural
90				79.451	120.668	174.095	120.668	Flexural

FIB- 36				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	7.5"	0.454	57.933	84.101	157.696	84.101	Flexural
60				57.933	84.283	157.826	84.283	Flexural
70				57.933	84.401	157.91	84.401	Flexural
80				57.933	84.484	157.969	84.484	Flexural
90				57.933	84.545	158.013	84.545	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.071	101.329	162.93	101.329	Flexural
60				79.071	101.482	163.031	101.482	Flexural
70				79.071	101.582	163.096	101.582	Flexural
80				79.071	101.651	163.142	101.651	Flexural
90				79.071	101.703	163.175	101.703	Flexural

B. Ultimate load capacity using FIB-36/ Thickness 8-inch

f'c=5ksi	FIB-36							
FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	8"	0.454	73.262	210.3	222.585	210.3	Flexural
60				73.262	210.714	222.718	210.714	Flexural
70				73.262	210.999	222.809	210.999	Flexural
80				73.262	211.209	222.876	211.209	Flexural
90				73.262	211.369	222.927	211.369	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	99.992	223.193	223.357	223.193	Flexural
60				125.631	223.556	223.474	223.474	Shear
70				125.631	223.806	223.554	223.554	Shear
80				125.631	223.99	223.613	223.613	Shear
90				125.631	224.13	223.658	223.658	Shear

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	8"	0.454	71.853	179.377	213.231	179.377	Flexural
60				71.853	179.762	213.374	179.762	Flexural
70				71.853	180.025	213.471	180.025	Flexural
80				71.853	180.215	213.541	180.215	Flexural
90				71.853	180.259	213.595	180.259	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	98.069	194.351	214.808	194.351	Flexural
60				98.069	194.692	214.934	194.692	Flexural
70				98.069	194.925	215.02	194.925	Flexural
80				98.069	195.082	215.082	195.082	Flexural
90				98.069	195.221	215.129	195.221	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	8"	0.454	71.007	150.746	202.471	150.746	Flexural
60				71.007	151.074	202.615	151.074	Flexural
70				71.007	151.294	202.711	151.294	Flexural
80				71.007	151.451	202.78	151.451	Flexural
90				71.007	151.57	202.831	151.57	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.916	167.737	205.068	167.737	Flexural
60				96.916	168.03	205.194	168.03	Flexural
70				96.916	168.226	205.278	168.226	Flexural
80				96.916	168.367	205.339	168.367	Flexural
90				96.916	168.472	205.384	168.472	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	8"	0.454	70.444	124.813	190.498	124.813	Flexural
60				70.444	125.083	190.641	125.083	Flexural
70				70.444	125.26	190.735	125.26	Flexural
80				70.444	125.385	190.801	125.385	Flexural
90				70.444	125.479	190.851	125.479	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.146	143.604	194.383	143.604	Flexural
60				96.146	143.846	194.506	143.846	Flexural
70				96.146	144.005	194.587	144.005	Flexural
80				96.146	144.118	194.644	144.118	Flexural
90				96.146	144.202	194.687	144.202	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	8"	0.454	70.041	101.529	177.078	101.529	Flexural
60				70.041	101.735	177.216	101.735	Flexural
70				70.041	101.869	177.304	101.869	Flexural
80				70.041	101.963	177.366	101.963	Flexural
90				70.041	102.032	177.412	102.032	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	95.597	122.258	182.905	122.258	Flexural
60				95.597	122.432	183.011	122.432	Flexural
70				95.597	122.544	183.08	122.544	Flexural
80				95.597	122.622	181.128	122.622	Flexural
90				95.597	122.68	183.164	122.68	Flexural

C. Ultimate load capacity using FIB-36/ Thickness 8.5-inch

f'c=5ksi	FIB-36							
FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	8.5"	0.454	87.821	242.831	245.2	242.831	Flexural
60				87.821	243.283	245.34	243.283	Flexural
70				87.821	243.595	245.436	243.595	Flexural
80				87.821	243.824	245.506	243.824	Flexural
90				87.821	243.999	245.56	243.999	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	119.863	258.949	246.248	246.248	Shear
60				119.863	259.345	246.371	246.371	Shear
70				119.863	259.619	246.455	246.455	Shear
80				119.863	259.819	246.517	246.517	Shear
90				119.863	259.972	246.564	246.564	Shear

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	8.5"	0.454	85.875	207.414	235.131	207.414	Flexural
60				85.875	207.827	235.278	207.827	Flexural
70				85.875	208.109	235.379	208.109	Flexural
80				85.875	208.312	235.451	208.312	Flexural
90				85.875	208.467	235.506	208.467	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	117.208	225.862	237.099	225.862	Flexural
60				117.208	226.228	237.228	226.228	Flexural
70				117.208	226.477	237.316	226.477	Flexural
80				117.208	226.657	237.38	226.657	Flexural
90				117.208	226.793	237.428	226.793	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	8.5"	0.454	84.708	175.259	223.695	175.259	Flexural
60				84.708	175.606	223.84	175.606	Flexural
70				84.708	175.839	223.938	175.839	Flexural
80				84.708	176.005	224.007	176.005	Flexural
90				84.708	176.13	224.06	176.13	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	115.615	195.951	226.804	195.951	Flexural
60				115.615	196.261	226.931	196.261	Flexural
70				115.615	196.469	227.016	196.469	Flexural
80				115.615	196.617	227.077	196.617	Flexural
90				115.615	196.728	227.122	196.728	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	8.5"	0.454	83.93	146.51	211.153	146.51	Flexural
60				83.93	146.793	211.296	146.793	Flexural
70				83.93	146.979	211.39	146.979	Flexural
80				83.93	147.111	211.456	147.111	Flexural
90				83.93	147.208	211.505	147.208	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	114.553	169.194	215.672	169.194	Flexural
60				114.553	169.448	215.795	169.448	Flexural
70				114.553	169.615	215.875	169.615	Flexural
80				114.553	169.733	215.931	169.733	Flexural
90				114.553	169.82	215.974	169.82	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	8.5"	0.454	83.374	120.794	197.264	120.794	Flexural
60				83.374	121.027	197.408	121.027	Flexural
70				83.374	121.177	197.502	121.177	Flexural
80				83.374	121.282	197.567	121.282	Flexural
90				83.374	121.359	197.615	121.359	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	113.794	145.353	203.692	145.353	Flexural
60				113.794	145.547	203.803	145.547	Flexural
70				113.794	145.672	203.875	145.672	Flexural
80				113.794	145.76	203.925	145.76	Flexural
90				113.794	145.825	203.962	145.825	Flexural

D. Ultimate load capacity using FIB-36/ Thickness 9-inch

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	9"	0.454	103.886	277.844	268.515	268.515	Shear
60				103.886	278.335	268.661	268.661	Shear
70				103.886	278.674	268.762	268.762	Shear
80				103.886	278.921	268.835	268.835	Shear
90				103.886	279.111	268.891	268.891	Shear
			ρ traditional	kip	kip	kip	kip	
50			0.63	141.79	297.617	269.874	269.874	Shear
60				141.79	298.046	270.002	270.002	Shear
70				141.79	298.343	270.09	270.09	Shear
80				141.79	298.559	270.154	270.154	Shear
90				141.79	298.725	270.203	270.203	Shear

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	9"	0.454	101.289	237.529	257.71	237.529	Flexural
60				101.289	237.969	257.862	237.969	Flexural
70				101.289	238.268	257.965	238.268	Flexural
80				101.289	238.485	258.039	238.485	Flexural
90				101.289	238.649	258.096	238.649	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	138.246	259.872	260.108	259.872	Flexural
60				138.246	260.262	260.24	260.24	Shear
70				138.246	260.526	260.33	260.33	Shear
80				138.246	260.718	260.395	260.395	Shear
90				138.246	260.863	260.445	260.445	Shear

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	9"	0.454	99.73	201.614	245.586	201.614	Flexural
60				99.73	201.979	245.733	201.979	Flexural
70				99.73	202.223	245.832	202.223	Flexural
80				99.73	202.398	245.902	202.398	Flexural
90				99.73	202.529	245.955	202.529	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	136.119	226.423	249.249	226.423	Flexural
60				136.119	226.748	249.377	226.748	Flexural
70				136.119	226.966	249.463	226.966	Flexural
80				136.119	227.122	249.524	227.122	Flexural
90				136.119	227.239	249.57	227.239	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	9"	0.454	98.692	169.9	232.463	169.9	Flexural
60				98.692	170.196	232.606	170.196	Flexural
70				98.692	170.39	232.699	170.39	Flexural
80				98.692	170.527	232.765	170.527	Flexural
90				98.692	170.629	232.814	170.629	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	134.701	196.887	237.659	196.887	Flexural
60				134.701	197.152	237.781	197.152	Flexural
70				134.701	197.326	237.861	197.326	Flexural
80				134.701	197.448	237.917	197.448	Flexural
90				134.701	197.54	237.959	197.54	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	9"	0.454	97.949	141.836	218.191	141.836	Flexural
60				97.949	142.075	218.331	142.075	Flexural
70				97.949	142.229	218.421	142.229	Flexural
80				97.949	142.336	218.484	142.336	Flexural
90				97.949	142.414	218.53	142.414	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	133.688	170.653	225.286	170.653	Flexural
60				133.688	170.868	225.403	170.868	Flexural
70				133.688	171.006	225.478	171.006	Flexural
80				133.688	171.103	225.531	171.103	Flexural
90				133.688	171.175	225.569	171.175	Flexural

