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A New Right Tailed Test of the Ratio of Variances

Elizabeth Rochelle Lesser

University of North Florida, n00836464@unf.edu

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A New Right Tailed Test of the Ratio of Variances

by

Elizabeth Rochelle Lesser

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This Thesis titled A New Right Tailed Tet of the Ratio of Variances is approved:

Ping Sa
Committee Chair

Pali Sen
Committee Member 1

Elena Buzaianu
Committee Member 2

Accepted for the Mathematics and Statistics Department:

Scott Hochwald
Department Chair

Accepted for the College of Arts and Sciences:

Daniel Moon
College Dean

Accepted for the University:

Dr. John Kantner
Dean of the Graduate School

DEDICATION

*I dedicate this Thesis to my Husband, Family, Dr. Ping Sa, and everyone who helped
along this journey. Could not have done it without you*

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ABSTRACT

It is important to be able to compare variances efficiently and accurately regardless of the parent populations. This study proposes a new right tailed test for the ratio of two variances using the Edgeworth's expansion. To study the Type I error rate and Power performance, simulation was performed on the new test with various combinations of symmetric and skewed distributions. It is found to have more controlled Type I error rates than the existing tests. Additionally, it also has sufficient power. Therefore, the newly derived test provides a good robust alternative to the already existing methods.

CHAPTER 1: INTRODUCTION

In many real world applications, analyzing variability is extremely important. The most common measurement of variability is standard deviation and/or variance. Although researchers are usually interested in comparing means, variance needs to be considered and controlled. If two means are compared from populations with unequal variabilities, the results could be incorrectly interpreted. Ott, Lyman, and Longnecker (2001) provided a case study on the Scholastic Assessment Test (SAT). The testing agency wanted to test a new method of administering the exam. A group of 182 high school students was randomly selected to participate in the study with 91 students randomly assigned to each of the two methods of administering the exam. The means of the final exam scores for the new and old methods were very close, but the standard deviation of the old method was significantly smaller than the standard deviation for the new method. If the differences in variabilities were overlooked, the testing agency might have compared means without accounting for unequal variances. Consequently, the conclusion may be different.

Additionally, comparing variabilities is greatly beneficial. By way of illustration, a soft drink firm is interested in evaluating their investment in a new type of canning machine (Ott, Lyman, and Longnecker, 2001). They will do so by determining whether the variability on the fills for the new machines is less than the variability on the current machines. 61 cans were selected from the output of both types of machines and the amount of fills is determined. If the new machine does have a smaller variance, then the

likelihood of over filling or under filling the cans of soda is greatly reduced. This saves money in the long run which is a good investment.

It is clear that the ability to compare variances between two populations accurately provides a lot of insight. The hypotheses for comparing two variances is

$$H_0: \sigma_1^2 = \sigma_2^2 \text{ vs. } H_a: \sigma_1^2 \neq \sigma_2^2 \quad (1.1)$$

To test (1.1), let $X_{i1}, X_{i2}, \dots, X_{in_i}$ be the random sample of size n_i from population i with a mean of μ_i and variance of σ_i^2 , $i = 1, 2$.

If both populations are normally distributed, the F test statistic can be used:

$$F^* = \frac{S_1^2}{S_2^2} \quad (1.2)$$

where $S_i^2 = \frac{\sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{n_i - 1}$ is the sample variance, and \bar{X}_i is the sample mean for $i = 1, 2$.

Under H_0 , F^* is distributed as $F_{(n_1-1, n_2-1)}$. When F^* is greater than $F_{(1-\frac{\alpha}{2}, n_1-1, n_2-1)}$ or when F^* is less than $F_{(\frac{\alpha}{2}, n_1-1, n_2-1)}$, the null hypothesis is rejected at the significant level of α .

One drawback of the F-test is that it is extremely sensitive from departures from normality. However, in real life, it is rare when both populations are normally distributed. Other methods for comparing two variances should be considered when the populations are not normal.

A well-known alternative to compare variances of non-normal populations is the modified Levene's test (Brown and Forsythe, 1974). To test the hypothesis test in (1.1), the modified Levene test statistic is calculated:

$$W_L = \frac{\frac{\sum_i n_i (\bar{z}_{i.} - \bar{z}_{..})^2}{1}}{\frac{\sum_i \sum_j (z_{ij} - \bar{z}_{i.})^2}{\sum_i (n_i - 1)}} \quad (1.3)$$

where $z_{ij} = |x_{ij} - \tilde{x}_i|$ is the absolute deviation from the median for sample i , $\bar{z}_{i.} =$

$$\sum_{j=1}^{n_i} \frac{z_{ij}}{n_i}, \text{ and } \bar{z}_{..} = \frac{\sum_{i=1}^2 \sum_{j=1}^{n_i} z_{ij}}{\sum_{i=1}^2 n_i}. W_L \text{ follows } F_{(1, n_1 + n_2 - 2)} \text{ under } H_0. \text{ When } W_L \text{ is}$$

greater than the critical value of $F_{(1 - \alpha, 1, n_1 + n_2 - 2)}$, the null hypothesis is rejected.

According to Brown and Forsythe (1974), the modified Levene's test performs conservatively when the two populations are either Gaussian with small sample sizes, long tailed, or Cauchy. However, it can 'maintain its size near the five-percent level of significance' for the chi-square distribution with four degrees of freedom. Only limited Type I error rates and Power simulations were studied in their article.

Bonett (2006) provided a method to construct a $(1 - \alpha) * 100\%$ confidence interval for the ratio of the two standard deviations, $\frac{\sigma_1}{\sigma_2}$. The natural log of the ratio of variances was used in their derivation instead of the regular ratio because the natural log of the sample variance was proven to approach normality at a faster rate than the original sample variance for growing sample sizes. The interval is constructed as follows:

$$\exp \left\{ \ln \left(\frac{cs_1^2}{s_2^2} \right) \pm z_{1 - \frac{\alpha}{2}} \right\} \quad (1.4)$$

$$\text{where } c = \left\{ \frac{n_1 / (n_1 - z_{1-\frac{\alpha}{2}})}{n_2 / (n_2 - z_{1-\frac{\alpha}{2}})} \right\}, \text{ se} = \left\{ \frac{\bar{Y}_{4p} - \frac{n_1 - 3}{n_1}}{n_1 - 1} + \frac{\bar{Y}_{4p} - \frac{n_2 - 3}{n_2}}{n_2 - 1} \right\}^{\frac{1}{2}}, \bar{Y}_{4p} =$$

$\frac{(n_1 + n_2) \sum_i \sum_j (x_{ij} - m_i)^4}{[\sum_i \sum_j (x_{ij} - \bar{x}_i)^2]^2}$ is the pooled kurtosis estimate, and m_i is the sample trimmed

mean with proportion equal to $\frac{1}{\{2(n_i - 4)^{\frac{1}{2}}\}}$, $i = 1, 2$. If the interval does not include one,

the variances are concluded to be unequal.

Bonett's simulations showed that the confidence interval has a coverage probability that is roughly $(1 - \alpha) * 100\%$ when the population distributions are the same, regardless of the sample sizes. However, when the two populations were not identical, coverage probability suffered when the first distribution was less skewed than the second, and sample sizes of 30 and 10 were selected from the first and second populations, respectively. Moreover, only a handful of simulations with different populations and equal variances were investigated.

Rajić and Stanojević (2013) proposed a $(1 - \alpha) * 100\%$ confidence interval for the ratio of two variances using Edgeworth's expansion and Johnson's transformation. The confidence interval is constructed using:

$$P(T_{\text{Rajic}} \leq x) = \Phi(x) + \frac{1}{\sqrt{n_1}} p(x) \phi(x) + O(n_1^{-1})$$

where $\phi(x)$ and $\Phi(x)$ are the probability density function and cumulative distribution function of the standard normal variable,

$$p(x) = \frac{M'_3}{6} (2x^2 + 1),$$

$$M'_3 = E\left(\left(\frac{1}{n_1}\right) \sum_{i=1}^{n_1} X_i'^3\right),$$

$$X_i' = \frac{\frac{(X_{1i} - \bar{X}_1)^2}{\sum_{j=1}^{n_2} (X_{2j} - \bar{X}_2)^2} - \frac{(n_1-1)(n_2+1)\sigma_1^2}{n_1(n_2-1)^2\sigma_2^2}}{\sqrt{E\left(\frac{(X_{1i} - \bar{X}_1)^2}{\sum_{j=1}^{n_2} (X_{2j} - \bar{X}_2)^2} - \frac{(n_1-1)(n_2+1)\sigma_1^2}{n_1(n_2-1)^2\sigma_2^2}\right)^2}} \text{ for } i = 1, 2, \dots, n_1,$$

$$\text{and } T_{\text{Rajic}} = \frac{\frac{S_1^2}{S_2^2} - \frac{\sigma_1^2}{\sigma_2^2} \left(\frac{n_2+1}{n_1+1}\right)}{\sqrt{\text{var}\left(\frac{S_1^2}{S_2^2}\right)}}.$$

They applied Johnson's transformation and it has the following form:

$$g(T_{\text{Rajic}}) = T_{\text{Rajic}} + \frac{1}{3} \frac{1}{\sqrt{n_1}} \hat{M}'_3 T_{\text{Rajic}}^2 + \frac{1}{6} \frac{1}{\sqrt{n_1}} \hat{M}'_3 \quad (1.5)$$

with the solution of $g(T_{\text{Rajic}}) = x$, up to term of order $\frac{1}{\sqrt{n_1}}$ is:

$$T_{\text{Rajic}} = g^{-1}(x) = x - \frac{1}{3} \frac{1}{\sqrt{n_1}} \hat{M}'_3 x^2 - \frac{1}{6} \frac{1}{\sqrt{n_1}} \hat{M}'_3 \quad (1.6)$$

where \hat{M}'_3 is the moment estimator of M'_3 . There is no closed form for this confidence interval.

In their simulation study, the proposed confidence interval has low coverage probabilities compared to the nominal level for all the cases considered. Furthermore, the study is very limited and displays simulations that were run on a few selected distributions with unequal variances.

As seen throughout Chapter 1, there does not exist an adequate method of comparing the ratio of two variances for any two populations. Therefore, this research is

on the development of a new test for testing the equality of variances. At this point, we would like to focus on the right-tailed test:

$$H_0: \sigma_1^2 = \sigma_2^2 \text{ vs. } H_a: \sigma_1^2 > \sigma_2^2 \quad (1.7)$$

In Chapter 2, the proposed methods and their derivations are provided. The Type I error rates and Power simulation study are in Chapter 3 with the summary of the results in Chapter 4. The conclusion is found in Chapter 5.

CHAPTER 2: THE PROPOSED TEST STATISTIC

Let $X_{1i}, X_{1i}, \dots, X_{1n_1}$ be a random sample of size n_1 from population one and let $X_{2i}, X_{2i}, \dots, X_{2n_2}$ of size n_2 be a random sample from population two with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 , respectively. The derivation of the proposed test statistic utilizes the Edgeworth Expansion and Edgeworth Inversion formula.

2.1: Edgeworth Expansion

Edgeworth expansion is an approximation to the distribution of any estimate $\hat{\theta}$ of the unknown quantity θ_0 . The distribution function of $(\hat{\theta} - \theta_0)$ can be expanded as:

$$P\left(\frac{(\hat{\theta} - \theta_0)}{\sigma_{\hat{\theta}}} \leq x\right) = \Phi(x) + \frac{1}{\sqrt{n}} p_1(x) \phi(x) \dots + \frac{1}{\sqrt{n}} p_i(x) \phi(x) + \dots, \quad (2.1)$$

where $\phi(x) = \exp\left\{-\frac{x^2}{2}\right\} (2\pi)^{-\frac{1}{2}}$ is the standard normal density function, $\Phi(x)$ is the standard normal distribution function, and $p_i(x)$ are polynomials with coefficients depending on the cumulants of $(\hat{\theta} - \theta_0)$ (Hall, 1992).

When estimating $\sigma_{\hat{\theta}}$, the distribution function of $(\hat{\theta} - \theta_0)$ can be expanded as:

$$P\left(\frac{(\hat{\theta} - \theta_0)}{\hat{\sigma}_{\hat{\theta}}} \leq x\right) = \Phi(x) + \frac{1}{\sqrt{n}} q_1(x) \phi(x) \dots + \frac{1}{\sqrt{n}} q_i(x) \phi(x) + \dots, \quad (2.2)$$

where $q_i(x)$ are polynomials with coefficients depending on the cumulants of $(\hat{\theta} - \theta_0)$ (Kendall et al., 1994).

2.2: Edgeworth Inversion Formula

Hall (1983) provided the inversion formula:

$$P\left(\frac{(\hat{\theta}-\theta_0)}{\sigma_{\hat{\theta}}} \leq x + \frac{1}{6}\gamma p(x)\right) = \Phi(x) + o(n^{-\frac{1}{2}}) \quad (2.3)$$

where $p(x) = (1 + 2x^2)$ and γ is the coefficient of skewness of $\hat{\theta}$. Also, when estimating $\sigma_{\hat{\theta}}$, the inversion formula becomes:

$$P\left(\frac{(\hat{\theta}-\theta_0)}{\hat{\sigma}_{\hat{\theta}}} \leq x + \frac{1}{6}\gamma q(x)\right) = \Phi(x) + o(n^{-\frac{1}{2}}) \quad (2.4)$$

where $q(x) = (x^2 - 1)$.

2.3: Derivation of Proposed Test Statistic

To test $H_0: \sigma_1^2 = \sigma_2^2$ vs. $H_a: \sigma_1^2 > \sigma_2^2$, consider $\hat{\theta} = \frac{S_1^2}{S_2^2}$ and define

$$Z = \frac{(\hat{\theta}-\theta_0)}{\sigma_{\hat{\theta}}} = \frac{\frac{S_1^2}{S_2^2} - E\left(\frac{S_1^2}{S_2^2}\right)}{\sqrt{\text{var}\left(\frac{S_1^2}{S_2^2}\right)}}. \quad (2.5)$$

where S_i^2 is the unbiased estimator for σ_i^2 . The approximate expectation for the ratio of two random variables is given by Grossman and Norton (1981) as follows:

$$E\left(\frac{X}{Y}\right) \cong \frac{E(X)}{E(Y)} \left[1 + \frac{\text{Var}(Y)}{E(X)^2} - \frac{\text{cov}(X,Y)}{E(X)E(Y)}\right] \text{ for } Y > 0 \quad (2.6)$$

Therefore, the expected value for the ratio of two sample variances is approximated by:

$$E\left(\frac{S_1^2}{S_2^2}\right)$$

$$\begin{aligned}
&\cong \frac{E(S_1^2)}{E(S_2^2)} \left[1 + \frac{\text{Var}(S_2^2)}{E(S_1^2)^2} - \frac{\text{cov}(S_1^2, S_2^2)}{E(S_1^2)E(S_2^2)} \right] \\
&= \frac{\sigma_1^2}{\sigma_2^2} \left[1 + \frac{\frac{K_{4(2)} + 2\sigma_2^4}{n_2 - n_2 - 1}}{\sigma_1^4} \right] \\
&= \frac{\sigma_1^2}{\sigma_2^2} \left(1 + \left[\frac{1}{\sigma_1^4} \left(\frac{2\sigma_2^4}{n_2 - 1} \right) \right] + \left[\frac{1}{\sigma_1^4} \left(\frac{K_{4(2)}}{n_2} \right) \right] \right) \\
&= \frac{\sigma_1^2}{\sigma_2^2} + \frac{\sigma_1^2}{\sigma_2^2} \left[\frac{1}{\sigma_1^4} \left(\frac{2\sigma_2^4}{n_2 - 1} \right) \right] + \frac{\sigma_1^2}{\sigma_2^2} \left[\frac{1}{\sigma_1^4} \left(\frac{K_{4(2)}}{n_2} \right) \right] \\
&= \frac{\sigma_1^2}{\sigma_2^2} + \frac{\sigma_1^2}{\sigma_2^2} \left[\frac{\sigma_2^4}{\sigma_1^4} \left(\frac{2}{n_2 - 1} \right) \right] + \frac{\sigma_1^2}{\sigma_2^2} \left[\frac{1}{\sigma_1^4} \left(\frac{K_{4(2)}}{n_2} \right) \right] \\
&= \frac{\sigma_1^2}{\sigma_2^2} + \left[\frac{\sigma_2^2}{\sigma_1^2} \left(\frac{2}{n_2 - 1} \right) \right] + \frac{\sigma_1^2}{\sigma_2^2} \left[\frac{1}{\sigma_1^4} \left(\frac{K_{4(2)}}{n_2} \right) \right] \tag{2.7}
\end{aligned}$$

where $K_{4(i)}$ is the fourth cumulant of population i (Kendall, Stuart, Ord, Arnold, & O'Hagan, 1994).

Additionally, the approximate variance for the ratio of two random variables (Stuart, Ord, & Arnold, 1999) is given by:

$$V\left(\frac{X}{Y}\right) \cong \frac{E(X)^2}{E(Y)^2} \left[\frac{\text{Var}(X)}{E(X)^2} - 2 \left(\frac{\text{cov}(X,Y)}{E(X)E(Y)} \right) + \frac{\text{Var}(Y)}{E(Y)^2} \right] \tag{2.8}$$

Consequently, $\sigma_{\hat{\theta}}^2 = \text{var}\left(\frac{S_1^2}{S_2^2}\right)$ is approximated by:

$$\frac{E(S_1^2)^2}{E(S_2^2)^2} \left[\frac{\text{Var}(S_1^2)}{E(S_1^2)^2} - 2 \left(\frac{\text{cov}(S_1^2, S_2^2)}{E(S_1^2)E(S_2^2)} \right) + \frac{\text{Var}(S_2^2)}{E(S_2^2)^2} \right]$$

$$\begin{aligned}
&= \frac{\sigma_1^4}{\sigma_2^4} \left[\frac{\frac{K_{4(1)} + 2\sigma_1^4}{n_1 + n_1 - 1}}{\sigma_1^4} + \frac{\frac{K_{4(2)} + 2\sigma_2^4}{n_2 + n_2 - 1}}{\sigma_2^4} \right] \\
&= \frac{\sigma_1^4}{\sigma_2^4} \left[\frac{K_{4(1)}}{\sigma_1^4 n_1} + \frac{K_{4(2)}}{\sigma_2^4 n_2} + \frac{2}{n_1 - 1} + \frac{2}{n_2 - 1} \right] \\
&= \frac{\sigma_1^4}{\sigma_2^4} \left[\frac{K_{4(1)}}{\sigma_1^4 n_1} + \frac{K_{4(2)}}{\sigma_2^4 n_2} \right] + \frac{\sigma_1^4}{\sigma_2^4} \left(\frac{2}{n_1 - 1} + \frac{2}{n_2 - 1} \right) \\
&= \frac{1}{\sigma_2^4} \left[\frac{K_{4(1)}}{n_1} + \frac{\sigma_1^4 K_{4(2)}}{\sigma_2^4 n_2} \right] + 2 \frac{\sigma_1^4}{\sigma_2^4} \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right) \tag{2.9}
\end{aligned}$$

With the approximations in (2.7) and (2.9), the original Z statistic from (2.5) looks like:

$$Z = \frac{\frac{S_1^2}{S_2^2} - \left(\frac{\sigma_1^2}{\sigma_2^2} + \left[\frac{\sigma_2^2}{\sigma_1^2} \left(\frac{2}{n_2 - 1} \right) \right] + \frac{\sigma_1^2}{\sigma_2^2} \left[\frac{1}{\sigma_1^4} \left(\frac{K_{4(2)}}{n_2} \right) \right] \right)}{\sqrt{\frac{1}{\sigma_2^4} \left[\frac{K_{4(1)}}{n_1} + \frac{\sigma_1^4 K_{4(2)}}{\sigma_2^4 n_2} \right] + 2 \frac{\sigma_1^4}{\sigma_2^4} \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}} \tag{2.10}$$

Under H_0 : $\sigma_1^2 = \sigma_2^2$, Z can be reduced to:

$$Z = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2 - 1} \right) \right] + \left[\frac{1}{\sigma_1^4} \left(\frac{K_{4(2)}}{n_2} \right) \right] \right)}{\sqrt{\frac{1}{\sigma_2^4} \left[\frac{K_{4(1)}}{n_1} + \frac{K_{4(2)}}{n_2} \right] + 2 \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}} \tag{2.11}$$

When $\sigma_{\hat{\theta}}$ is unknown in (2.5), the T statistic is considered with σ_i^2 estimated by S_i^2 and

$K_{4(i)}$ estimated by $\hat{K}_{4(i)}$ in (2.12):

$$T = \frac{(\hat{\theta} - \theta_0)}{\hat{\sigma}_{\hat{\theta}}} = \frac{\frac{S_1^2}{S_2^2} - E\left(\frac{S_1^2}{S_2^2}\right)}{\sqrt{\widehat{\text{var}}\left(\frac{S_1^2}{S_2^2}\right)}} = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2 - 1} \right) \right] + \left[\frac{1}{S_1^4} \left(\frac{\hat{K}_{4(2)}}{n_2} \right) \right] \right)}{\sqrt{\frac{1}{S_2^4} \left[\frac{\hat{K}_{4(1)}}{n_1} + \frac{\hat{K}_{4(2)}}{n_2} \right] + 2 \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}} \tag{2.12}$$

where $\hat{K}_{4(i)} = \frac{n_i(n_i+1)S'_4 - 3(n_i-1)S_2'^2}{(n_i-1)(n_i-2)3}$ and $S'_k = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^k$ for $k = 2, 4$ (Kendall

et al., 1994). Since $\hat{K}_{4(i)}$ may be negative and lead to a negative $\widehat{\text{var}}(\frac{S_1^2}{S_2^2})$, action needs to

be taken. Whenever $\widehat{\text{var}}(\frac{S_1^2}{S_2^2}) < 0$, $\frac{1}{S_2^4} [\frac{\hat{K}_{4(1)}}{n_1} + \frac{\hat{K}_{4(2)}}{n_2}]$ is set to 0 which makes the

denominator of (2.12) equivalent to the denominator in Rajić and Stanojević's test

statistic, $T_{\text{Rajić}}$.

From preliminary simulations, T in (2.12) did not provide an advantage for larger sample sizes. As the sample size increased, the Type I error rate didn't always decrease.

Thus, replacing n_i in the denominator with the smallest sample size denoted by n_{\min}

resolved the issue:

$$T1 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^4} \left(\frac{\hat{K}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left[\frac{\hat{K}_{4(1)}}{n_1} + \frac{\hat{K}_{4(2)}}{n_2}\right] + 2\left(\frac{1}{n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.13)$$

Furthermore, $(S_1^2)^2$ is replaced by the consistent estimator in the numerator, $(S_1^{2*})^2 =$

$\left(\frac{\sum_{j=1}^{n_1} (X_{1j} - \bar{X}_1)^2}{n_1}\right)^2$ which may provide stricter control over the Type I error rates:

$$T2 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^{4*}} \left(\frac{\hat{K}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left[\frac{\hat{K}_{4(1)}}{n_1} + \frac{\hat{K}_{4(2)}}{n_2}\right] + 2\left(\frac{1}{n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.14)$$

In addition to T , $T1$, and $T2$, there are different ways to construct the test statistic for the

variable $\frac{S_1^2}{S_2^2}$.

$$T3 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^{4*}} \left(\frac{\hat{R}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\frac{\hat{R}_{4(1)}}{n_1} + \frac{\hat{R}_{4(2)}}{n_2}\right) + 2 \frac{S_1^2}{S_2^2} \left(\frac{1}{n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.15)$$

$$T4 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^4} \left(\frac{\hat{R}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\frac{\hat{R}_{4(1)}}{n_1} + \frac{\hat{R}_{4(2)}}{n_2}\right) + 2 \frac{S_1^2}{S_2^2} \left(\frac{1}{n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.16)$$

$$T5 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\frac{\hat{R}_{4(1)}}{n_1} + \frac{\hat{R}_{4(2)}}{n_2}\right) + 2 \frac{S_1^2}{S_2^2} \left(\frac{1}{n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.17)$$

$$T6 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^{4*}} \left(\frac{\hat{R}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\frac{\hat{R}_{4(1)}}{n_1} + \frac{\hat{R}_{4(2)}}{n_2}\right) + 2 \left(\frac{S_1^2}{S_2^2 n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.18)$$

$$T7 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^4} \left(\frac{\hat{R}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\frac{\hat{R}_{4(1)}}{n_1} + \frac{\hat{R}_{4(2)}}{n_2}\right) + 2 \left(\frac{S_1^2}{S_2^2 n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.19)$$

$$T8 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\frac{\hat{R}_{4(1)}}{n_1} + \frac{\hat{R}_{4(2)}}{n_2}\right) + 2 \left(\frac{S_1^2}{S_2^2 n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.20)$$

$$T9 = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2-1}\right)\right] + \left[\frac{1}{S_1^{4*}} \left(\frac{\hat{R}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\left(1 + \frac{\hat{\eta}_1}{2}\right) \frac{\hat{R}_{4(1)}}{n_1} + \left(1 + \frac{\hat{\eta}_2}{2}\right) \frac{\hat{R}_{4(2)}}{n_2} \right) + 2 \frac{S_1^2}{S_2^2} \left(\frac{1}{n_{\min}-1} + \frac{1}{n_{\min}-1}\right)}} \quad (2.21)$$

$$T_{10} = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2 - 1}\right)\right] + \left[\frac{1}{S_1^4} \left(\frac{\hat{K}_{4(2)}}{n_2}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\left(1 + \frac{\hat{\eta}_1}{2}\right) \frac{\hat{K}_{4(1)}}{n_1} + \left(1 + \frac{\hat{\eta}_2}{2}\right) \frac{\hat{K}_{4(2)}}{n_2} \right) + 2 \frac{S_1^2}{S_2^2} \left(\frac{1}{n_{\min} - 1} + \frac{1}{n_{\min} - 1} \right)}} \quad (2.22)$$

$$T_{11} = \frac{\frac{S_1^2}{S_2^2} - \left(1 + \left[\left(\frac{2}{n_2 - 1}\right)\right]\right)}{\sqrt{\frac{1}{S_2^4} \left(\left(1 + \frac{\hat{\eta}_1}{2}\right) \frac{\hat{K}_{4(1)}}{n_1} + \left(1 + \frac{\hat{\eta}_2}{2}\right) \frac{\hat{K}_{4(2)}}{n_2} \right) + 2 \frac{S_1^2}{S_2^2} \left(\frac{1}{n_{\min} - 1} + \frac{1}{n_{\min} - 1} \right)}} \quad (2.23)$$

where $\hat{\eta}_i = \frac{\hat{K}_{4(i)}}{S_1^4}$ is a correction term based on simulation study. It was originally used for

the robust chi-square statistic which has the form $\frac{(n_i - 1) \hat{d} S_i^2}{\sigma_i^2}$ where $\hat{d} = \left(1 + \frac{\hat{\eta}_i}{2}\right)^{-1}$.

2.4: Derivation of the Proposed Decision Rule

The derived test statistics from the previous section need a decision rule to perform the hypothesis test in (1.7).

Applying the inversion formulas in (2.3) and (2.4), the considered decision rules for every form of T, is to reject H_0 when:

$$T > T_{(1-\alpha, n_1 + n_2 - 2)} + \frac{1}{6} \gamma \left(1 + 2 T_{(1-\alpha, n_1 + n_2 - 2)}^2\right) \quad (2.24)$$

and

$$T > T_{(1-\alpha, n_1 + n_2 - 2)} + \frac{1}{6} \gamma \left(T_{(1-\alpha, n_1 + n_2 - 2)}^2 - 1\right) \quad (2.25)$$

$$\text{where } \gamma = E\left(\left(\frac{1}{n_1}\right) \sum_{i=1}^{n_1} X_i'^3\right) \text{ with } X_i' = \frac{\frac{(X_{1i} - \bar{X}_1)^2}{\sum_{j=1}^{n_2} (X_{2j} - \bar{X}_2)^2} - \frac{(n_1 - 1)(n_2 + 1)\sigma_1^2}{n_1(n_2 - 1)^2\sigma_2^2}}{\sqrt{E\left(\frac{(X_{1i} - \bar{X}_1)^2}{\sum_{j=1}^{n_2} (X_{2j} - \bar{X}_2)^2} - \frac{(n_1 - 1)(n_2 + 1)\sigma_1^2}{n_1(n_2 - 1)^2\sigma_2^2}\right)^2}}.$$

From preliminary simulation studies, the decision rules in (2.24) and (2.25) had inflated Type I error rates when the first sample size was larger than the second. This was a result of a drastically reduced critical value of T due to the large sample size and a larger test statistic from the variability in the smaller sample. When the reduced critical value was compared to the larger test statistic, the tests rejected the null hypothesis more often. Thus, to control the Type I error, $T_{(1-\alpha, n_1 + n_2 - 2)}$ was replaced by $T_{(1-\alpha, 2n_{\min} - 2)}$:

$$T > T_{(1-\alpha, 2n_{\min} - 2)} + \frac{1}{6}\gamma(1 + 2T_{(1-\alpha, 2n_{\min} - 2)}^2) \quad (2.26)$$

and

$$T > T_{(1-\alpha, 2n_{\min} - 2)} + \frac{1}{6}\gamma(T_{(1-\alpha, 2n_{\min} - 2)}^2 - 1) \quad (2.27)$$

Other decision rules that did not incorporate the Edgeworth Inversion formulas were also considered and compared:

$$T > T_{(1-\alpha, 2n_{\min} - 2)} \quad (2.28)$$

$$T > Z_{(1-\alpha)} \quad (2.29)$$

CHAPTER 3: SIMULATION

The purpose of the simulation study is to compare Type I error rates and Power performance of different right tailed tests for equal variances. The simulations were run in R. The proposed test statistics are compared to all the existing methods which include: F, Bonett, Modified Levene's, and Rajić. The following sections will summarize the Type I error and Power simulation procedure, the parent distributions considered, and the reconfigured right tailed test statistics from Chapter 1. Pseudocode and program code for each test statistic is in the Appendix.