E. Ultimate load capacity using FIB-36/ Thickness 9.5-inch

f'c=5ksi	FIB-36							
FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	9.5"	0.454	121.501	315.352	292.524	292.524	Shear
60				121.501	315.881	292.676	292.676	Shear
70				121.501	316.245	292.781	292.781	Shear
80				121.501	316.512	292.857	292.857	Shear
90				121.501	316.715	292.915	292.915	Shear
			ρ traditional	kip	kip	kip	kip	
50			0.63	165.833	339.229	294.231	294.231	Shear
60				165.833	339.691	294.363	294.363	Shear
70				165.833	340.009	294.454	294.454	Shear
80				165.833	340.242	294.521	294.521	Shear
90				165.833	340.42	294.572	294.572	Shear

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	9.5"	0.454	118.126	269.729	280.966	269.729	Flexural
60				118.126	270.195	281.122	270.195	Flexural
70				118.126	270.512	281.227	270.512	Flexural
80				118.126	270.741	281.303	270.741	Flexural
90				118.126	270.914	281.361	270.914	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	161.226	296.404	283.833	283.833	Shear
60				161.226	296.816	283.968	283.968	Shear
70				161.226	297.096	284.06	284.06	Shear
80				161.226	297.298	284.126	284.126	Shear
90				161.226	297.451	284.176	284.176	Shear

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	9.5"	0.454	116.101	229.817	268.144	229.817	Flexural
60				116.101	230.2	268.293	230.2	Flexural
70				116.101	230.455	268.392	230.455	Flexural
80				116.101	230.638	268.464	230.638	Flexural
90				116.101	230.775	268.517	230.775	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	158.462	259.17	272.403	259.17	Flexural
60				158.462	259.511	272.532	259.511	Flexural
70				158.462	259.738	272.618	259.738	Flexural
80				158.462	259.901	272.68	259.901	Flexural
90				158.462	260.023	272.726	260.023	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	9.5"	0.454	114.751	194.99	254.431	194.99	Flexural
60				114.751	195.298	254.573	195.298	Flexural
70				114.751	195.5	254.666	195.5	Flexural
80				114.751	195.642	254.731	195.642	Flexural
90				114.751	195.748	254.78	195.748	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	156.62	226.699	260.347	226.699	Flexural
60				156.62	226.975	260.468	226.975	Flexural
70				156.62	227.156	260.547	227.156	Flexural
80				156.62	227.283	260.602	227.283	Flexural
90				156.62	227.378	260.644	227.378	Flexural

FIB- 36				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	9.5"	0.454	113.787	164.396	239.703	164.396	Flexural
60				113.787	164.645	239.842	164.645	Flexural
70				113.787	164.804	239.931	164.804	Flexural
80				113.787	164.916	239.992	164.916	Flexural
90				113.787	164.998	240.038	164.998	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	155.303	198.172	247.675	198.172	Flexural
60				155.303	198.395	247.79	198.395	Flexural
70				155.303	198.538	247.863	198.538	Flexural
80				155.303	198.637	247.914	198.637	Flexural
90				155.303	198.71	247.951	229.457	Flexural

Appendix B: Effects of different variables in the Ultimate Capacity Using an AASHTO TYPE III girder.

A. Ultimate load capacity using AASHTO TYPE III/ Thickness 7.5-inch

f'c=5ksi	AASHTO TYPE 3							
AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	139.254	185.305	139.254	Flexural
60				60.165	139.351	185.346	139.351	Flexural
70				60.165	139.417	185.374	139.417	Flexural
80				60.165	139.466	185.395	139.466	Flexural
90				60.165	139.503	185.411	139.503	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	153.7	187.535	153.7	Flexural
60				82.118	153.788	187.572	153.788	Flexural
70				82.118	153.848	187.597	153.848	Flexural
80				82.118	153.892	187.615	153.892	Flexural
90				82.118	153.926	187.629	153.926	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	7.5"	0.454	59.189	108.674	171.654	108.674	Flexural
60				59.189	108.737	171.688	108.737	Flexural
70				59.189	108.78	171.711	108.78	Flexural
80				59.189	108.811	171.728	108.811	Flexural
90				59.189	108.834	171.741	108.834	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	80.784	125.022	175.314	125.022	Flexural
60				80.784	125.08	175.344	125.08	Flexural
70				80.784	125.119	175.364	125.119	Flexural
80				80.784	125.147	175.379	125.147	Flexural
90				80.784	125.168	175.39	125.168	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	7.5"	0.454	58.602	84.058	156.974	84.058	Flexural
60				58.602	84.101	157.004	84.101	Flexural
70				58.602	84.129	157.024	84.129	Flexural
80				58.602	84.149	157.039	84.149	Flexural
90				58.602	84.164	157.049	84.164	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.985	101.876	162.521	101.876	Flexural
60				79.985	101.916	162.547	101.916	Flexural
70				79.985	101.942	162.565	101.942	Flexural
80				79.985	101.961	162.577	101.961	Flexural
90				79.985	101.976	162.586	101.976	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	7.5"	0.454	58.212	68.758	145.047	68.758	Flexural
60				58.212	68.774	145.062	68.774	Flexural
70				58.212	68.785	145.072	68.785	Flexural
80				58.212	68.792	145.078	68.792	Flexural
90				58.212	68.797	145.083	68.797	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.451	88.491	153.38	88.491	Flexural
60				79.451	88.505	153.391	88.505	Flexural
70				79.451	88.514	153.398	88.514	Flexural
80				79.451	88.52	153.403	88.52	Flexural
90				79.451	88.525	153.407	88.525	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	7.5"	0.454	57.933	62.878	139.653	62.878	Flexural
60				57.933	62.884	139.66	62.884	Flexural
70				57.933	62.888	139.664	62.888	Flexural
80				57.933	62.891	139.667	62.891	Flexural
90				57.933	62.893	139.669	62.893	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.071	83.313	149.478	83.313	Flexural
60				79.071	83.319	149.483	83.319	Flexural
70				79.071	83.322	149.486	83.322	Flexural
80				79.071	83.325	149.488	83.325	Flexural
90				79.071	83.327	149.489	83.327	Flexural

B. Ultimate load capacity using AASHTO TYPE III/ Thickness 8-inch

f'c=5ksi	AASHTO TYPE 3							
AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	8"	0.454	73.262	162.085	205.205	162.085	Flexural
60				73.262	162.185	205.246	162.185	Flexural
70				73.262	162.254	205.274	162.254	Flexural
80				73.262	162.304	205.294	162.304	Flexural
90				73.262	162.342	205.31	162.342	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	99.992	180.205	207.982	180.205	Flexural
60				99.992	180.296	208.018	180.296	Flexural
70				99.992	180.358	208.043	180.358	Flexural
80				99.992	180.404	208.061	180.404	Flexural
90				99.992	180.438	208.074	180.438	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	8"	0.454	71.853	127.931	190.821	127.931	Flexural
60				71.853	127.996	190.855	127.996	Flexural
70				71.853	128.04	190.877	128.04	Flexural
80				71.853	128.071	190.894	128.071	Flexural
90				71.853	128.095	190.906	128.095	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	98.069	148.108	195.192	148.108	Flexural
60				98.069	148.167	195.221	148.167	Flexural
70				98.069	148.207	195.241	148.207	Flexural
80				98.069	148.236	195.255	148.236	Flexural
90				98.069	148.258	195.266	148.258	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	8"	0.454	71.007	100.77	175.644	100.77	Flexural
60				71.007	100.814	175.673	100.814	Flexural
70				71.007	100.843	175.692	100.843	Flexural
80				71.007	100.864	175.706	100.864	Flexural
90				71.007	100.879	175.717	100.879	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.916	122.585	182.076	122.585	Flexural
60				96.916	122.624	182.1	122.624	Flexural
70				96.916	122.65	182.116	122.65	Flexural
80				96.916	122.669	182.127	122.669	Flexural
90				96.916	122.683	182.136	122.683	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	8"	0.454	70.444	82.925	162.727	82.925	Flexural
60				79.836	82.942	162.742	82.942	Flexural
70				79.836	82.954	162.752	82.954	Flexural
80				79.836	82.962	162.759	82.962	Flexural
90				79.836	82.968	162.764	82.968	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.146	106.794	172.116	106.794	Flexural
60				96.146	106.809	172.127	106.809	Flexural
70				96.146	106.818	172.134	106.818	Flexural
80				96.146	106.825	172.139	106.825	Flexural
90				96.146	106.83	172.143	106.83	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	8"	0.454	70.041	75.899	156.79	75.899	Flexural
60				70.041	75.906	156.797	75.906	Flexural
70				70.041	75.91	156.801	75.91	Flexural
80				70.041	75.913	156.804	75.913	Flexural
90				70.041	75.916	156.807	75.916	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	95.597	100.598	167.843	100.598	Flexural
60				95.597	100.605	167.848	100.605	Flexural
70				95.597	100.608	167.851	100.608	Flexural
80				95.597	100.611	167.853	100.611	Flexural
90				95.597	100.613	167.855	100.613	Flexural

C. Ultimate load capacity using AASHTO TYPE III/ Thickness 8.5-inch

f'c=5ksi	AASHTO TYPE 3							
AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	8.5"	0.454	87.821	186.767	225.723	186.767	Flexural
60				87.821	186.87	225.764	186.87	Flexural
70				87.821	186.941	225.792	186.941	Flexural
80				87.821	186.992	225.812	186.992	Flexural
90				87.821	187.032	225.828	187.032	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	119.863	209.043	229.101	209.043	Shear
60				119.863	209.136	229.137	209.136	Flexural
70				119.863	209.2	229.162	209.2	Flexural
80				119.863	209.246	229.179	209.246	Flexural
90				119.863	209.282	229.193	209.282	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	8.5"	0.454	85.875	148.836	210.623	148.836	Flexural
60				85.875	148.902	210.656	148.902	Flexural
70				85.875	148.947	210.678	148.947	Flexural
80				85.875	148.98	210.694	148.98	Flexural
90				85.875	149.004	210.707	149.004	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	117.208	173.298	215.758	173.298	Flexural
60				117.208	173.358	215.787	173.358	Flexural
70				117.208	173.399	215.806	173.399	Flexural
80				117.208	173.428	215.819	173.428	Flexural
90				117.208	173.451	215.83	173.451	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	8.5"	0.454	84.708	119.019	194.948	119.019	Flexural
60				84.708	119.064	194.976	119.064	Flexural
70				84.708	119.093	194.995	119.093	Flexural
80				84.708	119.115	195.009	119.115	Flexural
90				84.708	119.13	195.019	119.13	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	115.615	145.218	202.283	145.218	Flexural
60				115.615	145.259	202.306	145.259	Flexural
70				115.615	145.286	202.322	145.286	Flexural
80				115.615	145.305	202.333	145.305	Flexural
90				115.615	145.319	202.341	145.319	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	8.5"	0.454	83.93	98.538	181.102	98.538	Flexural
60				83.93	98.557	181.118	98.557	Flexural
70				83.93	98.57	181.128	98.57	Flexural
80				83.93	98.579	181.135	98.579	Flexural
90				83.93	98.585	181.14	98.585	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	114.553	126.963	191.583	126.963	Flexural
60				114.553	126.979	191.594	126.979	Flexural
70				114.553	126.99	191.602	126.99	Flexural
80				114.553	126.997	191.607	126.997	Flexural
90				114.553	127.003	191.61	127.003	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	8.5"	0.454	83.374	90.237	174.599	90.237	Flexural
60				83.374	90.245	174.606	90.245	Flexural
70				83.374	90.25	174.61	90.25	Flexural
80				83.374	90.253	174.613	90.253	Flexural
90				83.374	90.256	174.615	90.256	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	113.794	119.63	186.924	119.63	Flexural
60				113.794	119.636	186.929	119.636	Flexural
70				113.794	119.641	186.932	119.641	Flexural
80				113.794	119.644	186.934	119.644	Flexural
90				113.794	119.646	186.936	119.646	Flexural

D. Ultimate load capacity using AASHTO TYPE III/ Thickness 9-inch

f'c=5ksi		AASHTO TYPE 3						
AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	9"	0.454	103.886	213.325	246.862	213.325	Flexural
60				103.886	213.431	246.902	213.431	Flexural
70				103.886	213.503	246.93	213.503	Flexural
80				103.886	213.556	246.95	213.556	Flexural
90				103.886	213.596	246.965	213.596	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	141.79	240.256	250.895	240.256	Flexural
60				141.79	240.351	250.93	240.351	Flexural
70				141.79	240.417	250.954	240.417	Flexural
80				141.79	240.464	250.972	240.464	Flexural
90				141.79	240.5	250.985	240.5	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	9"	0.454	101.289	171.411	231.06	171.411	Flexural
60				101.289	171.479	231.093	171.479	Flexural
70				101.289	171.525	231.115	171.525	Flexural
80				101.289	171.558	231.13	171.558	Flexural
90				101.289	171.583	231.142	171.583	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	138.246	200.626	237.013	200.626	Flexural
60				138.246	200.688	237.04	200.688	Flexural
70				138.246	200.729	237.059	200.729	Flexural
80				138.246	200.759	237.072	200.759	Flexural
90				138.246	200.782	237.083	200.782	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	9"	0.454	99.73	138.812	214.884	138.812	Flexural
60				99.73	138.858	214.912	138.858	Flexural
70				99.73	138.888	214.93	138.888	Flexural
80				99.73	138.91	214.91	138.91	Flexural
90				99.73	138.926	214.953	138.926	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	136.119	169.858	223.181	169.858	Flexural
60				136.119	169.9	223.203	169.9	Flexural
70				136.119	169.927	223.218	169.927	Flexural
80				136.119	169.947	223.229	169.947	Flexural
90				136.119	169.961	223.237	169.961	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	9"	0.454	98.692	115.624	200.171	115.624	Flexural
60				98.692	115.644	200.187	115.644	Flexural
70				98.692	115.658	200.197	115.658	Flexural
80				98.692	115.668	200.204	115.668	Flexural
90				98.692	115.675	200.209	115.675	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	134.701	149.031	211.781	149.031	Flexural
60				134.701	149.048	211.792	149.048	Flexural
70				134.701	149.06	211.8	149.06	Flexural
80				134.701	149.068	211.805	149.068	Flexural
90				134.701	149.074	211.809	149.074	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	9"	0.454	97.949	105.914	193.078	105.914	Flexural
60				97.949	105.923	193.085	105.923	Flexural
70				97.949	105.928	193.089	105.928	Flexural
80				97.949	105.932	193.092	105.932	Flexural
90				97.949	105.935	193.095	105.935	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	133.688	140.434	206.72	140.434	Flexural
60				133.688	140.442	206.725	140.442	Flexural
70				133.688	140.446	206.728	140.446	Flexural
80				133.688	140.449	206.73	140.449	Flexural
90				133.688	140.452	206.732	140.452	Flexural

E. Ultimate load capacity using AASHTO TYPE III/ Thickness 9.5-inch

f'c=5ksi	AASHTO TYPE 3							
AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	9.5"	0.454	121.501	241.786	268.62	241.786	Flexural
60				121.501	241.894	268.66	241.894	Flexural
70				121.501	241.967	268.687	241.967	Flexural
80				121.501	242.021	268.707	242.021	Flexural
90				121.501	242.062	268.722	242.062	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	165.833	273.89	273.363	273.363	Shear
60				165.833	273.987	273.397	273.397	Shear
70				165.833	274.053	273.421	273.421	Shear
80				165.833	274.102	273.438	273.438	Shear
90				165.833	274.138	273.451	273.451	Shear

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	9.5"	0.454	118.126	195.678	252.133	195.678	Flexural
60				118.126	195.747	252.165	195.747	Flexural
70				118.126	195.794	252.187	195.794	Flexural
80				118.126	195.828	252.202	195.828	Flexural
90				118.126	195.853	252.214	195.853	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	161.226	230.128	258.956	230.128	Flexural
60				161.226	230.191	258.983	230.191	Flexural
70				161.226	230.233	259.001	230.233	Flexural
80				161.226	230.263	259.014	230.263	Flexural
90				161.226	230.286	259.286	230.286	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	9.5"	0.454	116.101	160.168	235.455	160.168	Flexural
60				116.101	160.214	235.482	160.214	Flexural
70				116.101	160.245	235.5	160.245	Flexural
80				116.101	160.267	235.512	160.267	Flexural
90				116.101	160.283	235.522	160.283	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	158.462	196.764	244.764	196.764	Flexural
60				158.462	196.564	244.786	196.564	Flexural
70				158.462	196.592	244.801	196.592	Flexural
80				158.462	196.612	244.811	196.612	Flexural
90				158.462	196.627	244.819	196.627	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	9.5"	0.454	114.751	134.205	219.931	134.205	Flexural
60				114.751	134.228	219.947	134.228	Flexural
70				114.751	134.242	219.957	134.242	Flexural
80				114.751	134.25	219.964	134.25	Flexural
90				114.751	134.26	219.97	134.26	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	156.62	173.029	232.708	173.029	Flexural
60				156.62	173.048	232.719	173.048	Flexural
70				156.62	173.06	232.727	173.06	Flexural
80				156.62	173.069	232.732	173.069	Flexural
90				156.62	173.075	232.736	173.075	Flexural

AASHTO TYPE III				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	9.5"	0.454	113.787	122.95	212.226	122.95	Flexural
60				113.787	122.96	212.234	122.96	Flexural
70				113.787	122.965	212.238	122.965	Flexural
80				113.787	122.969	212.241	122.969	Flexural
90				113.787	122.972	212.244	122.972	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	155.303	163.039	227.23	163.039	Flexural
60				155.303	163.047	227.235	163.047	Flexural
70				155.303	163.052	227.239	163.052	Flexural
80				155.303	163.055	227.241	163.055	Flexural
90				155.303	163.058	227.242	163.058	Flexural

Appendix C: Effects of different variables in the Ultimate Capacity Using Steel W44x335 girder.