3.1: Parent Distributions Examined

The parent distributions considered for the Type I error and Power simulation studies are:

- Normal described by $\text{Normal}(\mu, \sigma^2)$ with $\mu = 0$ and $\sigma^2 = 0.083, 0.5, 1, 2, 3, 4, 6, 8$
- Studentized T described by $T(\gamma)$ with γ degrees of freedom = 3, 4, 5, 6. The expected value and variance for the studentized T distribution are $\mu = 0$ and $\sigma^2 = \frac{\gamma}{\gamma-2}$.
- Gamma described by $\text{Gamma}(\alpha, \beta)$ with shape parameter $\alpha = 0.5, 1, 3/2, 2, 3, 4, 5, 10, 15, 20$ and scale parameter $\beta = 0.5, 1, 2, \sqrt{3}, 3, 4, 10$. The expected value and variance for the Gamma distribution are $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$.

- Exponential described by $\text{Exp}(\lambda)$ with $\lambda = 1$. The expected value and variance for the Exponential distribution are $\mu = \lambda$ and $\sigma^2 = \lambda^2$.
- Weibull described by $\text{Weibull}(\lambda, k)$ with shape parameter $\lambda = 1, 2, 5, 10$ and scale parameter $k = 1$. The expected value and variance for the Weibull distribution are $\mu = \lambda \Gamma(1 + \frac{1}{k})$, and $\sigma^2 = \lambda^2 [\Gamma(1 + \frac{2}{k}) - (\Gamma(1 + \frac{1}{k}))^2]$.
- Beta described by $\text{Beta}(\alpha, \beta)$ with shape parameters $\alpha = 0.5, 1, 2, 3, 5$ and $\beta = 0.5, 1, 2, 3, 4$. The expected value and variance for the Beta distribution are $\mu = \frac{\alpha}{\alpha + \beta}$ and $\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$.
- Chi-Squared described by $\text{Chisq}(\gamma)$ with γ degrees of freedom = 1, 3, 9. The expected value and variance for the Chi-Squared distribution are $\mu = \gamma$ and $\sigma^2 = 2\gamma$.
- Log-Normal described by $\text{LogNorm}(\mu, \sigma^2)$ with $\mu = 0$ and $\sigma^2 = 1$. The expected value and variance for the LogNormal distribution are $\exp^{\frac{\mu + \sigma^2}{2}}$ and $(\exp^{\sigma^2} - 1)\exp^{2\mu + \sigma^2}$, respectively.

For each pair of populations considered, sample sizes of (10, 10), (20, 20), (30, 30), (30, 10) and (10, 30) were examined with nominal levels of 0.05 and 0.01.

3.2: Type I Error Simulation Study

To study the Type I error rate, different combinations of symmetric and skewed distributions were run 10,000 times each for sample sizes (10, 10), (20, 20), (30, 30), (30, 10), and (10, 30). The Type I error rate was calculated as the number of times H_0 was rejected divided by 10,000. Results are found in tables I, II, III, and IV of the Appendix for α levels of 0.05 and 0.1.

3.3: Power Simulation Study

For Power performance, a few distributions and sample size combinations were considered and simulated 10,000 times. The first population is assumed to have variance K-times larger than the variance of the second population where $K = 2, 3, 4$. Results are found in tables V and VI of the Appendix for α level of 0.05.

3.4: Simulation Code Summary

The program requires input from the user that contains information about the parent distributions from which the two independent samples are drawn, the sample sizes, the simulation size, the alpha value, K for the power study, and the seed. After all the information is input, it collects the random samples of data, calculates all the test statistics, and compares it to the critical value for the corresponding test statistic's distribution. If the test statistic is greater than the critical value, it increases the number of times the hypothesis is rejected by one. The above process is repeated based on the number of times specified by the user. Once all the simulations are performed, the number of times the hypothesis is rejected is divided by the simulation size to retrieve a proportion for each test. If the distributions from which the samples are drawn have the same variance, the proportion measures the Type I error rate. If the distributions from

which the samples are drawn do not have the same variance, the proportion measures the Power.

In order to accurately compare the existing methods, from Chapter 1, to the proposed hypotheses tests, they were repurposed into right tailed hypotheses tests. The following sections summarize the existing method reconfigurations.

3.4.1: F Right Tailed Test Statistic

The right tailed hypothesis F-test test statistic is:

$$F^* = \frac{S_1^2}{S_2^2} \quad (3.1)$$

If F^* is greater than $F_{(1-\alpha, n_1-1, n_2-1)}$, the null hypothesis in (1.7) is rejected.

3.4.2: Bonett Right Tailed Test Statistic and Pseudocode

The right tailed hypothesis Bonett test statistic is:

$$Z^* = \frac{\ln\left(\frac{\sigma_1^2}{\sigma_2^2}\right) - \ln\left(\left(\frac{\frac{n_1}{n_1-z_1-\alpha}}{\frac{n_2}{n_2-z_1-\alpha}}\right) \frac{S_1^2}{S_2^2}\right)}{\left\{ \frac{\bar{y}_{4p} - \frac{n_1-3}{n_1}}{n_1-1} + \frac{\bar{y}_{4p} - \frac{n_2-3}{n_2}}{n_2-1} \right\}^{\frac{1}{2}}} \quad (3.2)$$

When Z^* is greater than $Z_{1-\alpha}$, the null hypothesis in (1.7) is rejected.

3.4.3: Modified Levene's Right Tailed Test Statistic

The right tailed hypothesis test statistic for the modified Levene's Test is:

$$T^* = \sqrt{W_L} = \sqrt{\frac{\frac{\sum_i n_i (\bar{Z}_i - \bar{Z}_{..})^2}{1}}{\frac{\sum_i \sum_j (Z_{ij} - \bar{Z}_i.)^2}{\sum_i (n_i - 1)}}} \quad (3.3)$$

When T^* is greater than the critical value of $T_{(1-\alpha, n_1 + n_2 - 2)}$, the null hypothesis in (1.7) is rejected.

3.4.4: Rajić and Stanojević Right Tailed Test Statistic

Rajić and Stanojević provided the following Z test statistic:

$$Z = \frac{\frac{S_1^2}{S_2^2} - \frac{\sigma_1^2}{\sigma_2^2} \left(1 + \left[\left(\frac{2}{n_2 - 1} \right) \right] \right)}{\sqrt{2 \frac{\sigma_1^4}{\sigma_2^4} \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}} \quad (3.4)$$

Estimating σ_i^2 with the sample variance for population i, S_i^2 , T_{Rajic} is considered:

$$T_{\text{Rajic}} = \frac{\frac{S_1^2}{S_2^2} - \frac{\sigma_1^2}{\sigma_2^2} \left(\frac{n_2 + 1}{n_1 + 1} \right)}{\sqrt{2 \frac{S_1^4}{S_2^4} \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}} \quad (3.5)$$

Under H_0 , T_{Rajic} reduces to:

$$T_{\text{Rajic}} = \frac{\frac{S_1^2}{S_2^2} - \left(\frac{n_2 + 1}{n_1 + 1} \right)}{\sqrt{2 \frac{S_1^4}{S_2^4} \left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}} \quad (3.6)$$

Rajić and Stanojević then applied Johnson's transformation using (3.6) to get the following test statistic:

$$g(T_{\text{Rajic}}) = T_{\text{Rajic}} + \frac{1}{3} \frac{1}{\sqrt{n_1}} \hat{M}_3' T_{\text{Rajic}}^2 + \frac{1}{6} \frac{1}{\sqrt{n_1}} \hat{M}_3' \quad (3.7)$$

For the right tailed hypothesis test, when $g(T_{\text{Rajic}})$ is greater than the critical value of $Z_{1-\alpha}$, the null hypothesis in (1.7) is rejected.

CHAPTER 4: RESULTS

In this chapter, the Type I error rates and Power are compared between the proposed test statistics and existing methods. From preliminary simulation studies, T3, T6, and T9 with the decision rule in (2.27) produce the best Type I error rates and Power. Therefore, for the ease of comparison and analysis of the different statistics the other derived test statistics and decision rules from Chapter 2 will not be mentioned discussed.

4.1: Type I Error Rate Comparisons

Results for the Type I error rates for identical distributions are found in Tables 1 and 3 of Appendix A, and results for the Type I error rates for two different parent distributions are found in Tables 2 and 4 of Appendix for nominal levels of 0.05 and 0.1, respectively.

One can immediately notice that the F-test fails almost all cases except when the parent distributions are Normal, Beta, or some cases of Weibull. It appears to perform the best when the distributions have a small kurtosis and skew.

Additionally, Rajić's test also produces conservative Type I error rates when the distributions are Normal, Beta, Weibull, and some cases of Gamma. However, it is completely out of control for most combinations and sample sizes. For a lot of cases with the same distribution combination, the Type I error rate can be extremely conservative, or

inflated depending on the sample size. Although smaller sample sizes perform better, it is still uncontrollable.

4.1.1: Identical Populations

When the two parent distributions are the same, Bonett and W_L usually have error rates around α for sample sizes (10, 10), (20, 20), and (30, 30). Yet, when both distributions are LogNorm(0, 1), the Type I error rates with sample size (10, 10) for Bonett and W_L are 0.0716 and 0.0432. W_L appears to perform better than the Bonett test.

Furthermore, Bonett's error rates tend to inflate for right skewed parent distributions with sample sizes (30, 10). For example, when both distributions are Chisq(1) at a 0.05 significance level, Bonett's Type I error rate for sample sizes (30, 10) and (10, 30) is 0.0914 and 0.0213, respectively. Clearly, Bonett fails this test case when the sample size is (30, 10). Also, the difference between the sample sizes (30, 10) and (10, 30) indicates that the Bonett has inflated Type I error rates even though the sample size is larger than (10, 10). This was a disadvantage that the proposed tests are corrected for in (2.13).

On the other hand, T3, T6, and T9 produce more conservative, consistent, and controlled Type I error rates. T3's Type I error rates fall right in the middle of the conservative T9 and slightly uncontrolled T6.

Moreover, it can be noted that as sample size increases, the Type I error rates of the three proposed tests usually decreases. This is a good characteristic that Bonett and W_L do not have. It can be used to quickly stabilize inflated Type I error rates that usually occur for smaller sample sizes.

For instance, when both parent distributions are Gamma(5, 1) and the sample size is (10, 10), the Type I error rate for Bonett, W_L , and T6 is 0.0463, 0.0459, and 0.0532. T6's Type I error rate decreases to 0.026 while Bonett and W_L 's error rates increase to 0.0508 and 0.0518 when the sample size is increased to (30, 30).

However, when both of the parent distributions have a noticeable negative kurtosis, T6 struggles to continually deflate the Type I error rate like T3 and T9. A good illustration of this effect is when the populations are beta distributions with small shape parameters like Beta(0.5, 0.5) and a kurtosis of -0.25. T6's type I error rate increases as the sample size increases.

It becomes evident that the T3 and T9 have more control than T6, Bonett, and W_L when the distributions are identical.

4.1.2: Different Populations

Comparatively, when the two parent distributions have like shapes with different parameters, Bonett, W_L , T3, T6, and T9 perform similarly to having two identical parent distributions. For example, when the first parent distribution is Gamma(5, 1) and the second parent distribution is Normal(0, 5), the respective Type I error rates at the nominal level of 0.05 are 0.0335, 0.0309, 0.0275, 0.0407, and 0.0162. The populations have close skew and kurtosis values to one another.

Contrariwise, when the first parent distribution is roughly symmetric like Normal(0, 0.5) and the second is right skewed and/or long tailed like Gamma(0.5, 1), Bonett and W_L completely fail to control the Type I error rates. Further investigation revealed, it is common for sample variances from a distribution with a small skew value to

be significantly higher than sample variances from a more skewed distribution. Even though the two population variances are the same, this can cause tests to reject the null hypothesis at an alarming frequency. For the combination of $\text{Normal}(0, 0.5)$ and $\text{Gamma}(0.5, 1)$, Bonett's Type I error rate is approximately 0.2 for all sample sizes while W_L begins rejecting at a rate of 0.2 for sample size (10, 10). Interestingly, as the sample size increases, W_L rejects more often, and fails to detect that the variances are equal. By sample size (30, 30), W_L 's Type I error rate increases to 0.3087 which is worse than the F-test and Rajić's test.

Investigating further, when the distributions are $\text{Normal}(0, 1)$ and $\text{Exp}(1)$, Bonett and W_L continue experiencing issues. Bonett's Type I error rate is around 0.13 for sample sizes of (10, 10), (20, 20), and (30, 30). W_L 's Type I error rate begins at 0.1222 for a sample size of (10, 10) and reaches 0.1735 by sample size (30, 30). Again, W_L 's Type I error is completely out of control. It is clear that Bonett shows more control in this situation than W_L . Regardless, both tests fail in this scenario.

T3, T6, and T9 also have inflated error rates when the first distribution is symmetric and the second is skewed. However, one can see that they have significantly more control than Bonett and W_L , especially for distributions $\text{Normal}(0, 0.5)$ and $\text{Gamma}(0.5, 1)$. At a significance level of 0.05, T3, T6, and T9 have Type I error rates of 0.1621, 0.1887, and 0.128 for sample size (10, 10). T9 has the best rate starting from small sample sizes. Additionally, T3, T6, and T9's Type I error rates decrease as sample size increases. Thus, their respective error rates at sample size (30, 30) reduce to 0.1127, 0.1404, and 0.0861. This is about half of the rejection rate of Bonett and W_L for the same distribution combination.

Of all distributions and sample sizes considered, T3 behaves the best. Although T9 has the largest advantage when the cases are extreme like Normal(0, 0.5) and Gamma(0.5, 1), the parent distributions are usually unknown. Since T9 is extremely conservative for all other cases, it will most likely fail to reject the null hypothesis, especially when the first variance is larger.

On the other hand, T6 is not conservative enough. Its Type I error rates are very close to Bonett and W_L . This means that in extreme scenarios it will most likely go out of control. Additionally, its Type I error rate doesn't decrease for larger sample sizes with parent distributions like Beta(0.5, 0.5). This displays less control than T3.

It is clear that T3 is the happy medium between T6 and T9. It has the most reasonable Type I error rates out of all the tests, and consistently decreases the Type I error rate as the sample size increases.

4.2: Power Performance Comparisons

Results for Power for identical parent distributions and different parent distributions are found in Tables 5 and 6 of Appendix A, respectively.

It is visible that the F-test has the largest power. This is expected in response to its inflated Type I error rates for non-normal populations.

Additionally, Rajić's test also has large power. It's small for sample sizes of (10, 10) and increases as the sample size and magnitude of ratios increase.

On the other side of the spectrum, T9 has the lowest power in this study because of its conservative nature.

Since F, Rajić's, and T9 consequently produce Power results that are not meaningful, they are not examined further in this Power analysis.

Furthermore, it is evident that T3 and T6 have lower power than Bonett and W_L . The robust nature of the two proposed tests costs in Power. Therefore, the ability to reject the null hypothesis of T3 and T6 are of interest in the following sections.

4.2.1: Identical Populations

When the parent distributions are identical and symmetric, T3 and T6's power rates are close to those of Bonett and W_L . Considering two standard normal parent distributions where the first distribution's variance is K times larger than the second, their power is only about 0.05 less than Bonett and W_L for sample sizes of (10, 10).

However, T3 and T6's power does not keep up at the same rate as sample sizes increase. For example, when $K = 3$, Bonett and W_L 's Power increases to 0.8855 and 0.8382 by a sample size of (30, 30). Meanwhile, T3 and T6's peak rejection rates are 0.6596 and 0.7563. This is anticipated since Bonett and W_L had slightly more inflated Type I error rate. Also, 0.6596 and 0.7563 is adequate power to test the ratio of variances.

When the two identical parent distributions are skewed, T3 and T6 have more difficulty rejecting the null hypothesis when the first population's variance is K times larger than the second. For instance, when the first distribution is Chisq(3) and the second distribution is Chisq(1) with a sample size of (30, 30) and $K = 3$, Bonett and W_L reject the null hypothesis at rates of 0.5796 and 0.7515. T3 and T6 reject at rates of 0.2744 and 0.3153. Neither test performs as well as they do when the distributions are symmetric, and there is a larger difference between the existing and proposed tests. The Bonett and

W_L appear to have a larger ability to detect the differences in variances. Nonetheless, the proposed tests still have sufficient power. It is clearly not the best, but if five tests are conducted on variances where the first variance is 3 times larger than the second for skewed distributions, Bonett and the proposed tests will arrive at approximately the same conclusion.

Additionally, the proposed tests' power increases as the sample size and magnitude of the ratios increase. This is another good property that the proposed tests have. Despite their conservative nature, larger sample sizes help T3 and T6 recognize when the variances are unequal with more accuracy. Therefore, T3 and T6 have sufficient ability to test whether the first distribution's variance is larger than the second.

4.2.2: Different Populations

When the parent distributions are different, the proposed tests perform similarly compared to Bonett and W_L . The Power is expectedly higher when the first distribution is symmetric like Normal(0, 6) and the second is heavier tailed like T(4). Although the proposed tests still produce lower Power, they are consistently not far behind Bonett and W_L .

Considering only the power results that were discussed, T3 and T6 are the recommended tests.

4.3: Further Discussion

In view of the Type I error and Power results, T3 is recommended over T6 and T9. Although T3 does not have the best power, it has sufficient capability to test the ratio

of variances. In the real world, controlling Type I error plays a bigger role than Power as long as the test has sufficient capability to reject the null hypothesis when necessary.

For example, in manufacturing, having a more robust test like T3 instead of T6 is preferred. It creates less false alarms on the production line. False alarms take time away from employees performing important tasks to fix possible discrepancies. This can cost the employer thousands of dollars.

If a discrepancy is indicated by a test like T3, then most likely the ratio of variances is significantly large as seen in the Power study. Thus, a fix or adjustment is definitely required. Otherwise, if the test is too sensitive and signals that a fix is required when it is not, time and money are wasted on problems that may not exist.

Therefore, T3 is the ideal comparison of variances out of all of the proposed and existing methods discussed while considering its possible real world applications.

CHAPTER 5: CONCLUSION

In this study, right tailed tests were derived using the Edgeworth Expansion to compare the ratio of two variances. The new tests were compared to the following established methods: F, Bonett, Modified Levene's, and Rajić. The presented simulation study found that T3 is preferred over existing methods. It had the best balance between Type I error rate and Power out of the new tests.

Furthermore, T3 performs consistently with a more controlled Type I error rate than Bonett and W_L , regardless of sample size. Although it lacks sensitivity, it still has enough power to reject the null hypothesis when the variances are unequal.

In the real world, the parent populations will not be known, and in many cases only small random samples can be collected. Therefore, using T3 to compare variances provides a more robust method than any of the proposed and existing ones. T3 is ideal in these situations. Future research for similarly derived test statistics should take into account robust estimates of location and new approximations for the coefficient of skewness for ratio of variances

APPENDIX A: DATA TABLES

Table 1

Type Error Rates with $\alpha = 0.05$ for Identical Distributions

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Normal (0,1)	10, 10	0.0515	0.0424	0.0391	0.0049	0.0334	0.0508	0.0209
II: Normal (0,1)	20,20	0.0494	0.0447	0.04	0.0223	0.0198	0.0342	0.0122
	30,30	0.0491	0.0476	0.044	0.0278	0.0164	0.0307	0.0105
Skew(0, 0)	30,10	0.0513	0.0522	0.0548	0.0169	0.0085	0.02	0.0069
Kurtosis(0, 0)	10,30	0.0506	0.0373	0.0321	0.0112	0.0088	0.0192	0.0022
I: T(3)	10, 10	0.1566	0.0569	0.0406	0.075	0.0272	0.0386	0.0198
II: T(3)	20,20	0.1855	0.0574	0.0434	0.1563	0.0101	0.017	0.0076
	30,30	0.208	0.0551	0.0445	0.1928	0.0073	0.0101	0.0046
Skew(UD, UD)	30,10	0.195	0.0567	0.0361	0.145	0.0066	0.0144	0.0048
Kurtosis(∞, ∞)	10,30	0.1438	0.0356	0.0521	0.0973	0.0061	0.0116	0.0032
I: T(4)	10, 10	0.1235	0.0547	0.0448	0.049	0.0331	0.0472	0.0244
II: T(4)	20,20	0.1398	0.0503	0.0422	0.1087	0.0112	0.0212	0.008
	30,30	0.1549	0.0549	0.049	0.1399	0.0088	0.0153	0.0065
Skew(0, 0)	30,10	0.1391	0.0594	0.046	0.0911	0.0079	0.0199	0.006
Kurtosis(∞, ∞)	10,30	0.1166	0.0397	0.0445	0.0699	0.0089	0.0149	0.0059
I: T(5)	10, 10	0.1004	0.0491	0.0416	0.0324	0.0281	0.043	0.0195
II: T(5)	20,20	0.1214	0.052	0.0476	0.0871	0.017	0.0295	0.0131
	30,30	0.133	0.0533	0.0477	0.1112	0.0102	0.0171	0.0078
Skew(0, 0)	30,10	0.1153	0.0575	0.0487	0.0667	0.0097	0.0218	0.0077
Kurtosis(6, 6)	10,30	0.1102	0.0432	0.0454	0.0569	0.0086	0.017	0.004
I: T(6)	10, 10	0.0903	0.0458	0.0406	0.0244	0.0296	0.045	0.0207
II: T(6)	20,20	0.1081	0.502	0.0425	0.0703	0.015	0.026	0.0104
	30,30	0.1143	0.0537	0.046	0.0923	0.0123	0.0216	0.0101
Skew(0, 0)	30,10	0.0953	0.054	0.0476	0.0496	0.0099	0.02	0.0082
Kurtosis(3, 3)	10,30	0.099	0.0432	0.043	0.0489	0.0081	0.0192	0.0041

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(0.5, 0.5)	10, 10	0.2323	0.0671	0.0564	0.1485	0.0486	0.06	0.0388
II: Gamma(0.5,0.5)	20,20	0.2329	0.0588	0.0521	0.2104	0.0186	0.0252	0.0128
	30,30	0.2348	0.0526	0.0466	0.2272	0.0093	0.0123	0.0049
Skew(2.83, 2.83)	30,10	0.2689	0.0914	0.0355	0.2322	0.0179	0.0261	0.011
Kurtosis(12, 12)	10,30	0.1864	0.0213	0.0588	0.1451	0.0157	0.0209	0.101
I: Gamma(1, 0.5)	10, 10	0.1629	0.0526	0.0484	0.0804	0.046	0.058	0.0336
II: Gamma(1, 0.5)	20,20	0.1758	0.0513	0.0515	0.1478	0.0192	0.0284	0.015
	30,30	0.1827	0.0493	0.0466	0.1725	0.011	0.0166	0.0074
Skew(2, 2)	30,10	0.1852	0.0715	0.0462	0.1428	0.0156	0.0275	0.0134
Kurtosis(6, 6)	10,30	0.1523	0.0267	0.0517	0.1007	0.0132	0.0218	0.0085
I: Gamma(2, 2)	10, 10	0.1098	0.0433	0.0442	0.0402	0.0388	0.0518	0.0246
II: Gamma(2, 2)	20,20	0.1252	0.0502	0.0499	0.0944	0.0229	0.034	0.0171
	30,30	0.1366	0.0519	0.0511	0.121	0.0138	0.0228	0.0095
Skew(1.41, 1.41)	30,10	0.1179	0.0553	0.0504	0.0802	0.0113	0.0228	0.0088
Kurtosis(3, 3)	10,30	0.1135	0.0323	0.0444	0.0644	0.0126	0.0233	0.0058
I: Gamma(3, 3)	10, 10	0.0923	0.0466	0.0445	0.0282	0.0384	0.053	0.0276
II: Gamma(3, 3)	20,20	0.1014	0.0492	0.0477	0.0716	0.0213	0.0362	0.0144
	30,30	0.1138	0.053	0.052	0.0969	0.014	0.0262	0.0096
Skew(1.16, 1.16)	30,10	0.0942	0.0571	0.0541	0.0527	0.0106	0.0259	0.0084
Kurtosis(2, 2)	10,30	0.1016	0.037	0.0434	0.0507	0.0134	0.0241	0.0061
I: Gamma(4, 4)	10, 10	0.0797	0.0409	0.0392	0.0221	0.0355	0.0507	0.0234
II: Gamma(4, 4)	20,20	0.0907	0.0478	0.0431	0.0638	0.0202	0.0334	0.0137
	30,30	0.0977	0.0531	0.0489	0.0792	0.0135	0.024	0.0101
Skew(1, 1)	30,10	0.0879	0.0557	0.0538	0.0467	0.0104	0.0224	0.0087
Kurtosis(1.5, 1.5)	10,30	0.0857	0.034	0.0409	0.0398	0.0125	0.02	0.0051
I: Gamma(10, 10)	10, 10	0.0637	0.0422	0.0405	0.0102	0.0334	0.051	0.02
II: Gamma(10, 10)	20,20	0.0663	0.0466	0.0426	0.0373	0.0207	0.037	0.014
	30,30	0.0679	0.049	0.0437	0.0479	0.014	0.0266	0.01
Skew(0.63, 0.63)	30,10	0.0616	0.0505	0.0537	0.0269	0.0085	0.0206	0.007
Kurtosis(0.6, 0.6)	10,30	0.0645	0.0379	0.035	0.0237	0.011	0.0209	0.0039
I: Gamma(5, 1)	10, 10	0.0778	0.0463	0.0459	0.0182	0.0388	0.0532	0.0243
II: Gamma(5, 1)	20,20	0.0831	0.0471	0.0428	0.0527	0.0207	0.0376	0.0144
	30,30	0.0905	0.0508	0.0518	0.0722	0.0144	0.026	0.0103
Skew(0.89, 0.89)	30,10	0.0797	0.0533	0.0544	0.0415	0.0112	0.0229	0.0096
Kurtosis(1.2, 1.2)	10,30	0.0792	0.0356	0.0387	0.0354	0.0124	0.0217	0.0048