A. Ultimate load capacity using W44x335/ Thickness 7.5-inch

f'c=5ksi	W44X335							
W44X335				Flexure	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	133.28	182.716	133.28	Flexural
60				60.165	133.57	182.75	133.57	Flexural
70				60.165	133.409	182.773	133.409	Flexural
80				60.165	133.447	182.79	133.447	Flexural
90				60.165	133.476	182.803	133.476	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	148.291	185.242	148.291	Flexural
60				82.118	148.36	185.272	148.36	Flexural
70				82.118	148.407	185.292	148.407	Flexural
80				82.118	148.442	185.307	148.442	Flexural
90				82.118	148.468	185.319	148.468	Flexural

W44X335				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	7.5"	0.454	59.189	103.234	168.622	103.234	Flexural
60				59.189	103.282	168.65	103.282	Flexural
70				59.189	103.315	168.668	103.315	Flexural
80				59.189	103.338	168.682	103.338	Flexural
90				59.189	103.356	168.692	103.356	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	80.784	120.067	172.689	120.067	Flexural
60				80.784	120.111	172.713	120.111	Flexural
70				80.784	120.14	172.729	120.14	Flexural
80				80.784	120.162	172.741	120.162	Flexural
90				80.784	120.178	172.749	120.178	Flexural

W44X335				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	7.5"	0.454	58.602	79.274	153.427	79.274	Flexural
60				58.602	79.305	153.45	79.305	Flexural
70				58.602	79.325	153.466	79.325	Flexural
80				58.602	79.34	153.477	79.34	Flexural
90				58.602	79.35	153.485	79.35	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.985	97.689	159.699	97.689	Flexural
60				79.985	97.715	159.717	97.715	Flexural
70				79.985	97.732	159.729	97.732	Flexural
80				79.985	97.745	159.738	97.745	Flexural
90				79.985	97.754	159.744	97.754	Flexural

W44X335				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	7.5"	0.454	58.212	66.7	143.099	66.7	Flexural
60				58.212	66.68	143.109	66.68	Flexural
70				58.212	66.687	143.115	66.687	Flexural
80				58.212	66.692	143.12	66.692	Flexural
90				58.212	66.695	143.123	66.695	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.451	86.706	151.957	86.706	Flexural
60				79.451	86.714	151.764	86.714	Flexural
70				79.451	86.72	151.969	86.72	Flexural
80				79.451	86.724	151.972	86.724	Flexural
90				79.451	86.727	151.974	86.727	Flexural

W44X335				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	7.5"	0.454	57.933	61.881	138.616	61.881	Flexural
60				57.933	61.886	138.62	61.886	Flexural
70				57.933	61.888	138.623	61.888	Flexural
80				57.933	61.89	138.625	61.89	Flexural
90				57.933	61.891	138.627	61.891	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.071	82.46	148.744	82.46	Flexural
60				79.071	82.463	148.747	82.463	Flexural
70				79.071	82.466	148.749	82.466	Flexural
80				79.071	82.467	148.751	82.467	Flexural
90				79.071	82.468	148.752	82.468	Flexural

B. Ultimate load capacity using W44x335/ Thickness 8-inch

f'c=5ksi		W44X335						
W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	8"	0.454	73.262	155.277	202.277	155.277	Flexural
60				73.262	155.356	202.396	155.356	Flexural
70				73.262	155.41	202.419	155.41	Flexural
80				73.262	155.449	202.435	155.449	Flexural
90				73.262	155.479	202.448	155.479	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	99.992	174.052	205.48	174.052	Flexural
60				99.992	174.123	205.509	174.123	Flexural
70				99.992	174.172	205.529	174.172	Flexural
80				99.992	174.207	205.544	174.207	Flexural
90				99.992	174.245	205.555	174.245	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	8"	0.454	71.853	121.82	187.57	121.82	Flexural
60				71.853	121.869	187.597	121.869	Flexural
70				71.853	121.903	187.616	121.903	Flexural
80				71.853	121.927	187.629	121.927	Flexural
90				71.853	121.945	187.639	121.945	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	98.069	142.554	192.397	142.554	Flexural
60				98.069	142.599	192.42	142.599	Flexural
70				98.069	142.629	192.436	142.629	Flexural
80				98.069	142.651	192.447	142.651	Flexural
90				98.069	142.668	192.455	142.668	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	8"	0.454	71.007	95.419	171.927	95.419	Flexural
60				71.007	95.452	171.951	95.452	Flexural
70				71.007	95.475	171.967	95.475	Flexural
80				71.007	95.49	171.978	95.49	Flexural
90				71.007	95.502	171.986	95.502	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.916	117.725	179.015	117.725	Flexural
60				96.916	117.753	179.033	117.753	Flexural
70				96.916	117.772	179.046	117.772	Flexural
80				96.916	117.786	179.054	117.786	Flexural
90				96.916	117.796	179.061	117.796	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	8"	0.454	74.444	80.443	160.568	80.443	Flexural
60				74.444	80.454	160.578	80.454	Flexural
70				74.444	80.461	160.585	80.461	Flexural
80				74.444	80.467	160.589	80.467	Flexural
90				74.444	80.471	160.593	80.471	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.146	104.681	170.546	104.681	Flexural
60				96.146	104.691	170.554	104.691	Flexural
70				96.146	104.697	170.558	104.697	Flexural
80				96.146	104.702	170.562	104.702	Flexural
90				96.146	104.705	170.564	104.705	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	8"	0.454	70.041	74.715	155.643	74.715	Flexural
60				70.041	74.72	155.647	74.72	Flexural
70				70.041	74.723	155.65	74.723	Flexural
80				70.041	74.725	155.652	74.725	Flexural
90				70.041	74.726	155.654	74.726	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	95.597	99.59	167.035	99.59	Flexural
60					99.593	167.039	99.593	Flexural
70					99.596	167.041	99.596	Flexural
80					99.598	167.042	99.598	Flexural
90					99.599	167.043	99.599	Flexural

C. Ultimate load capacity using W44x335/ Thickness 8.5-inch

f'c=5ksi	W44X335							
W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	8.5"	0.454	87.821	179.085	222.626	179.085	Flexural
60				87.821	179.165	222.659	179.165	Flexural
70				87.821	179.22	222.681	179.22	Flexural
80				87.821	179.26	222.698	179.26	Flexural
90				87.821	179.291	222.71	179.291	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	119.863	202.11	226.39	202.11	Flexural
60				119.863	202.182	226.419	202.182	Flexural
70				119.863	202.232	226.439	202.232	Flexural
80				119.863	202.268	226.453	202.268	Flexural
90				119.863	202.296	226.464	202.296	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	8.5"	0.454	85.875	142.027	207.152	142.027	Flexural
60				85.875	142.077	207.179	142.077	Flexural
70				85.875	142.111	207.197	142.111	Flexural
80				85.875	142.136	207.209	142.136	Flexural
90				85.875	142.154	207.219	142.154	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	117.208	167.12	212.792	167.12	Flexural
60				117.208	167.166	212.815	167.166	Flexural
70				117.208	167.197	212.83	167.197	Flexural
80				117.208	167.219	212.841	167.219	Flexural
90				117.208	167.236	212.849	167.236	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	8.5"	0.454	84.708	113.135	191.094	113.135	Flexural
60				84.708	113.168	191.117	113.168	Flexural
70				84.708	113.19	191.132	113.19	Flexural
80				84.708	113.205	191.142	113.205	Flexural
90				84.708	113.217	191.15	113.217	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	115.615	139.815	199.082	139.815	Flexural
60				115.615	139.846	199.1	139.846	Flexural
70				115.615	139.866	199.113	139.866	Flexural
80				115.615	139.881	199.122	139.881	Flexural
90				115.615	139.892	199.128	139.892	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	8.5"	0.454	83.93	95.622	178.722	95.622	Flexural
60				83.93	95.634	178.732	95.634	Flexural
70				83.93	95.642	178.739	95.642	Flexural
80				83.93	95.648	178.744	95.648	Flexural
90				83.93	95.652	178.747	95.652	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	114.553	124.491	189.861	124.491	Flexural
60				114.553	124.501	189.868	124.501	Flexural
70				114.553	124.508	189.873	124.508	Flexural
80				114.553	124.513	189.876	124.513	Flexural
90				114.553	124.516	189.879	124.516	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	8.5"	0.454	83.374	88.847	173.35	88.847	Flexural
60				83.374	88.852	1783.34	88.852	Flexural
70				83.374	88.855	173.342	88.855	Flexural
80				83.374	88.857	173.344	88.857	Flexural
90				83.374	88.859	173.346	88.859	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	113.794	118.45	186.038	118.45	Flexural
60				113.794	118.454	186.042	118.454	Flexural
70				113.794	118.456	186.044	118.456	Flexural
80				113.794	118.458	186.045	118.458	Flexural
90				113.794	118.46	186.046	118.46	Flexural

D. Ultimate load capacity using W44x335/ Thickness 9-inch

f'c=5ksi		W44X335						
W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	9"	0.454	103.886	204.73	243.507	204.73	Flexural
60				103.886	204.812	243.54	204.812	Flexural
70				103.886	204.868	243.562	204.868	Flexural
80				103.886	204.909	243.578	204.909	Flexural
90				103.886	204.94	243.59	204.94	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	141.79	232.509	247.975	232.509	Shear
60				141.79	232.583	248.003	232.583	Flexural
70				141.79	232.634	248.023	232.634	Flexural
80				141.79	232.67	248.037	232.67	Flexural
90				141.79	232.698	248.048	232.698	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	9"	0.454	101.289	163.877	227.369	163.877	Flexural
60				101.289	163.928	227.395	163.928	Flexural
70				101.289	163.963	227.412	163.963	Flexural
80				101.289	163.988	227.425	163.988	Flexural
90				101.289	164.006	227.434	164.006	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	138.246	193.802	233.876	193.802	Flexural
60				138.246	193.848	233.898	193.848	Flexural
70				138.246	193.88	233.912	193.88	Flexural
80				138.246	193.902	233.923	193.902	Flexural
90				138.246	193.919	233.931	193.919	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	9"	0.454	99.73	132.317	210.849	132.317	Flexural
60				99.73	132.351	210.871	132.351	Flexural
70				99.73	132.373	210.885	132.373	Flexural
80				99.73	132.389	210.895	132.389	Flexural
90				99.73	132.401	210.903	132.401	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	136.119	163.979	219.886	163.979	Flexural
60				136.119	164.01	219.904	164.01	Flexural
70				136.119	164.03	219.915	164.03	Flexural
80				136.119	164.045	219.924	164.045	Flexural
90				136.119	164.056	219.93	164.056	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	9"	0.454	98.692	112.232	197.559	112.232	Flexural
60				98.692	112.245	197.57	112.245	Flexural
70				98.692	112.254	197.576	112.254	Flexural
80				98.692	112.26	197.581	112.26	Flexural
90				98.692	112.264	197.585	112.264	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	134.701	146.166	209.899	146.166	Flexural
60				134.701	146.177	209.907	146.177	Flexural
70				134.701	146.185	209.911	146.185	Flexural
80				134.701	146.19	209.915	146.19	Flexural
90				134.701	146.194	209.917	146.194	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	9"	0.454	97.949	104.297	191.692	104.297	Flexural
60				97.949	104.303	191.697	104.303	Flexural
70				97.949	104.306	191.7	104.306	Flexural
80				97.949	104.308	191.702	104.308	Flexural
90				97.949	104.31	191.703	104.31	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	133.688	139.067	205.753	139.067	Flexural
60				133.688	139.071	205.756	139.071	Flexural
70				133.688	139.074	205.758	139.074	Flexural
80				133.688	139.076	205.759	139.076	Flexural
90				133.688	139.078	205.76	139.078	Flexural

E. Ultimate load capacity using W44x335/ Thickness 9.5-inch

f'c=5ksi	W44X335							
W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	6'	9.5"	0.454	121.501	232.242	265.007	232.242	Flexural
60				121.501	232.325	265.039	232.325	Flexural
70				121.501	232.381	265.061	232.381	Flexural
80				121.501	232.423	265.077	232.423	Flexural
90				121.501	232.454	265.089	232.454	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	165.833	265.297	270.234	265.297	Shear
60				165.833	265.372	270.262	265.372	Flexural
70				165.833	265.423	270.281	265.423	Flexural
80				165.833	265.46	270.295	265.46	Flexural
90				165.833	265.489	270.305	265.489	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	8'	9.5"	0.454	118.126	187.393	248.221	187.393	Flexural
60				118.126	187.445	248.246	187.445	Flexural
70				118.126	187.481	248.263	187.481	Flexural
80				118.126	187.506	248.275	187.506	Flexural
90				118.126	187.525	248.284	187.525	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	161.226	222.635	255.648	222.635	Flexural
60				161.226	222.682	255.67	222.682	Flexural
70				161.226	222.714	255.684	222.714	Flexural
80				161.226	222.737	255.694	222.737	Flexural
90				161.226	222.754	255.702	222.754	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	10'	9.5"	0.454	116.101	153.05	231.24	153.05	Flexural
60				116.101	153.084	231.261	153.084	Flexural
70				116.101	153.107	231.275	153.107	Flexural
80				116.101	153.123	231.285	153.123	Flexural
90				116.101	153.135	231.292	153.135	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	158.462	190.081	241.332	190.081	Flexural
60				158.462	190.112	241.349	190.112	Flexural
70				158.462	190.133	241.36	190.133	Flexural
80				158.462	190.147	241.368	190.147	Flexural
90				158.462	190.158	241.374	190.158	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	12'	9.5"	0.454	114.751	130.296	217.078	130.296	Flexural
60				114.751	130.31	217.089	130.31	Flexural
70				114.751	130.32	217.095	130.32	Flexural
80				114.751	130.326	217.1	130.326	Flexural
90				114.751	130.331	217.104	130.331	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	156.62	169.738	230.66	169.738	Flexural
60				156.62	169.75	230.667	169.75	Flexural
70				156.62	169.758	230.672	169.758	Flexural
80				156.62	169.763	230.676	169.763	Flexural
90				156.62	169.768	230.678	169.768	Flexural

W44X335				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ empirical	kip	kip	kip	kip	
50	14'	9.5"	0.454	113.787	121.086	210.716	121.086	Flexural
60				113.787	121.092	210.718	121.092	Flexural
70				113.787	121.096	210.721	121.096	Flexural
80				113.787	121.098	210.723	121.098	Flexural
90				113.787	121.1	210.724	121.1	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	155.303	161.467	226.177	161.467	Flexural
60				155.303	161.472	226.181	161.472	Flexural
70				155.303	161.475	226.183	161.475	Flexural
80				155.303	161.478	226.184	161.478	Flexural
90				155.303	161.479	226.185	229.457	Flexural

Appendix D: Effects of different variables in the Ultimate Capacity Using a Steel Built-up Section.

A. Ultimate load capacity using a steel built-up section/ Thickness 7.5-inch

f'c=5ksi	STEEL BUILT UP SECTION							
EFFECT OF SPAN				Flexure	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	6'	7.5"	0.454	60.165	155.352	191.788	155.352	Flexural
60				60.165	155.526	191.854	155.526	Flexural
70				60.165	155.646	191.9	155.646	Flexural
80				60.165	155.734	191.934	155.734	Flexural
90				60.165	155.8	191.959	155.8	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	82.118	168.199	193.299	168.199	Flexural
60				82.118	168.355	193.358	168.355	Flexural
70				82.118	168.462	193.399	168.462	Flexural
80				82.118	168.541	193.429	168.541	Flexural
90				82.118	168.601	193.451	168.601	Flexural

EFFECT OF SPAN				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	8'	7.5"	0.454	59.189	124.345	179.59	124.345	Flexural
60				59.189	124.471	179.65	124.471	Flexural
70				59.189	124.557	179.69	124.557	Flexural
80				59.189	124.619	179.72	124.619	Flexural
90				59.189	124.666	179.742	124.666	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	80.784	139.243	182.255	139.243	Flexural
60				80.784	139.357	182.308	139.357	Flexural
70				80.784	139.434	182.344	139.434	Flexural
80				80.784	139.491	182.369	139.491	Flexural
90				80.784	139.533	182.389	139.533	Flexural

EFFECT OF SPAN				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	10'	7.5"	0.454	58.602	98.304	166.316	98.304	Flexural
60				58.602	98.394	166.37	98.394	Flexural
70				58.602	98.454	166.406	98.454	Flexural
80				58.602	98.497	166.432	98.497	Flexural
90				58.602	98.529	166.451	98.529	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.985	114.884	170.46	114.884	Flexural
60				79.985	114.966	170.507	114.966	Flexural
70				79.985	115.02	170.538	115.02	Flexural
80				79.985	115.059	170.56	115.059	Flexural
90				79.985	115.088	170.577	115.088	Flexural

EFFECT OF SPAN				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	12'	7.5"	0.454	58.12	77.143	152.173	77.143	Flexural
60				58.12	77.195	152.214	77.195	Flexural
70				58.12	77.229	152.241	77.229	Flexural
80				58.12	77.253	152.26	77.253	Flexural
90				58.12	77.271	152.274	77.271	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.451	95.639	158.716	95.639	Flexural
60				79.451	95.684	158.748	95.684	Flexural
70				79.451	95.712	158.768	95.712	Flexural
80				79.451	95.733	158.783	95.733	Flexural
90				79.451	95.748	158.793	95.748	Flexural

EFFECT OF SPAN				Flexure	Flexural punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	14'	7.5"	0.454	57.933	66.967	143.685	66.967	Flexural
60				57.933	66.989	143.706	66.989	Flexural
70				57.933	67.002	143.719	67.002	Flexural
80				57.933	67.012	143.728	67.012	Flexural
90				57.933	67.019	143.735	67.019	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	79.071	86.809	152.377	86.809	Flexural
60				79.071	86.827	152.391	86.827	Flexural
70				79.071	86.839	152.401	86.839	Flexural
80				79.071	86.847	152.408	86.847	Flexural
90				79.071	86.853	152.413	86.853	Flexural