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(2, $\sqrt{3}$)	10, 10	0.1098	0.0433	0.0442	0.0402	0.0388	0.0517	0.0246
II: Gamma(2, $\sqrt{3}$)	20,20	0.1252	0.0502	0.0499	0.0944	0.0229	0.034	0.0171
	30,30	0.1366	0.0519	0.0511	0.121	0.0138	0.0228	0.0095
Skew(1.41, 1.41)	30,10	0.1179	0.0553	0.0504	0.0802	0.0113	0.0228	0.0088
Kurtosis(3.5, 3.5)	10,30	0.1135	0.0323	0.0444	0.0644	0.0126	0.0233	0.0058
I: Gamma(3/2, 2)	10, 10	0.1319	0.0495	0.0493	0.056	0.0404	0.056	0.0294
II: Gamma(3/2, 2)	20,20	0.147	0.0492	0.0502	0.1173	0.0196	0.0298	0.0144
	30,30	0.15	0.0529	0.0516	0.1377	0.0122	0.0209	0.0092
Skew(1.63, 1.63)	30,10	0.1497	0.0644	0.0501	0.1046	0.0137	0.0272	0.011
Kurtosis(4, 4)	10,30	0.1273	0.0331	0.0519	0.0792	0.0152	0.0238	0.008
I: Exp(1)	10, 10	0.163	0.051	0.0473	0.0791	0.043	0.0562	0.0343
II: Exp(1)	20,20	0.1725	0.0526	0.0506	0.1464	0.0211	0.0315	0.0162
	30,30	0.1803	0.0499	0.0469	0.1698	0.0091	0.0146	0.0069
Skew(2, 2)	30,10	0.1815	0.0702	0.0446	0.1398	0.0144	0.0252	0.0106
Kurtosis(6, 6)	10,30	0.1463	0.0279	0.0521	0.1	0.0139	0.0217	0.0075
I: Weibull(1, 1)	10, 10	0.1581	0.0526	0.049	0.0811	0.0441	0.0602	0.0349
II: Weibull(1, 1)	20,20	0.1777	0.0567	0.0541	0.1503	0.0212	0.0322	0.0141
	30,30	0.1896	0.0561	0.0519	0.1794	0.0128	0.0181	0.0083
Skew(2, 2)	30,10	0.1896	0.0189	0.0519	0.1794	0.0128	0.0181	0.0083
Kurtosis(-0.5, -0.5)	10,30	0.1581	0.0355	0.049	0.0811	0.0441	0.0602	0.0349
I: Weibull(2, 1)	10, 10	0.0551	0.0409	0.0404	0.0089	0.0375	0.0509	0.0209
II: Weibull(2, 1)	20,20	0.0588	0.0512	0.0448	0.0307	0.024	0.0447	0.0128
	30,30	0.0592	0.049	0.0439	0.0389	0.0169	0.0317	0.0109
Skew(6.35, 6.35)	30,10	0.0592	0.0172	0.0439	0.0389	0.0169	0.0317	0.0109
Kurtosis(-0.004, -0.004)	10,30	0.0551	0.024	0.0404	0.0089	0.0375	0.0509	0.0209
I: Weibull(5, 1)	10, 10	0.0454	0.0408	0.037	0.0041	0.035	0.049	0.0187
II: Weibull(5, 1)	20,20	0.0468	0.0477	0.0429	0.0194	0.0233	0.0406	0.0139
	30,30	0.0467	0.0482	0.042	0.0271	0.0143	0.0292	0.0103
Skew(-27.32, -27.32)	30,10	0.0467	0.0151	0.042	0.0271	0.0143	0.0292	0.0103
Kurtosis(0,0)	10,30	0.0454	0.0224	0.037	0.0041	0.035	0.049	0.0187
I: Weibull(10, 1)	10, 10	0.0619	0.0416	0.0394	0.0115	0.035	0.0505	0.0192
II: Weibull(10, 1)	20,20	0.0705	0.0492	0.0445	0.037	0.0217	0.0378	0.0141
	30,30	0.0692	0.0465	0.0424	0.0486	0.0122	0.0243	0.0091
Skew(-425.3, -425.3)	30,10	0.0692	0.0157	0.0424	0.0486	0.0122	0.0243	0.0091
Kurtosis(0,0)	10,30	0.0619	0.0224	0.0394	0.0115	0.035	0.0505	0.0192

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Beta(0.5, 0.5)	10, 10	0.0087	0.0211	0.0315	0.0007	0.0308	0.0284	0.0036
II: Beta(0.5, 0.5)	20,20	0.0029	0.026	0.0309	0.0002	0.0269	0.0536	0.0011
	30,30	0.0018	0.0341	0.0326	0.0001	0.0294	0.0659	0.0004
Skew(0, 0)	30,10	0.0098	0.0334	0.0731	0.0019	0.0064	0.0137	0.0031
Kurtosis(-0.25, -0.25)	10,30	0.0015	0.0277	0.015	0	0.0122	0.0206	0
I: Beta(1, 1)	10, 10	0.0148	0.0274	0.0336	0.0003	0.031	0.0384	0.0071
II: Beta(1, 1)	20,20	0.009	0.0339	0.0318	0.0006	0.0249	0.0476	0.0034
	30,30	0.0075	0.0391	0.039	0.0009	0.0241	0.0516	0.0022
Skew(0, 0)	30,10	0.0136	0.0395	0.0646	0.0016	0.0069	0.0145	0.0029
Kurtosis(-1.2, -1.2)	10,30	0.0059	0.0296	0.0193	0.0001	0.0119	0.0217	0
I: Beta(2, 2)	10, 10	0.0241	0.0355	0.0348	0.0009	0.0306	0.0474	0.0127
II: Beta(2, 2)	20,20	0.0224	0.0443	0.0422	0.0032	0.0261	0.0465	0.0084
	30,30	0.0151	0.0428	0.0399	0.0039	0.0184	0.0405	0.0059
Skew(0, 0)	30,10	0.0266	0.0494	0.0656	0.0038	0.0085	0.0201	0.0055
Kurtosis(-0.85, -0.85)	10,30	0.0189	0.0337	0.0249	0.0003	0.0116	0.0211	0.0004
I: Beta(3, 3)	10, 10	0.0298	0.0349	0.0373	0.0012	0.0299	0.0485	0.016
II: Beta(3, 3)	20,20	0.0264	0.0439	0.042	0.0059	0.0247	0.0475	0.0107
	30,30	0.023	0.0433	0.0401	0.0081	0.0188	0.0355	0.0076
Skew(0, 0)	30,10	0.03	0.0471	0.0598	0.0071	0.0092	0.0208	0.0073
Kurtosis(-0.67, -0.67)	10,30	0.0249	0.0358	0.0271	0.0017	0.0139	0.0199	0.001
I: Beta(1, 2)	10, 10	0.0371	0.0404	0.042	0.0019	0.0384	0.0478	0.0158
II: Beta(1, 2)	20,20	0.0323	0.0456	0.0438	0.0078	0.0307	0.0486	0.0103
	30,30	0.0267	0.0443	0.0441	0.0124	0.0202	0.0411	0.0089
Skew(0.57, 0.57)	30,10	0.041	0.0569	0.0673	0.0106	0.0115	0.0276	0.0078
Kurtosis(-0.6, -0.6)	10,30	0.029	0.0353	0.0306	0.0024	0.016	0.0248	0.0006
I: Beta(1, 3)	10, 10	0.0608	0.0431	0.0459	0.0109	0.0426	0.0538	0.0244
II: Beta(1, 3)	20,20	0.0586	0.0464	0.0443	0.0281	0.0268	0.0443	0.0141
	30,30	0.0558	0.0469	0.0452	0.0362	0.0182	0.0307	0.0118
Skew(0.6, 0.6)	30,10	0.0652	0.0598	0.0619	0.0281	0.0131	0.0274	0.0101
Kurtosis(0.095, 0.095)	10,30	0.0529	0.0311	0.0337	0.0137	0.0138	0.0239	0.0036
I: Beta(2, 3)	10, 10	0.0313	0.0393	0.0396	0.0021	0.0345	0.0478	0.0152
II: Beta(2, 3)	20,20	0.0295	0.0459	0.0392	0.0077	0.0264	0.0492	0.0118
	30,30	0.0284	0.0494	0.0424	0.0089	0.0195	0.0412	0.0082
Skew(0.29, 0.29)	30,10	0.0307	0.0473	0.06	0.0067	0.0091	0.0222	0.0064
Kurtosis(-0.64, -0.64)	10,30	0.0279	0.0387	0.0297	0.0016	0.0132	0.0231	0.0009

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Beta(5, 4)	10, 10	0.0395	0.0414	0.0406	0.0032	0.0367	0.0512	0.0181
II: Beta(5, 4)	20,20	0.0325	0.0433	0.0387	0.0087	0.0212	0.0434	0.0105
	30,30	0.032	0.0502	0.045	0.0134	0.0179	0.0377	0.0091
Skew(-0.129, -0.129)	30,10	0.0359	0.0511	0.0588	0.009	0.0092	0.0226	0.0068
Kurtosis(-0.48, -0.48)	10,30	0.033	0.0385	0.029	0.0036	0.0112	0.0217	0.0016
I: Chisq(1)	10, 10	0.2323	0.0671	0.0564	0.1485	0.0486	0.0601	0.0388
II: Chisq(1)	20,20	0.2329	0.0588	0.0521	0.2104	0.0186	0.0252	0.0128
	30,30	0.2348	0.0526	0.0466	0.2272	0.0093	0.0123	0.0049
Skew(2.8, 2.8)	30,10	0.2689	0.0914	0.0355	0.2322	0.0179	0.0261	0.011
Kurtosis(12, 12)	10,30	0.1864	0.0213	0.0588	0.1451	0.0157	0.0209	0.0101
I: Chisq(3)	10, 10	0.1319	0.0495	0.0493	0.056	0.0404	0.056	0.0294
II: Chisq(3)	20,20	0.147	0.0492	0.0502	0.1173	0.0196	0.0298	0.0144
	30,30	0.15	0.0529	0.0516	0.1377	0.0122	0.0209	0.0092
Skew(1.633, 1.633)	30,10	0.1497	0.0644	0.0501	0.1046	0.0137	0.0272	0.011
Kurtosis(4, 4)	10,30	0.1273	0.0331	0.0519	0.0792	0.0152	0.0238	0.008
I: Chisq(9)	10, 10	0.077	0.0438	0.045	0.0171	0.036	0.0524	0.0227
II: Chisq(9)	20,20	0.0936	0.0512	0.0483	0.0611	0.0244	0.0415	0.0162
	30,30	0.0891	0.0504	0.0469	0.0707	0.0135	0.0242	0.0102
Skew(0.942, 0.942)	30,10	0.082	0.0502	0.0529	0.0414	0.0095	0.0213	0.0077
Kurtosis(1.333, 1.33)	10,30	0.0807	0.0353	0.0376	0.0368	0.0096	0.0219	0.004
I: LogNorm(0, 1)	10, 10	0.2443	0.0716	0.0432	0.1698	0.0388	0.0487	0.0306
II: LogNorm(0,1)	20,20	0.2767	0.0681	0.0407	0.2605	0.013	0.0175	0.0084
	30,30	0.304	0.0694	0.0485	0.3016	0.0066	0.0091	0.0036
Skew(7.4, 7.4)	30,10	0.3265	0.0944	0.0273	0.2938	0.0107	0.018	0.0077
Kurtosis(110.9, 110.9)	10,30	0.1997	0.0235	0.0626	0.1644	0.011	0.0147	0.0072

Table 2

Type I Error Rates with $\alpha = 0.05$ for Different Distributions

Distribution	n_1, n_2	F	Bonett	W _L	Rajić	T3	T6	T9
I: Gamma(2, $\sqrt{3}$) II: Gamma(3/2, 2) Skew(1.41, 1.63) Kurtosis(3.5, 4)	10, 10	0.1269	0.0586	0.0556	0.0458	0.0458	0.0631	0.0337
	20,20	0.1424	0.0586	0.0581	0.1086	0.0246	0.0399	0.019
	30,30	0.1431	0.0602	0.064	0.1284	0.0179	0.029	0.0133
	30,10	0.1419	0.0725	0.0648	0.0953	0.0172	0.0312	0.0134
	10,30	0.1195	0.036	0.0528	0.0693	0.0151	0.0246	0.0082
I: Gamma(3/2, 2) II: Gamma(2, $\sqrt{3}$) Skew(1.63, 1.41) Kurtosis(4, 3.5)	10, 10	0.1147	0.0392	0.0394	0.0454	0.0363	0.0502	0.0267
	20,20	0.1362	0.0467	0.0443	0.1062	0.0194	0.0297	0.0138
	30,30	0.1442	0.0457	0.0426	0.1321	0.0111	0.0189	0.0086
	30,10	0.1238	0.0486	0.0386	0.086	0.0084	0.0169	0.0072
	10,30	0.1224	0.029	0.0393	0.0719	0.0117	0.0212	0.0062
I: Normal (0, 2) II: T(4) Skew(0, 0) Kurtosis(0, ∞)	10, 10	0.1237	0.0998	0.0975	0.0219	0.0767	0.1024	0.0551
	20,20	0.134	0.1166	0.1206	0.0738	0.0578	0.0894	0.0425
	30,30	0.1474	0.1326	0.1518	0.1022	0.0578	0.0856	0.0455
	30,10	0.1343	0.1344	0.1398	0.0535	0.0333	0.0643	0.0288
	10,30	0.1082	0.0699	0.0818	0.0339	0.0253	0.044	0.0109
I: T(4) II: Normal (0, 2) Skew(0, 0) Kurtosis(∞ , 0)	10, 10	0.0688	0.022	0.0161	0.0252	0.011	0.0198	0.0081
	20,20	0.0809	0.02	0.0129	0.062	0.0032	0.0068	0.0027
	30,30	0.0859	0.0177	0.0111	0.0787	0.0026	0.0043	0.0024
	30,10	0.0667	0.0178	0.0146	0.0441	0.0016	0.0057	0.0014
	10,30	0.0815	0.0227	0.0199	0.0442	0.0031	0.0076	0.0013
I: Gamma(1,1) II: Normal (0,1) Skew(2, 0) Kurtosis(6, 0)	10, 10	0.0809	0.014	0.0151	0.0312	0.0154	0.0215	0.0097
	20,20	0.0959	0.014	0.0099	0.0766	0.0044	0.0085	0.0028
	30,30	0.1085	0.0136	0.0077	0.1019	0.0026	0.0046	0.0022
	30,10	0.0676	0.0105	0.0063	0.0454	0.0015	0.0029	0.001
	10,30	0.103	0.0151	0.0204	0.0659	0.0059	0.0116	0.003
I: Normal (0,1) II: Gamma(1,1) Skew(0, 2) Kurtosis(0, 6)	10, 10	0.1589	0.1286	0.1259	0.041	0.1005	0.1287	0.0773
	20,20	0.1641	0.1368	0.1477	0.1013	0.0772	0.1104	0.0588
	30,30	0.1616	0.1327	0.1722	0.1175	0.0664	0.0923	0.0526
	30,10	0.1823	0.1797	0.1852	0.0936	0.0642	0.1073	0.0561
	10,30	0.1155	0.0637	0.0809	0.0438	0.0289	0.0481	0.0157