B. Ultimate load capacity using a steel built-up section / Thickness 8-inch

f'c=5ksi		STEEL BUILT UP SECTION						
STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	6'	8"	0.454	73.262	180.661	212.412	180.661	Flexural
60				73.262	180.845	212.48	180.845	Flexural
70				73.262	180.971	212.527	180.971	Flexural
80				73.262	181.064	212.561	181.064	Flexural
90				73.262	181.134	212.586	181.134	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	99.992	196.907	214.352	196.907	Flexural
60				99.992	197.071	214.412	197.071	Flexural
70				99.992	197.185	214.453	197.185	Flexural
80				99.992	197.267	214.484	197.267	Flexural
90				99.992	197.33	214.506	197.33	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	8'	8"	0.454	71.853	145.671	199.421	145.671	Flexural
60				71.853	145.803	199.48	145.803	Flexural
70				71.853	145.892	199.521	145.892	Flexural
80				71.853	145.956	199.55	145.956	Flexural
90				71.853	145.005	199.572	145.005	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	98.069	164.174	202.665	164.174	Flexural
60				98.069	164.293	202.717	164.293	Flexural
70				98.069	164.374	202.753	164.374	Flexural
80				98.069	164.432	202.778	164.432	Flexural
90				98.069	164.476	202.798	164.476	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	10'	8"	0.454	71.007	116.763	185.536	116.763	Flexural
60				71.007	116.857	185.589	116.857	Flexural
70				71.007	116.919	185.625	116.919	Flexural
80				71.007	116.963	185.65	116.963	Flexural
90				71.007	116.996	185.669	116.996	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.916	137.114	190.408	137.114	Flexural
60				96.916	137.198	190.453	137.198	Flexural
70				96.916	137.255	190.483	137.255	Flexural
80				96.916	137.295	190.505	137.295	Flexural
90				96.916	137.325	190.521	137.325	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	12'	8"	0.454	70.444	92.947	170.673	92.947	Flexural
60				70.444	93.004	170.716	93.004	Flexural
70				70.444	93.042	170.743	93.042	Flexural
80				70.444	93.068	170.763	93.068	Flexural
90				70.444	93.088	170.777	93.088	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	96.146	115.299	178.036	115.299	Flexural
60				96.146	115.348	178.068	115.348	Flexural
70				96.146	115.38	178.089	115.38	Flexural
80				96.146	115.402	178.104	115.402	Flexural
90				96.146	115.419	178.115	115.419	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	14'	8"	0.454	70.041	80.775	161.27	80.775	Flexural
60				70.041	80.799	161.291	80.799	Flexural
70				70.041	80.814	161.305	80.814	Flexural
80				70.041	80.825	161.314	80.825	Flexural
90				70.041	80.833	161.321	80.833	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	95.597	104.747	171.047	104.747	Flexural
60				95.597	104.768	171.063	104.768	Flexural
70				95.597	104.781	171.072	104.781	Flexural
80				95.597	104.79	171.079	104.79	Flexural
90				95.597	104.797	171.084	104.797	Flexural

C. Ultimate load capacity using a steel built-up section / Thickness 8.5-inch

f'c=5ksi	STEEL BUILT UP SECTION							
STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	6'	8.5"	0.454	87.821	207.972	233.67	207.972	Flexural
60				87.821	208.165	233.739	208.165	Flexural
70				87.821	208.297	233.786	208.297	Flexural
80				87.821	208.394	233.82	208.394	Flexural
90				87.821	208.467	233.846	208.467	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	119.863	228.082	236.088	228.082	Flexural
60				119.863	228.254	236.148	228.254	Flexural
70				119.863	228.372	236.19	228.372	Flexural
80				119.863	228.458	236.22	228.458	Flexural
90				119.863	228.524	236.243	228.524	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	8'	8.5"	0.454	85.875	168.744	219.892	168.744	Flexural
60				85.875	168.88	219.951	168.88	Flexural
70				85.875	168.972	219.992	168.972	Flexural
80				85.875	169.039	220.021	169.039	Flexural
90				85.875	169.089	220.043	169.089	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	117.208	191.295	223.765	191.295	Flexural
60				117.208	191.418	223.817	191.418	Flexural
70				117.208	191.501	223.852	191.501	Flexural
80				117.208	191.561	223.877	191.561	Flexural
90				117.208	191.607	223.896	191.607	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	10'	8.5"	0.454	84.708	136.823	205.396	136.823	Flexural
60				84.708	136.919	205.448	136.919	Flexural
70				84.708	136.983	205.483	136.983	Flexural
80				84.708	137.029	205.508	137.029	Flexural
90				84.708	137.063	205.526	137.063	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	115.615	161.374	211.044	161.374	Flexural
60				115.615	161.461	211.088	161.461	Flexural
70				115.615	161.519	211.118	161.519	Flexural
80				115.615	161.56	211.139	161.56	Flexural
90				115.615	161.591	211.154	161.591	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	12'	8.5"	0.454	83.93	110.372	189.91	110.372	Flexural
60				83.93	110.435	189.953	110.435	Flexural
70				83.93	110.477	189.982	110.477	Flexural
80				83.93	110.506	190.002	110.506	Flexural
90				83.93	110.527	190.017	110.527	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	114.553	136.966	198.117	136.966	Flexural
60				114.553	137.02	198.15	137.02	Flexural
70				114.553	137.054	198.171	137.054	Flexural
80				114.553	137.079	198.187	137.079	Flexural
90				114.553	137.097	198.198	137.097	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	14'	8.5"	0.454	83.374	95.984	179.551	95.984	Flexural
60				83.374	96.011	179.573	96.011	Flexural
70				83.374	96.028	179.587	96.028	Flexural
80				83.374	96.039	179.596	96.039	Flexural
90				83.374	96.048	179.603	96.048	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	126.799	124.5	190.451	124.5	Flexural
60				126.799	124.522	190.466	124.522	Flexural
70				126.799	124.536	190.476	124.536	Flexural
80				126.799	124.546	190.483	124.546	Flexural
90				126.799	124.554	190.489	124.554	Flexural

D. Ultimate load capacity using a steel built-up section / Thickness 9-inch

f'c=5ksi		STEEL BUILT UP SECTION						
STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	6'	9"	0.454	103.886	237.302	255.56	237.302	Flexural
60				103.886	237.502	255.629	237.502	Flexural
70				103.886	237.64	255.677	237.64	Flexural
80				103.886	237.74	255.711	237.74	Flexural
90				103.886	237.817	255.737	237.817	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	141.79	261.758	258.505	258.505	Shear
60				141.79	261.937	258.565	258.565	Shear
70				141.79	262.06	258.607	258.607	Shear
80				141.79	262.15	258.637	258.637	Shear
90				141.79	262.218	258.66	258.66	Shear

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	8'	9"	0.454	101.289	193.579	241.003	193.579	Flexural
60				101.289	193.719	241.063	197.498	Flexural
70				101.289	193.814	241.102	193.814	Flexural
80				101.289	193.883	241.131	193.883	Flexural
90				101.289	193.935	241.153	193.935	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	138.246	220.636	245.556	220.636	Flexural
60				138.246	220.763	235.498	220.763	Flexural
70				138.246	220.848	245.642	220.848	Flexural
80				138.246	220.910	245.667	220.91	Flexural
90				138.246	220.957	245.685	220.957	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	10'	9"	0.454	99.73	158.497	225.897	158.497	Flexural
60				99.73	158.596	225.949	158.596	Flexural
70				99.73	158.661	225.983	158.661	Flexural
80				99.73	158.708	226.007	158.708	Flexural
90				99.73	158.743	226.025	158.743	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	136.119	187.69	232.371	187.69	Flexural
60				136.119	187.78	232.414	187.78	Flexural
70				136.119	187.839	232.443	187.839	Flexural
80				136.119	187.881	232.463	187.881	Flexural
90				136.119	187.913	232.479	187.913	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	12'	9"	0.454	98.692	129.448	209.879	129.448	Flexural
60				98.692	129.517	209.924	129.517	Flexural
70				98.692	129.562	209.954	129.562	Flexural
80				98.692	129.593	209.975	129.593	Flexural
90				98.692	129.617	209.99	129.617	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	134.701	160.676	218.956	160.676	Flexural
60				134.701	160.733	218.99	160.733	Flexural
70				134.701	160.771	219.012	160.771	Flexural
80				134.701	160.798	219.028	160.798	Flexural
90				134.701	160.817	219.039	160.817	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	14'	9"	0.454	97.949	112.619	198.526	112.619	Flexural
60				97.949	112.648	198.549	112.648	Flexural
70				97.949	112.666	198.563	112.666	Flexural
80				97.949	112.679	198.573	112.679	Flexural
90				97.949	112.688	198.58	112.688	Flexural
			ρ traditional	kip			kip	
50			0.63	133.688	146.095	210.585	146.095	Flexural
60				133.688	146.119	210.601	146.119	Flexural
70				133.688	146.135	210.612	146.135	Flexural
80				133.688	146.146	210.619	146.146	Flexural
90				133.688	146.154	210.624	146.154	Flexural

E. Ultimate load capacity using a steel built-up section / Thickness 9.5-inch

f'c=5ksi	STEEL BUILT UP SECTION							
STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	6'	9.5"	0.454	121.501	268.668	278.08	268.668	Flexural
60				121.501	268.876	278.149	268.876	Flexural
70				121.501	269.018	278.197	269.018	Flexural
80				121.501	269.122	278.232	269.122	Flexural
90				121.501	269.201	278.258	269.201	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	165.833	297.975	281.601	281.601	Shear
60				165.833	298.16	281.662	281.662	Shear
70				165.833	298.287	281.703	281.703	Shear
80				165.833	298.38	281.734	281.734	Shear
90				165.833	298.45	281.757	281.757	Shear

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	8'	9.5"	0.454	118.126	220.195	262.755	220.195	Flexural
60				118.126	220.339	262.814	220.339	Flexural
70				118.126	220.436	262.853	220.436	Flexural
80				118.126	220.507	262.882	220.507	Flexural
90				118.126	220.56	262.904	220.56	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	161.226	252.229	268.036	252.229	Flexural
60				161.226	252.359	268.087	252.359	Flexural
70				161.226	252.447	268.121	252.447	Flexural
80				161.226	252.21	268.146	252.21	Flexural
90				161.226	252.558	268.164	252.558	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	10'	9.5"	0.454	161.101	181.802	247.042	181.802	Flexural
60				161.101	181.903	247.092	181.903	Flexural
70				161.101	181.97	247.126	181.97	Flexural
80				161.101	182.018	247.15	182.018	Flexural
90				161.101	182.054	247.167	182.054	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	158.462	216.09	254.39	216.09	Flexural
60				158.462	216.183	254.432	216.183	Flexural
70				158.462	216.242	254.461	216.242	Flexural
80				158.462	216.285	254.481	216.285	Flexural
90				158.462	216.318	254.496	216.318	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	12'	9.5"	0.454	114.751	150.691	230.579	150.691	Flexural
60				114.751	150.275	230.625	150.275	Flexural
70				114.751	150.324	230.655	150.324	Flexural
80				114.751	150.358	230.677	150.358	Flexural
90				114.751	150.384	230.692	150.384	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	156.62	186.46	240.552	186.46	Flexural
60				156.62	186.523	240.586	186.523	Flexural
70				156.62	186.563	240.609	186.563	Flexural
80				156.62	186.592	240.625	186.592	Flexural
90				156.62	186.613	240.637	186.613	Flexural