Distribution	n_1, n_2	F	Bonett	WL	Rajić	T3	T6	T9
I: Gamma(5,1) II: Normal (0,5) Skew(0.89, 0) Kurtosis(1.2, 0)	10, 10	0.0597	0.0335	0.0309	0.0137	0.0275	0.0407	0.0162
	20,20	0.0653	0.0364	0.0338	0.403	0.0143	0.0273	0.0079
	30,30	0.0662	0.0337	0.0302	0.0504	0.0085	0.0164	0.0048
	30,10	0.0567	0.0368	0.0369	0.0249	0.0041	0.0143	0.0034
	10,30	0.0697	0.0308	0.0317	0.0284	0.0094	0.0173	0.0028
I: Normal (0,5) II: Gamma(5,1) Skew(0, 0.89) Kurtosis(0, 1.2)	10, 10	0.0683	0.055	0.0516	0.0086	0.0457	0.0625	0.0263
	20,20	0.0741	0.0684	0.0594	0.0341	0.0323	0.0571	0.022
	30,30	0.0751	0.0724	0.0674	0.433	0.0193	0.0373	0.0163
	30,10	0.0759	0.0737	0.0766	0.0243	0.0146	0.0341	0.0125
	10,30	0.0645	0.0491	0.0423	0.0169	0.014	0.0251	0.0048
I:Normal(0, 0.5) II:Gamma(0.5, 1) Skew(0, 2.83) Kurtosis(0, 12)	10, 10	0.2513	0.2055	0.2091	0.11	0.1621	0.1887	0.128
	20,20	0.2454	0.1989	0.2564	0.1753	0.1284	0.1612	0.1021
	30,30	0.2495	0.193	0.3087	0.2009	0.1127	0.1404	0.0861
	30,10	0.2825	0.2745	0.315	0.1865	0.14	0.1861	0.1253
	10,30	0.1885	0.0969	0.1568	0.0942	0.0568	0.0812	0.0339
I:Gamma(0.5, 1) II: Normal(0, 0.5) Skew(2.83, 0) Kurtosis(12, 0)	10, 10	0.0872	0.0076	0.0077	0.0432	0.0083	0.0115	0.0055
	20,20	0.1136	0.0083	0.0032	0.0969	0.0022	0.0036	0.0015
	30,30	0.1246	0.0077	0.0028	0.1226	0.0007	0.0014	0.0006
	30,10	0.084	0.0051	0.0007	0.0651	0.0002	0.0008	0.0002
	10,30	0.1126	0.0086	0.0123	0.0791	0.0039	0.0071	0.0027
I: Exp(1) II: Normal (0,1) Skew(2, 0) Kurtosis(6, 0)	10, 10	0.0742	0.0143	0.0161	0.0279	0.0147	0.0235	0.009
	20,20	0.0909	0.0117	0.0085	0.0737	0.0036	0.0083	0.0027
	30,30	0.0999	0.0136	0.0079	0.0934	0.0017	0.0037	0.0013
	30,10	0.067	0.0127	0.0071	0.0445	0.0016	0.0038	0.0014
	10,30	0.0953	0.0161	0.0211	0.0578	0.0068	0.0106	0.0026
I: Normal (0,1) II: Exp(1) Skew(0, 2) Kurtosis(0, 6)	10, 10	0.1665	0.1314	0.1222	0.0462	0.1007	0.1276	0.0777
	20,20	0.1674	0.1395	0.1507	0.1049	0.0836	0.1128	0.0657
	30,30	0.1672	0.1351	0.1735	0.1226	0.0702	0.0948	0.0549
	30,10	0.1895	0.1847	0.188	0.1034	0.0723	0.1133	0.0631
	10,30	0.1209	0.0699	0.0883	0.0482	0.0308	0.0507	0.0182
I: T(4) II: Gamma(2, 1) Skew(0, 1.41) Kurtosis(∞ , 3)	10, 10	0.1084	0.0423	0.0315	0.0416	0.025	0.0366	0.0168
	20,20	0.1196	0.041	0.0316	0.0919	0.0107	0.0179	0.0082
	30,30	0.1279	0.0399	0.0284	0.1146	0.0042	0.0096	0.0037
	30,10	0.1243	0.0497	0.0393	0.0834	0.0075	0.0171	0.0059
	10,30	0.1036	0.0317	0.032	0.0624	0.0055	0.0113	0.0026

Distribution	n_1, n_2	F	Bonett	WL	Rajić	T3	T6	T9
I: Gamma(2,1) II: T(4) Skew(1.41, 0) Kurtosis(3, ∞)	10, 10	0.1248	0.051	0.0542	0.0455	0.0448	0.0601	0.029
	20,20	0.1571	0.0657	0.0685	0.1201	0.0281	0.0442	0.0202
	30,30	0.1616	0.0696	0.0767	0.1446	0.0168	0.0295	0.012
	30,10	0.1439	0.0693	0.0643	0.0917	0.0143	0.0282	0.0121
	10,30	0.1262	0.0393	0.0573	0.0765	0.0171	0.0266	0.0078
I: Normal(0, 1/12) II: Beta(1, 1) Skew(0, 0) Kurtosis(0, -0.85)	10, 10	0.024	0.0228	0.0185	0.0016	0.0193	0.0315	0.0086
	20,20	0.0204	0.0219	0.0125	0.0064	0.0088	0.0197	0.0039
	30,30	0.0206	0.0281	0.0136	0.0097	0.0067	0.0178	0.0034
	30,10	0.0177	0.0191	0.0218	0.0044	0.0022	0.0066	0.0019
	10,30	0.0324	0.0424	0.0166	0.0053	0.0063	0.0147	0.0008
I: Beta(1, 1) II: Normal(0, 1/12) Skew(0, 0) Kurtosis(-0.85, 0)	10, 10	0.0395	0.0595	0.0672	0.0015	0.054	0.0604	0.0209
	20,20	0.0327	0.0704	0.0857	0.0069	0.053	0.0712	0.0156
	30,30	0.0326	0.0755	0.1101	0.0084	0.0455	0.0764	0.0133
	30,10	0.0445	0.0956	0.1354	0.0069	0.021	0.0452	0.0116
	10,30	0.0219	0.0329	0.0408	0.001	0.0191	0.0274	0.0003
I: Chisq(1) II: T(4) Skew(2.83, 0) Kurtosis(12, ∞)	10, 10	0.1339	0.0185	0.0163	0.07	0.0184	0.0256	0.0129
	20,20	0.1691	0.0212	0.0097	0.1493	0.0059	0.0102	0.0042
	30,30	0.1855	0.0259	0.0101	0.1807	0.0032	0.0051	0.0017
	30,10	0.1538	0.0209	0.0042	0.1231	0.0018	0.0046	0.0012
	10,30	0.1498	0.0142	0.028	0.1096	0.0082	0.0123	0.0054
I: T(4) II: Chisq(1) Skew(0, 2.83) Kurtosis(∞ , 12)	10, 10	0.2229	0.1372	0.1286	0.1173	0.082	0.1048	0.0661
	20,20	0.2262	0.1206	0.1432	0.1855	0.0475	0.0621	0.0343
	30,30	0.2294	0.1102	0.1614	0.2085	0.0305	0.0427	0.019
	30,10	0.2772	0.1791	0.1746	0.2147	0.0598	0.087	0.0478
	10,30	0.1726	0.0565	0.0969	0.1105	0.0212	0.0337	0.0141
I: Chisq(1) II: Normal(0, 2) Skew(2.83, 0) Kurtosis(12, 0)	10, 10	0.0872	0.0076	0.0077	0.0432	0.0083	0.0115	0.0055
	20,20	0.1136	0.0083	0.0032	0.0939	0.0022	0.0036	0.0015
	30,30	0.1246	0.0077	0.0028	0.1226	0.0007	0.0014	0.0006
	30,10	0.084	0.0051	0.0007	0.0651	0.0002	0.0008	0.0002
	10,30	0.1126	0.0086	0.0123	0.0791	0.0039	0.0071	0.0027
I: Normal(0, 2) II: Chisq(1) Skew(0, 2.83) Kurtosis(0, 12)	10, 10	0.2513	0.2055	0.2091	0.11	0.1621	0.1888	0.128
	20,20	0.2454	0.1989	0.2564	0.1753	0.1284	0.1612	0.1021
	30,30	0.2495	0.193	0.3087	0.2009	0.1127	0.1404	0.0861
	30,10	0.2825	0.2745	0.315	0.1865	0.14	0.1861	0.1253
	10,30	0.1885	0.0969	0.1568	0.0942	0.0568	0.0811	0.0339

Table 3

Type I Error Rates with $\alpha = 0.1$ for Identical Distributions

Distribution	n_1, n_2	F	Bonett	WL	Rajić	T3	T6	T9
I: Normal (0,1)	10,10	0.1003	0.0918	0.0873	0.0411	0.0726	0.0952	0.0516
II: Normal (0,1)	20,20	0.095	0.0941	0.0858	0.062	0.0542	0.0802	0.043
	30,30	0.1013	0.1024	0.0945	0.0696	0.0551	0.0757	0.046
Skew(0, 0)	30,10	0.0973	0.1	0.1211	0.0568	0.0364	0.0622	0.0313
Kurtosis(0, 0)	10,30	0.0998	0.0903	0.0653	0.0505	0.0298	0.0482	0.016
I: T(3)	10, 10	0.2131	0.1226	0.0941	0.154	0.0651	0.0795	0.0481
II: T(3)	20,20	0.2423	0.1268	0.0992	0.2158	0.0467	0.0595	0.0311
	30,30	0.265	0.1278	0.1017	0.2535	0.0377	0.0482	0.0195
Skew(UD, UD)	30,10	0.269	0.1292	0.1011	0.2326	0.045	0.0674	0.0288
Kurtosis(∞, ∞)	10,30	0.1832	0.0979	0.0924	0.1518	0.023	0.034	0.0153
I: T(4)	10, 10	0.1848	0.1151	0.0973	0.1193	0.0715	0.0874	0.0546
II: T(4)	20,20	0.1997	0.1138	0.0934	0.1693	0.049	0.0636	0.0339
	30,30	0.2152	0.1145	0.0958	0.1972	0.0468	0.0596	0.0316
Skew(0, 0)	30,10	0.2149	0.1253	0.1135	0.1684	0.0491	0.074	0.0328
Kurtosis(∞, ∞)	10,30	0.1674	0.0953	0.0854	0.1256	0.0249	0.0377	0.0175
I: T(5)	10, 10	0.1592	0.1071	0.0928	0.094	0.0697	0.09	0.0505
II: T(5)	20,20	0.1831	0.1157	0.1009	0.1513	0.0553	0.0721	0.0434
	30,30	0.1903	0.1161	0.1026	0.1725	0.0489	0.063	0.0352
Skew(0, 0)	30,10	0.1857	0.1175	0.1145	0.1387	0.0471	0.0734	0.0363
Kurtosis(6, 6)	10,30	0.1613	0.1042	0.0885	0.1148	0.0294	0.0453	0.0198
I: T(6)	10, 10	0.1512	0.0992	0.0897	0.0843	0.0675	0.0881	0.0501
II: T(6)	20,20	0.1673	0.1102	0.0944	0.1316	0.0538	0.0724	0.0411
	30,30	0.1722	0.1151	0.0991	0.1508	0.0503	0.0661	0.0399
Skew(0, 0)	30,10	0.1616	0.1108	0.1094	0.1166	0.0451	0.0702	0.0355
Kurtosis(3, 3)	10,30	0.1497	0.0992	0.0834	0.1017	0.0297	0.0456	0.0198
I: Gamma(0.5,0.5)	10, 10	0.2866	0.132	0.1163	0.2386	0.0933	0.1068	0.0696
II: Gamma(0.5,0.5)	20,20	0.286	0.1202	0.1063	0.2698	0.0572	0.0679	0.0331
	30,30	0.2889	0.1137	0.1008	0.2826	0.0461	0.0527	0.021
Skew(2.83, 2.83)	30,10	0.3334	0.1613	0.0945	0.3074	0.0758	0.0963	0.0377
Kurtosis(12, 12)	10,30	0.2304	0.0691	0.1024	0.2032	0.0361	0.0455	0.0247

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(1, 0.5)	10, 10	0.2225	0.1112	0.1036	0.1658	0.0903	0.104	0.0673
II: Gamma(1, 0.5)	20,20	0.2314	0.1097	0.1049	0.2116	0.0602	0.0747	0.0436
	30,30	0.2424	0.109	0.1007	0.2332	0.0493	0.0606	0.0296
Skew(2, 2)	30,10	0.2573	0.1332	0.1115	0.2265	0.0685	0.0921	0.0453
Kurtosis(6, 6)	10,30	0.2039	0.0773	0.0943	0.1665	0.0356	0.0524	0.0242
I: Gamma(2, 2)	10, 10	0.1744	0.0989	0.0956	0.1071	0.0828	0.0991	0.06
II: Gamma(2, 2)	20,20	0.1857	0.1077	0.1021	0.1585	0.0635	0.0828	0.0428
	30,30	0.1868	0.1022	0.0985	0.1638	0.0462	0.0608	0.0357
Skew(1.41, 1.41)	30,10	0.1929	0.1159	0.1179	0.1507	0.0513	0.0761	0.0387
Kurtosis(3, 3)	10,30	0.1655	0.0833	0.0835	0.1225	0.0386	0.0521	0.0239
I: Gamma(3, 3)	10, 10	0.1493	0.0965	0.0935	0.0884	0.0788	0.1002	0.0575
II: Gamma(3, 3)	20,20	0.1659	0.1045	0.1001	0.1317	0.0617	0.083	0.0476
	30,30	0.1746	0.1089	0.1032	0.1582	0.0582	0.074	0.0445
Skew(1.16, 1.16)	30,10	0.1642	0.1097	0.1164	0.1208	0.0488	0.0744	0.0389
Kurtosis(2, 2)	10,30	0.1551	0.092	0.0826	0.1077	0.038	0.0551	0.0245
I: Gamma(4, 4)	10, 10	0.138	0.0909	0.0888	0.0742	0.0738	0.0935	0.0524
II: Gamma(4, 4)	20,20	0.1495	0.103	0.0926	0.1167	0.061	0.0809	0.0446
	30,30	0.1558	0.1073	0.1018	0.138	0.0566	0.0743	0.0451
Skew(1, 1)	30,10	0.1522	0.1098	0.1206	0.1086	0.0448	0.0739	0.0364
Kurtosis(1.5, 1.5)	10,30	0.1389	0.0838	0.0783	0.0928	0.0327	0.0508	0.0214
I: Gamma(10, 10)	10, 10	0.1182	0.0927	0.0879	0.0547	0.0738	0.0982	0.0541
II: Gamma(10, 10)	20,20	0.1225	0.1008	0.0922	0.086	0.0581	0.0821	0.0456
	30,30	0.1227	0.0982	0.0942	0.0995	0.0545	0.0719	0.045
Skew(0.63, 0.63)	30,10	0.1188	0.0995	0.1136	0.0768	0.0397	0.0613	0.0332
Kurtosis(0.6, 0.6)	10,30	0.1133	0.0853	0.068	0.0689	0.0331	0.0491	0.0201
I: Gamma(5, 1)	10, 10	0.1387	0.0978	0.0944	0.0709	0.079	0.1023	0.0565
II: Gamma(5, 1)	20,20	0.1443	0.1022	0.0934	0.1077	0.0598	0.0823	0.0452
	30,30	0.1489	0.1055	0.1002	0.1271	0.0545	0.073	0.0439
Skew(0.89, 0.89)	30,10	0.1421	0.1097	0.1213	0.0979	0.044	0.0705	0.0356
Kurtosis(1.2, 1.2)	10,30	0.1318	0.085	0.0734	0.085	0.0338	0.0495	0.0209
I: Gamma(2, $\sqrt{3}$)	10, 10	0.1744	0.0989	0.0956	0.1071	0.0828	0.0991	0.06
II: Gamma(2, $\sqrt{3}$)	20,20	0.1857	0.1077	0.1021	0.1585	0.0635	0.0828	0.0482
	30,30	0.1963	0.1068	0.1007	0.183	0.0549	0.0699	0.0401
Skew(1.41, 1.41)	30,10	0.1929	0.1159	0.1179	0.1507	0.0513	0.0761	0.0387
Kurtosis(3.5, 3.5)	10,30	0.1655	0.0833	0.0835	0.1225	0.0386	0.0521	0.0239