STEEL BUILT UP SECTION				Flexural	Flexural Arching punching capacity	Shear punching Capacity	Ultimate capacity	Type of failure
Length	Spacing	Thickness	ρ traditional	kip	kip	kip	kip	
50	14'	9.5"	0.454	113.787	130.701	218.195	130.701	Flexural
60				113.787	130.732	218.218	130.732	Flexural
70				113.787	130.752	218.232	130.752	Flexural
80				113.787	130.766	218.243	130.766	Flexural
90				113.787	130.776	218.25	130.776	Flexural
			ρ traditional	kip	kip	kip	kip	
50			0.63	155.303	169.561	231.45	169.561	Flexural
60				155.303	169.587	231.467	169.587	Flexural
70				155.303	169.604	231.477	169.604	Flexural
80				155.303	169.616	231.485	169.616	Flexural
90				155.303	169.624	231.49	169.624	Flexural

APPENDIX E: Ultimate capacity calculation algorithms for BS5400, ACI 318-05, BD81/02 and TRC approach

The subsequent equations present the detail of the procedure proposed by Taylor (2009) for predicting the ultimate strength of a laterally restrained brindge deck slabs.

INPUT

$\text{Span} := 50\text{ft}$ - Bridge Span

$L := 6\text{ft}$ - Spacing

$h := 7.5\text{in}$ - Deck Thickness

$f_y := 60\text{ksi}$ - Reinforcement yield strength

$f'c := 5\text{ksi}$ - Concrete compressive strength

$\rho := \frac{0.454}{100}$ - Reinforcement ratio at principal section

$\text{diam} := \frac{5}{8}\text{in}$ - Reinforcement diameter

$cl_c := 2\text{in}$ - Clear Cover

$d := h - cl_c - \frac{\text{diam}}{2} = 131.762 \cdot \text{mm}$ - Effective depth of slab

$c_x := 10\text{in}$ - Width of patch load parallel to slab span

$c_y := 20\text{in}$ - Width of patch load perpendicular to slab span

Support := S/S, F/E

- S/S: Simply Supported
- F/E: Fixed End

Support = 1

$load_{shape} :=$

Circular Load
Square Load
Rectangular Load

- Choose loaded area according to the load shape

$load_{shape} = 1$

Material of support beam

$Beam_m :=$

Concrete
Steel

1. Effective width of loaded slab

$$L_e := \frac{L}{2} - \frac{c_x}{2}$$

- half the span of the arch length

$$L_e = 787.4 \cdot mm$$

$$b_{eff} := c_y + 2 \cdot L_e + 2 \cdot h$$

- effective width of loaded slab

$$b_{eff} = 2.464 \times 10^3 \cdot mm$$

$$A_s := (b_{eff} \cdot d) \cdot \rho$$

- area steel reinforcement

$$A_s = 1.474 \times 10^3 \cdot mm^2$$

2. Stiffness Parameters

$$E_c := 4.23 \cdot \left(f'c \cdot \frac{N}{mm^2} \right)^{0.5} \cdot 1000$$

- concrete elastic modulus

$$E_c = 2.484 \times 10^4 \cdot \frac{N}{mm^2}$$

$$E_s := 200000 \frac{N}{mm^2}$$

$$A_{sl} := b_{eff} \cdot h$$

- area of slab

$$K_s := E_c \cdot \frac{A_{sl}}{L_e}$$

-stifness of slab within effective width

$$K_s = 1.48 \times 10^7 \cdot \frac{N}{mm}$$

$$I_{yb} := 81131 in^4$$

- Second moment of area of support beam about the vertical axis

$$I_{yb} = 3.377 \times 10^{10} \cdot mm^4$$

$$\zeta := \begin{vmatrix} 114.5 & \text{if Support} = 1 \\ 985 & \text{otherwise} \end{vmatrix}$$

- Constant for support condition

$$A_b := \zeta \cdot L_e \cdot \frac{I_{yb}}{b_{eff}^3}$$

- equivalent area of support beam

$$A_b = 2.036 \times 10^5 \cdot mm^2$$

$$K_b := \begin{vmatrix} A_b \cdot \dfrac{E_c}{L_e} & \text{if } Beam_m = 1 \\ A_b \cdot \dfrac{E_s}{L_e} & \text{otherwise} \end{vmatrix}$$

$$K_b = 6.421 \times 10^6 \cdot \frac{N}{mm}$$

- equivalent stiffness of support beam

$$A_d := (Span - b_{eff}) \cdot h$$

area of diaphragm + area of slab outside the effective width

$$A_d = 2.434 \times 10^6 \cdot mm^2$$

$$K_d := \frac{A_d \cdot E_c}{L_e}$$

- stiffness of slab within effective width

$K_d = 7.677 \times 10^7 \cdot \frac{N}{mm}$

$K_r := \frac{1}{\frac{1}{K_b} + \frac{1}{K_d}}$ - combined stiffness of restraint

$K_r = 5.925 \times 10^6 \cdot \frac{N}{mm}$

$\frac{K_r}{K_s} = 0.4$

---

3. Bending Capacity

$$\beta := \begin{cases} \left(1 - 0.003 \cdot \frac{f'c}{\frac{N}{mm^2}}\right) & \text{if } 1 - 0.003 \cdot \frac{(f'c)}{\frac{N}{mm^2}} < 0.9 \\ 0.9 & \text{otherwise} \end{cases}$$

- proportional depthof stress block (=0.9 in BS)

$\beta = 0.897$

$x := \frac{f_y \cdot A_s}{0.67 \cdot f'c \cdot \beta \cdot b_{eff}}$ - depth of concrete compression zone

$x = 11.95 \cdot mm$

$z := d - 0.5 \cdot \beta \cdot x$ - lever arm

$z = 126.405 \cdot mm$

$M_b := f_y \cdot A_s \cdot z$ - flexural moment of resitance

$M_b = 77.071 \cdot kN \cdot m$

$$P_b := \left(\frac{8}{L}\right)\cdot M_b$$

- predicted ultimate flexural capacity

$$P_b = 337.142\cdot kN$$

4. Arching Section

$$
\text{Iteration} := \begin{array}{|l}
\alpha \leftarrow 1 \\
d_1 \leftarrow \dfrac{h - 2x\cdot\beta}{2} \\
\text{error} \leftarrow 1 \\
\text{while error} > 0.001 \\
\quad \begin{array}{|l}
\text{check}_1 \leftarrow \alpha\cdot d_1 \\
A \leftarrow \alpha\cdot b_{eff}\cdot d_1 \\
L_r \leftarrow L_e\cdot\left(\dfrac{E_c\cdot A}{K_r\cdot L_e} + 1\right)^{\frac{1}{3}} \\
\varepsilon_u \leftarrow \begin{array}{|l} \left[\left[0.0043 - \left(\dfrac{f'c}{\frac{N}{mm^2}} - 60\right)\cdot 2.5\cdot 10^{-5}\right]\right] \text{ if } 0.0043 - \left(\dfrac{f'c}{\frac{N}{mm^2}} - 60\right)\cdot 2.5\cdot 10^{-5} \le 0.0 \\ 0.0043 \quad \text{otherwise} \end{array} \\
\varepsilon_c \leftarrow 2\cdot\varepsilon_u\cdot(1 - \beta) \\
R \leftarrow \dfrac{\varepsilon_c\cdot L_r^{\,2}}{4\cdot d_1^{\,2}} \\
u \leftarrow \begin{array}{|l} -0.15 + 0.36\sqrt{0.18 + 5.6\cdot R} \quad \text{if } R < 0.26 \\ 0.31 \quad \text{otherwise} \end{array} \\
\alpha \leftarrow 1 - \dfrac{u}{2}
\end{array}
\end{array}
$$

$$\left|\begin{array}{l} \left|\begin{array}{l} check_2 \leftarrow \alpha \cdot d_1 \\ error \leftarrow \dfrac{|check_1 - check_2|}{check_2} \end{array}\right. \\ \begin{pmatrix} \dfrac{A}{mm^2} \\ \dfrac{L_r}{mm} \\ R \\ u \\ \alpha \\ \dfrac{d_1}{mm} \end{pmatrix} \end{array}\right.$$

$A := Iteration_0 \cdot mm^2$ - Cross section area

$A = 2.015 \times 10^5 \cdot mm^2$

$L_r := Iteration_1 \cdot mm$ - half the span of the rigidly restrained arch

$L_r = 1.004 \times 10^3 \cdot mm$

$R := Iteration_2$ - McDowell'son-dimensional parameter (elastic deformation)

$R = 0.031$

$u := Iteration_3$ - McDowell's non-dimensional parameter (deflection)