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(3/2, 2)	10, 10	0.1944	0.1037	0.1008	0.1319	0.0855	0.1043	0.0667
II: Gamma(3/2, 2)	20, 20	0.2076	0.1084	0.1025	0.1824	0.0627	0.0805	0.0436
	30, 30	0.2111	0.104	0.1006	0.1994	0.0544	0.0667	0.0371
Skew(1.63, 1.63)	30, 10	0.2216	0.1232	0.1165	0.1848	0.0613	0.0872	0.0448
Kurtosis(4, 4)	10, 30	0.176	0.082	0.0889	0.139	0.0399	0.0553	0.0266
I: Exp(1)	10, 10	0.2206	0.1063	0.1025	0.1639	0.0868	0.1003	0.0656
II: Exp(1)	20, 20	0.2336	0.1074	0.1008	0.2128	0.0638	0.0757	0.0446
	30, 30	0.2388	0.1071	0.0986	0.2292	0.0481	0.0602	0.0282
Skew(2, 2)	30, 10	0.2519	0.1346	0.1117	0.2179	0.0658	0.0892	0.0439
Kurtosis(6, 6)	10, 30	0.1965	0.0751	0.0919	0.1602	0.0369	0.0495	0.0265
I: Weibull(1, 1)	10, 10	0.2195	0.1083	0.1029	0.162	0.0866	0.1038	0.0682
II: Weibull(1, 1)	20, 20	0.2439	0.1145	0.1059	0.2191	0.0663	0.0801	0.0456
	30, 30	0.2431	0.1113	0.1016	0.236	0.054	0.0639	0.0344
Skew(2, 2)	30, 10	0.2431	0.0593	0.1016	0.236	0.054	0.0639	0.0344
Kurtosis(-0.5, -0.5)	10, 30	0.2195	0.0767	0.1029	0.162	0.0866	0.1038	0.0682
I: Weibull(2, 1)	10, 10	0.1064	0.0901	0.0874	0.0468	0.0772	0.0977	0.0532
II: Weibull(2, 1)	20, 20	0.112	0.1045	0.0951	0.0764	0.0664	0.0902	0.0501
	30, 30	0.1114	0.1014	0.0925	0.088	0.0563	0.0757	0.0454
Skew(6.35, 6.35)	30, 10	0.1114	0.0493	0.0925	0.088	0.0563	0.0757	0.0454
Kurtosis(-0.004, -0.004)	10, 30	0.1064	0.0622	0.0874	0.0468	0.0772	0.0977	0.0532
I: Weibull(5, 1)	10, 10	0.0925	0.0856	0.0826	0.0374	0.0712	0.0934	0.0481
II: Weibull(5, 1)	20, 20	0.0946	0.1004	0.0942	0.0576	0.0598	0.0861	0.0475
	30, 30	0.0911	0.098	0.09	0.0645	0.0555	0.0742	0.0448
Skew(-27.32, -27.32)	30, 10	0.0911	0.0481	0.09	0.0645	0.0555	0.0742	0.0448
Kurtosis(0, 0)	10, 30	0.925	0.0594	0.0824	0.0374	0.0712	0.0934	0.0481
I: Weibull(10, 1)	10, 10	0.1175	0.0915	0.0868	0.055	0.0749	0.0959	0.0525
II: Weibull(10, 1)	20, 20	0.1242	0.1031	0.0952	0.0899	0.0624	0.0842	0.0496
	30, 30	0.1235	0.1021	0.0928	0.0986	0.0515	0.0703	0.042
Skew(-425.3, -425.3)	30, 10	0.1235	0.047	0.0928	0.0986	0.0515	0.0703	0.042
Kurtosis(0, 0)	10, 30	0.1175	0.0636	0.0868	0.055	0.0749	0.0959	0.0525
I: Beta(0.5, 0.5)	10, 10	0.023	0.0488	0.0691	0.0034	0.0629	0.0568	0.0122
II: Beta(0.5, 0.5)	20, 20	0.012	0.064	0.0711	0.002	0.0646	0.0912	0.0059
	30, 30	0.0102	0.0793	0.0795	0.0018	0.0733	0.1076	0.0054
	30, 10	0.0234	0.066	0.1396	0.0081	0.0155	0.026	0.0094
Skew(0, 0)	10, 30	0.0107	0.072	0.0347	0.0001	0.0283	0.0437	0
Kurtosis(-0.25, -0.25)								

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Beta(1, 1)	10, 10	0.0389	0.0636	0.0738	0.0081	0.0655	0.0773	0.0236
II: Beta(1, 1)	20,20	0.0256	0.0739	0.0772	0.0083	0.0596	0.0884	0.0179
	30,30	0.0259	0.0875	0.085	0.009	0.065	0.098	0.0192
Skew(0, 0)	30,10	0.0363	0.0783	0.1292	0.012	0.0196	0.0353	0.0128
Kurtosis(-1.2, -1.2)	10,30	0.0303	0.0753	0.0467	0.0033	0.0303	0.0475	0.0012
I: Beta(2, 2)	10, 10	0.0599	0.0749	0.0802	0.0167	0.0694	0.0914	0.0378
II: Beta(2, 2)	20,20	0.0538	0.09	0.0856	0.0234	0.0633	0.0918	0.0359
	30,30	0.046	0.0887	0.0873	0.0202	0.059	0.0838	0.0313
Skew(0, 0)	30,10	0.0618	0.0931	0.1302	0.0239	0.0278	0.0501	0.0228
Kurtosis(-0.85, -0.85)	10,30	0.0537	0.0821	0.0561	0.0142	0.0331	0.0514	0.0079
I: Beta(3, 3)	10, 10	0.0671	0.0791	0.0806	0.0226	0.0698	0.0885	0.0411
II: Beta(3, 3)	20,20	0.0628	0.0895	0.0859	0.0296	0.0618	0.0887	0.0398
	30,30	0.0604	0.0954	0.0884	0.0305	0.0566	0.0819	0.0373
Skew(0, 0)	30,10	0.0678	0.0939	0.1257	0.0294	0.0302	0.0513	0.0248
Kurtosis(-0.67, -0.67)	10,30	0.0656	0.0832	0.0587	0.0191	0.0336	0.0485	0.0096
I: Beta(1, 2)	10, 10	0.0821	0.0823	0.0871	0.0278	0.0827	0.0977	0.0468
II: Beta(1, 2)	20,20	0.071	0.0897	0.091	0.0387	0.0693	0.0942	0.0416
	30,30	0.0694	0.0913	0.092	0.04	0.0619	0.0856	0.0393
Skew(0.57, 0.57)	30,10	0.082	0.1045	0.1356	0.0421	0.0389	0.0638	0.0333
Kurtosis(-0.6, -0.6)	10,30	0.0724	0.0802	0.0658	0.0251	0.0399	0.0577	0.0109
I: Beta(1, 3)	10, 10	0.1128	0.0877	0.0936	0.0536	0.0835	0.101	0.0577
II: Beta(1, 3)	20,20	0.1101	0.0943	0.0907	0.0784	0.0673	0.0902	0.0512
	30,30	0.1033	0.0914	0.0928	0.0805	0.0567	0.0768	0.0466
Skew(0.6, 0.6)	30,10	0.1168	0.1118	0.1276	0.0766	0.0475	0.0763	0.0391
Kurtosis(0.095, 0.095)	10,30	0.1037	0.0756	0.0695	0.0559	0.0377	0.0595	0.0189
I: Beta(2, 3)	10, 10	0.0735	0.0835	0.0834	0.0216	0.0739	0.0944	0.0437
II: Beta(2, 3)	20,20	0.0675	0.0932	0.0911	0.0341	0.0648	0.0944	0.0421
	30,30	0.0644	0.0955	0.091	0.0379	0.0608	0.0848	0.0433
Skew(0.29, 0.29)	30,10	0.0711	0.0941	0.1254	0.0299	0.0303	0.0517	0.0256
Kurtosis(-0.64, -0.64)	10,30	0.069	0.0852	0.0611	0.0237	0.0363	0.0541	0.0111
I: Beta(5, 4)	10, 10	0.0832	0.0884	0.0859	0.0286	0.0769	0.0968	0.0484
II: Beta(5, 4)	20,20	0.0734	0.0957	0.0876	0.0383	0.0588	0.0881	0.0421
	30,30	0.0777	0.1027	0.0923	0.0457	0.0613	0.0855	0.0457
Skew(-0.129, -0.129)	30,10	0.0789	0.0976	0.1242	0.0376	0.0335	0.0559	0.0286
Kurtosis(-0.48, -0.48)	10,30	0.0781	0.0853	0.0626	0.0296	0.0328	0.0534	0.0132

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Chisq(1)	10, 10	0.2866	0.132	0.1163	0.2386	0.0933	0.1068	0.0696
II: Chisq(1)	20,20	0.286	0.1202	0.1063	0.2698	0.0572	0.0679	0.0331
	30,30	0.2889	0.1137	0.1008	0.2826	0.0461	0.0527	0.021
Skew(2.8, 2.8)	30,10	0.3334	0.1613	0.0945	0.3074	0.0758	0.0963	0.0377
Kurtosis(12, 12)	10,30	0.2304	0.0691	0.1024	0.2032	0.0361	0.0455	0.0247
I: Chisq(3)	10, 10	0.1944	0.1037	0.1008	0.1319	0.0855	0.1043	0.0667
II: Chisq(3)	20,20	0.2076	0.1084	0.1025	0.1824	0.0627	0.0805	0.0436
	30,30	0.2111	0.104	0.1006	0.1994	0.0544	0.0667	0.0371
Skew(1.633, 1.633)	30,10	0.2216	0.1232	0.1165	0.1848	0.0613	0.0872	0.0448
Kurtosis(4, 4)	10,30	0.176	0.082	0.0889	0.139	0.0399	0.0553	0.0266
I: Chisq(9)	10, 10	0.139	0.0932	0.0927	0.0719	0.0789	0.0993	0.0564
II: Chisq(9)	20,20	0.1505	0.1076	0.0998	0.1164	0.0646	0.0884	0.0498
	30,30	0.1468	0.1048	0.0968	0.1279	0.0557	0.0733	0.0431
Skew(0.942, 0.942)	30,10	0.143	0.1059	0.1201	0.1017	0.0413	0.0676	0.0332
Kurtosis(1.333, 1.33)	10,30	0.1312	0.0866	0.0729	0.0867	0.0334	0.0507	0.0205
I: LogNorm(0, 1)	10, 10	0.2963	0.1428	0.1039	0.2498	0.0793	0.0905	0.0607
II: LogNorm(0,1)	20,20	0.3256	0.1435	0.0992	0.3118	0.0478	0.0555	0.0258
	30,30	0.3488	0.1466	0.1099	0.3452	0.0426	0.0483	0.0142
Skew(7.4, 7.4)	30,10	0.3903	0.1772	0.0853	0.3684	0.0616	0.0791	0.0282
Kurtosis(110.9, 110.9)	10,30	0.2366	0.0829	0.1139	0.2119	0.0304	0.0375	0.0194

Table 4

Type I Error Rates with $\alpha = 0.1$ for Different Distributions

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(2, $\sqrt{3}$)	10, 10	0.1922	0.1151	0.1147	0.1231	0.0955	0.1133	0.0721
II: Gamma(3/2, 2)	20, 20	0.2051	0.1228	0.1198	0.1759	0.072	0.0919	0.0525
	30, 30	0.2058	0.1205	0.1217	0.1898	0.064	0.0765	0.0481
Skew(1.41, 1.63)	30, 10	0.2145	0.1384	0.1384	0.1739	0.0662	0.0936	0.0499
Kurtosis(3.5, 4)	10, 30	0.1716	0.0892	0.0964	0.1299	0.0433	0.0588	0.0275
I: Gamma(3/2, 2)	10, 10	0.1755	0.0932	0.0876	0.115	0.0767	0.0905	0.0566
II: Gamma(2, $\sqrt{3}$)	20, 20	0.1979	0.099	0.0911	0.1714	0.0594	0.0746	0.0451
	30, 30	0.2036	0.1011	0.0872	0.1939	0.0487	0.0615	0.0344
Skew(1.63, 1.41)	30, 10	0.1902	0.1029	0.0956	0.1544	0.0455	0.0682	0.032
Kurtosis(4, 3.5)	10, 30	0.1756	0.0773	0.0801	0.1339	0.0341	0.0484	0.0223
I: Normal (0, 2)	10, 10	0.2011	0.1786	0.173	0.1063	0.1424	0.1732	0.1117
II: T(4)	20, 20	0.2073	0.1916	0.2072	0.1536	0.1297	0.1606	0.1097
	30, 30	0.2257	0.2128	0.2475	0.1835	0.1384	0.1687	0.1223
Skew(0, 0)	30, 10	0.2252	0.2226	0.2583	0.146	0.1052	0.1501	0.0938
Kurtosis(0, ∞)	10, 30	0.1777	0.1331	0.1457	0.1056	0.0666	0.0927	0.0429
I: T(4)	10, 10	0.1081	0.0584	0.0431	0.0687	0.0313	0.0448	0.0242
II: Normal (0, 2)	20, 20	0.1257	0.0558	0.0367	0.1041	0.0199	0.0277	0.0138
	30, 30	0.1298	0.0508	0.0276	0.1183	0.0158	0.0212	0.0103
Skew(0, 0)	30, 10	0.1138	0.0468	0.0436	0.0885	0.0151	0.0239	0.0108
Kurtosis(∞ , 0)	10, 30	0.1234	0.0703	0.0434	0.0886	0.0127	0.0196	0.0098
I: Gamma(1, 1)	10, 10	0.1262	0.0424	0.0369	0.0836	0.036	0.0479	0.0256
II: Normal (0, 1)	20, 20	0.1465	0.0472	0.0312	0.1285	0.0209	0.0309	0.0155
	30, 30	0.1616	0.0471	0.0241	0.1567	0.0164	0.0233	0.0104
Skew(2, 0)	30, 10	0.1181	0.0323	0.0229	0.0937	0.0116	0.0198	0.0071
Kurtosis(6, 0)	10, 30	0.1459	0.0537	0.0448	0.1174	0.0195	0.0277	0.013
I: Normal (0, 1)	10, 10	0.2356	0.2077	0.2061	0.1389	0.1768	0.2053	0.1382
II: Gamma(1, 1)	20, 20	0.2382	0.2064	0.2425	0.1837	0.1548	0.1817	0.1282
	30, 30	0.2339	0.1972	0.2683	0.1946	0.1418	0.1647	0.1205
Skew(0, 2)	30, 10	0.2654	0.2584	0.3005	0.1931	0.1502	0.1976	0.1355
Kurtosis(0, 6)	10, 30	0.1825	0.1176	0.1425	0.1155	0.0682	0.0954	0.0486