$u = 0.065$

$\alpha := Iteration_4$ - proportion of d1 in contact with the support

$\alpha = 0.968$

$d_1 := Iteration_5 \cdot mm$ - half the arching depth

$d_1 = 84.536 \cdot mm$

$\alpha \cdot d_1 = 81.802 \cdot mm$

9. Arching Capacity

$$M_r := \begin{cases} \left(4.3 - 16.1 \cdot \sqrt{3.3 \cdot 10^{-4} + 0.1243R}\right) & \text{if } R < 0.26 \\ \dfrac{0.3615}{R} & \text{otherwise} \end{cases}$$

$M_r = 3.253$ - Moment ratio (non-dimensional)

$$M_a := 0.168 \cdot b_{eff} \cdot f'c \cdot {d_1}^2 \cdot M_r \cdot \frac{L_e}{L_r}$$

$$P_a := \frac{4}{L} \cdot M_a$$

$M_a = 260.189 \cdot kN \cdot m$ - arching moment of resistance

$P_a = 569.093 \cdot kN$ - predicted ultimate arching capacity

10. Flexural/ Arching punching capacity

$$P_{pf} := P_a + P_b$$

$P_{pf} = 203.73 \cdot kip$ - flexural punching capacity

11. Shear punching capacity

$$b_o := \begin{cases} \pi \cdot (c_x + d) & \text{if } load_{shape} = 1 \\ \text{otherwise} & \begin{cases} 4 \cdot (c_x + d) & \text{if } load_{shape} = 2 \\ 2 \cdot (c_x + c_y + 2 \cdot d) & \text{otherwise} \end{cases} \end{cases}$$

- Critical parameter

$$\rho_e := \left[\frac{(M_a + M_b)}{M_b}\right] \cdot \left(\frac{\frac{f_y}{\frac{N}{mm^2}}}{320}\right) \cdot \rho \cdot 100$$

- Effective reinforcement ratio at principal section

$$\rho_e = 2.568$$

$$P_{pv} := \frac{0.43}{1} \cdot \sqrt{f'c \cdot \frac{N}{mm^2}} \cdot b_o \cdot d \cdot (\rho_e)^{0.25}$$

$P_{pv} = 114.736 \cdot kip$ - shear punching capacity

12. Ultimate Capacity

$$P_p := \begin{cases} P_{pv} & \text{if } P_{pf} > P_{pv} \\ P_{pf} & \text{otherwise} \end{cases}$$

$$\text{Result} := \begin{array}{|l} \text{"Shear Type Failure"} \quad \text{if } P_{pv} < P_{pf} \\ \text{"Flexural Type Failure"} \quad \text{otherwise} \end{array}$$

$P_p = 114.736 \cdot \text{kip}$ - predicted ultimate capacity under proposed method

Result = "Shear Type Failure"

BS5400, ACI 318-05, and BD81/02 METHODS

INPUT

$h := 9.5\text{in}$ - Deck Thickness

$b := 1000\text{mm}$

$Span := 80\text{ft}$ - Bridge Span

$b = 39.37 \cdot \text{in}$

$L_s := 14\text{ft}$ - Spacing

$f_{cu} := 6250\text{psi} = 43.092 \cdot \frac{N}{mm^2}$

$f'c1 := 4000\text{psi}$ - Concrete compressive strength

$f'c := 0.8 \cdot f_{cu}$ - Concrete compressive strength

$f'c = 34.474 \cdot \frac{N}{mm^2}$

$A_s := 828.836\text{mm}^2$

$f_y := 60\text{ksi}$ - Reinforcement yield strength

$c_x := 10\text{in}$ - Width of patch load parallel to slab span

$c_y := 20\text{in}$ - Width of patch load perpendicular to slab span

$diam := \frac{5}{8}\text{in}$ - Reinforcement diameter

$cl_c := 2\text{in}$ - Clear Cover

$load_{shape} :=$

Circular Load
Square Load
Rectangular Load

- Choose loaded area according to the load shape

$load_{shape} = 1$

$load_{shape2} :=$

Circular Load
Square Load
Rectangular Load

- Choose loaded area according to the load shape

$load_{shape2} = \mathbf{1}$

$d := h - cl_c - \frac{diam}{\mathbf{2}} = \mathbf{182.563} \cdot mm$ - Effective depth of slab

$\rho := \frac{A_s}{(b \cdot d)} = \mathbf{4.54} \times \mathbf{10}^{-\mathbf{3}}$ - Reinforcement ratio at principal section

$\rho \cdot \mathbf{100} = \mathbf{0.454}$

BS METHOD

Flexural capacity under a concentrated load

$$M := A_s \cdot f_y \cdot d \cdot \left(\mathbf{1} - \frac{\mathbf{0.746} \cdot A_s \cdot f_y}{f_{cu} \cdot b \cdot d}\right)$$

$M = \mathbf{60.561} \cdot kN \cdot m$

from Pucher chart:

$P_{vf} := \frac{M}{\mathbf{0.08}m} = \mathbf{170.184} \cdot kip$

Shear capacity under a concentrated load

$$b_{o1} := \begin{cases} \pi \cdot (c_x + 3d) & \text{if } load_{shape} = \mathbf{1} \\ \text{otherwise} & \begin{cases} \mathbf{4} \cdot (c_x + 3d) & \text{if } load_{shape} = \mathbf{2} \\ \mathbf{2} \cdot (c_x + c_y + \mathbf{6} \cdot d) & \text{otherwise} \end{cases} \end{cases}$$

- Critical parameter at a distance of 1.5d

$$P_{vs} := \begin{cases} 0.79 \cdot \sqrt[3]{100 \cdot \dfrac{\left(\dfrac{A_s}{mm^2}\right)}{\left(\dfrac{b}{mm}\right) \cdot \dfrac{d}{mm}}} \cdot \sqrt[3]{\dfrac{\dfrac{f_{cu}}{\left(\dfrac{N}{mm^2}\right)}}{25}} \cdot \sqrt[4]{\dfrac{500}{\left(\dfrac{d}{mm}\right)}} \cdot b_{o1} \cdot d \cdot \dfrac{N}{mm^3} \cdot \dfrac{1m}{1000} & \text{if } \sqrt[4]{\dfrac{500}{\left(\dfrac{d}{mm}\right)}} < 1.5 \\ 0.79 \cdot \sqrt[3]{100 \cdot \dfrac{\left(\dfrac{A_s}{mm^2}\right)}{\left(\dfrac{b}{mm}\right) \cdot \dfrac{d}{mm}}} \cdot \sqrt[3]{\dfrac{\dfrac{f_{cu}}{\left(\dfrac{N}{mm^2}\right)}}{25}} \cdot 1.5 \cdot b_{o1} \cdot d \cdot \dfrac{N}{mm^3} \cdot \dfrac{1m}{1000} & \text{otherwise} \end{cases}$$

$$P_{vs} = 96.806 \cdot kip$$

ACI 318-05 METHOD

Flexural capacity under a concentrated load

$$\beta := 0.85 - 0.05 \cdot \left(\left(\frac{f'c - f'c1}{1000psi}\right)\right) = 0.8$$

$$M_{ACI} := \rho \cdot b \cdot f_y \cdot d^2 \cdot \left(1 - \frac{0.5 \cdot \rho \cdot f_y}{\beta \cdot f'c}\right)$$

$$M_{ACI} = 60.465 \, m \cdot kN$$

$$P_{avf} := \frac{M_{ACI}}{0.08m} = 169.914 \cdot kip$$

$$b_{o2} := \begin{cases} \pi \cdot (c_x + d) & \text{if } load_{shape2} = 1 \\ \text{otherwise} & \\ \quad \begin{cases} 4 \cdot (c_x + d) & \text{if } load_{shape2} = 2 \\ 2 \cdot (c_x + c_y + 2 \cdot d) & \text{otherwise} \end{cases} \end{cases}$$

- Critical parameter at a distance of 0.5d

Shear capacity under a concentrated load

$$P_{avs} := 4 \cdot \sqrt{\frac{f'c}{\frac{lbf}{in^2}}} \cdot b_{o2} \cdot d \cdot \frac{lbf}{in^2}$$

$$P_{avs} = 109.77 \cdot kip$$

BD81/02 METHOD

$$\frac{L_s}{h} = 17.684$$

$$\varepsilon_c := \left[-400 + 60 \cdot \frac{f'c}{\frac{N}{mm^2}} - 0.33 \cdot \left(\frac{f'c}{\frac{N}{mm^2}}\right)^2\right] \cdot 10^{-6}$$

$$R_1 := \frac{\varepsilon_c \cdot \left(\frac{L_s}{2}\right)^2}{h^2} = 0.1$$

$$k := 0.0525 \cdot \left(4.3 - 16.1 \cdot \sqrt{3.3 \cdot 10^{-4} + 0.1243 \cdot R_1}\right) = 0.13$$

$$\rho_e := \frac{k \cdot f'c \cdot h^2}{240 \cdot d^2 \cdot 1 \frac{N}{mm^2}} = 0.033$$

$$P_{BD81} := 1.52 \cdot \left(c_x + d\right) \cdot d \cdot \sqrt{\frac{f'c}{\frac{N}{mm^2}}} \cdot \left(100 \cdot \rho_e\right)^{0.25} \cdot \frac{N}{mm^2} = 215.055 \cdot kip$$

VITA

Andrea Toro was born . She completed her secondary school studies in . In 2010, she graduated from the Technological University of Central America (UNITEC) as a B.Sc. Civil Engineer with honors. Soon after graduation she volunteered for engineering companies to gain experience. In 2014, she came to the University of North Florida in Jacksonville, FL., to do her Masters in Science for Civil Engineering.