Distribution	n_1, n_2	F	Bonett	WL	Rajić	T3	T6	T9
I: Gamma(5,1)	10, 10	0.111	0.0742	0.0732	0.0552	0.0611	0.0803	0.0436
II: Normal (0,5)	20,20	0.1144	0.0826	0.0723	0.0875	0.0474	0.0659	0.0362
	30,30	0.1186	0.0834	0.0724	0.101	0.0387	0.0553	0.0297
Skew(0.89, 0)	30,10	0.1097	0.0786	0.0901	0.0717	0.0289	0.0481	0.0225
Kurtosis(1.2, 0)	10,30	0.1186	0.079	0.0659	0.0748	0.0278	0.0438	0.0159
I: Normal (0,5)	10, 10	0.1274	0.1154	0.1084	0.0551	0.0929	0.1193	0.067
II: Gamma(5,1)	20,20	0.1315	0.1284	0.1176	0.0893	0.0833	0.1064	0.0687
	30,30	0.132	0.1286	0.1248	0.0976	0.0745	0.0991	0.0642
Skew(0, 0.89)	30,10	0.1361	0.1375	0.1529	0.0845	0.0567	0.0871	0.0511
Kurtosis(0, 1.2)	10,30	0.1174	0.0998	0.0791	0.0629	0.0397	0.0594	0.0234
I:Normal(0,0.5)	10, 10	0.3249	0.288	0.3111	0.2298	0.2482	0.2782	0.2039
II: Gamma(0.5, 1)	20,20	0.3178	0.2692	0.364	0.2655	0.2148	0.2425	0.1784
	30,30	0.3195	0.2629	0.412	0.283	0.2015	0.2239	0.1583
Skew(0, 2.83)	30,10	0.3577	0.3477	0.4247	0.2927	0.241	0.2827	0.2172
Kurtosis(0, 12)	10,30	0.2514	0.1564	0.2315	0.1849	0.1178	0.1472	0.0851
I:Gamma(0.5, 1)	10, 10	0.1309	0.0243	0.0188	0.0958	0.0202	0.0277	0.014
II:Normal(0,0.5)	20,20	0.1598	0.0303	0.0125	0.1462	0.0119	0.0163	0.0065
	30,30	0.1732	0.0314	0.0145	0.1698	0.009	0.0119	0.0041
Skew(2.83, 0)	30,10	0.1372	0.0193	0.005	0.1157	0.0064	0.0112	0.0024
Kurtosis(12, 0)	10,30	0.1512	0.0367	0.0292	0.1272	0.0136	0.0203	0.0089
I: Exp(1)	10, 10	0.1245	0.0406	0.0379	0.0773	0.0374	0.0474	0.0271
II: Normal (0,1)	20,20	0.1462	0.0394	0.0277	0.1261	0.019	0.0266	0.0133
	30,30	0.1527	0.0454	0.0243	0.1465	0.0165	0.022	0.0096
Skew(2, 0)	30,10	0.1211	0.0358	0.0249	0.0969	0.0131	0.0211	0.0088
Kurtosis(6, 0)	10,30	0.1411	0.052	0.0437	0.11	0.0197	0.0282	0.0121
I: Normal (0,1)	10, 10	0.2381	0.2085	0.2046	0.1451	0.1762	0.2044	0.1398
II: Exp(1)	20,20	0.2371	0.2087	0.2343	0.185	0.1572	0.1845	0.1314
	30,30	0.2367	0.2019	0.2649	0.1991	0.1434	0.1673	0.1216
Skew(0, 2)	30,10	0.2673	0.2652	0.2998	0.2002	0.1565	0.2014	0.1419
Kurtosis(0, 6)	10,30	0.184	0.1265	0.1491	0.1189	0.0744	0.1013	0.051
I: T(4)	10, 10	0.1646	0.0965	0.0777	0.1054	0.0574	0.076	0.0432
II: Gamma(2, 1)	20,20	0.1724	0.0945	0.0728	0.1476	0.0398	0.0537	0.0291
	30,30	0.1812	0.0896	0.0679	0.1674	0.0347	0.0451	0.0209
Skew(0, 1.41)	30,10	0.1887	0.1041	0.0926	0.1501	0.0411	0.0624	0.0287
Kurtosis(∞ , 3)	10,30	0.1439	0.0805	0.0667	0.1115	0.021	0.0304	0.0137

Distribution	n_1, n_2	F	Bonett	WL	Rajić	T3	T6	T9
I: Gamma(2,1) II: T(4) Skew(1.41, 0) Kurtosis(3, ∞)	10, 10	0.1881	0.1132	0.1136	0.1212	0.0916	0.1133	0.0681
	20,20	0.2252	0.1358	0.1355	0.1918	0.0808	0.1006	0.0628
	30,30	0.2271	0.1361	0.1461	0.2109	0.0713	0.0873	0.0513
	30,10	0.2231	0.1405	0.1454	0.1773	0.0594	0.091	0.046
	10,30	0.1817	0.0982	0.1087	0.1369	0.0452	0.0625	0.0294
I: Normal(0, 1/12) II: Beta(1, 1) Skew(0, 0) Kurtosis(0, -0.85)	10, 10	0.0569	0.0543	0.0477	0.0184	0.0429	0.0652	0.0278
	20,20	0.0523	0.0603	0.0354	0.0278	0.0313	0.0541	0.0215
	30,30	0.056	0.0687	0.035	0.034	0.0365	0.0539	0.0252
	30,10	0.046	0.0469	0.0578	0.0206	0.0129	0.0246	0.0111
	10,30	0.0754	0.0964	0.0402	0.0314	0.0226	0.038	0.0097
I: Beta(1, 1) II: Normal(0, 1/12) Skew(0, 0) Kurtosis(-0.85, 0)	10, 10	0.086	0.1151	0.1266	0.0236	0.1067	0.123	0.0569
	20,20	0.0746	0.1243	0.1573	0.0318	0.0953	0.1316	0.0516
	30,30	0.075	0.1313	0.1906	0.0374	0.0974	0.1264	0.0529
	30,10	0.0934	0.1539	0.2277	0.0381	0.0561	0.0873	0.0414
	10,30	0.0602	0.0769	0.0858	0.0147	0.0458	0.067	0.0076
I: Chisq(1) II: T(4) Skew(2.83, 0) Kurtosis(12, ∞)	10, 10	0.1893	0.0523	0.0404	0.1426	0.0431	0.0524	0.032
	20,20	0.2246	0.0667	0.033	0.2067	0.0264	0.0356	0.0162
	30,30	0.2388	0.069	0.0314	0.2352	0.0235	0.0294	0.0108
	30,10	0.2248	0.0601	0.02	0.1985	0.0218	0.0354	0.0101
	10,30	0.1908	0.0503	0.0546	0.1658	0.0214	0.03	0.0147
I: T(4) II: Chisq(1) Skew(0, 2.83) Kurtosis(∞ , 12)	10, 10	0.2878	0.2126	0.2159	0.2144	0.1492	0.1724	0.1149
	20,20	0.2838	0.1886	0.2316	0.253	0.1129	0.1294	0.0803
	30,30	0.2873	0.176	0.2539	0.2685	0.098	0.1119	0.0606
	30,10	0.3486	0.2594	0.284	0.305	0.1545	0.1854	0.1131
	10,30	0.2274	0.1186	0.1566	0.1798	0.0517	0.0695	0.0368
I: Chisq(1) II: Normal(0, 2) Skew(2.83, 0) Kurtosis(12, 0)	10, 10	0.1309	0.0243	0.0188	0.0958	0.0202	0.0277	0.014
	20,20	0.1598	0.0303	0.0125	0.1462	0.0119	0.0163	0.0065
	30,30	0.1732	0.0314	0.0145	0.1698	0.009	0.0119	0.0041
	30,10	0.1372	0.0193	0.005	0.1157	0.0064	0.0112	0.0024
	10,30	0.1512	0.0367	0.0292	0.1272	0.0136	0.0203	0.0089
I: Normal(0, 2) II: Chisq(1) Skew(0, 2.83) Kurtosis(0, 12)	10, 10	0.3249	0.288	0.3111	0.2298	0.2482	0.2782	0.2039
	20,20	0.3178	0.2692	0.364	0.2655	0.2148	0.2425	0.1784
	30,30	0.3195	0.2629	0.412	0.283	0.2015	0.2239	0.1583
	30,10	0.3577	0.3477	0.4247	0.2927	0.241	0.2827	0.2172
	10,30	0.2514	0.1564	0.2315	0.1849	0.1178	0.1472	0.0851

Table 5

Power Simulation with $\alpha = 0.05$ for Identical Distributions

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Normal (0,2)	10, 10	0.2491	0.2011	0.1829	0.0507	0.1521	0.1975	0.1151
II: Normal (0,1)	20,20	0.4335	0.3943	0.3469	0.2725	0.2228	0.3089	0.1779
	30,30	0.5803	0.5568	0.4974	0.4657	0.3056	0.4093	0.2681
Skew(0, 0)								
Kurtosis(0, 0)								
K = 2								
I: Normal (0,3)	10, 10	0.4617	0.3793	0.3338	0.1367	0.2833	0.3475	0.239
II: Normal (0,1)	20,20	0.758	0.7138	0.6524	0.6075	0.4898	0.5905	0.4357
	30,30	0.905	0.8855	0.8382	0.8425	0.6596	0.7563	0.6126
Skew(0, 0)								
Kurtosis(0, 0)								
K = 3								
I: Normal (0,4)	10, 10	0.6311	0.5303	0.4693	0.2359	0.3922	0.4707	0.3499
II: Normal (0,1)	20,20	0.9037	0.8682	0.8244	0.8133	0.6674	0.7506	0.6201
	30,30	0.9795	0.972	0.9523	0.961	0.8415	0.8948	0.7987
Skew(0, 0)								
Kurtosis(0, 0)								
K = 4								
I: Exp(2)	10, 10	0.3438	0.1495	0.1399	0.1994	0.1127	0.1364	0.0909
II: Exp(1)	20,20	0.4638	0.2158	0.2258	0.4129	0.0926	0.1208	0.0706
	30,30	0.5422	0.2773	0.2993	0.5185	0.0822	0.1094	0.0574
Skew(2, 2)								
Kurtosis(6, 6)								
K = 2								
I: Exp(3)	10, 10	0.4801	0.247	0.227	0.3004	0.1726	0.2026	0.1435
II: Exp(1)	20,20	0.6502	0.3842	0.4095	0.6005	0.176	0.2099	0.1363
	30,30	0.7526	0.4975	0.5598	0.7344	0.1836	0.2223	0.1266
Skew(2, 2)								
Kurtosis(6, 6)								
K = 3								
I: Exp(4)	10, 10	0.5745	0.3284	0.3024	0.3832	0.2219	0.2644	0.1923
II: Exp(1)	20,20	0.5297	0.5297	0.5571	0.7261	0.2436	0.2857	0.1913
	30,30	0.8678	0.6577	0.7278	0.8504	0.27	0.3154	0.193
Skew(2, 2)								
Kurtosis(6, 6)								
K = 4								

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(2, 1)	10, 10	0.3625	0.2187	0.2096	0.1835	0.1672	0.199	0.135
II: Gamma(1, 1)	20,20	0.4867	0.3093	0.3447	0.4156	0.1634	0.2009	0.1242
	30,30	0.5668	0.3738	0.4508	0.5297	0.1604	0.2001	0.123
Skew(1.41, 2) Kurtosis(3, 6)								
K = 2								
I: Gamma(3, 1)	10, 10	0.531	0.3813	0.3683	0.2885	0.2851	0.332	0.2451
II: Gamma(1, 1)	20,20	0.7055	0.5562	0.6213	0.6246	0.3401	0.3961	0.2796
	30,30	0.8093	0.6755	0.7802	0.7743	0.3981	0.452	0.3145
Skew(1.16, 2) Kurtosis(2, 6)								
K = 3								
I: Gamma(4, 1)	10, 10	0.6481	0.512	0.4988	0.3777	0.3826	0.44	0.3403
II: Gamma(1, 1)	20,20	0.8362	0.7307	0.7876	0.7692	0.4963	0.5572	0.4171
	30,30	0.9182	0.8385	0.9152	0.8917	0.5773	0.629	0.4677
Skew(1, 2) Kurtosis(1.5, 6)								
K = 4								
I: Chisq(2)	10, 10	0.4087	0.223	0.2177	0.2637	0.1565	0.1839	0.1289
II: Chisq (1)	20,20	0.5108	0.2848	0.3478	0.4669	0.1244	0.1523	0.091
	30,30	0.5819	0.3202	0.4408	0.5596	0.1066	0.1308	0.0701
Skew(2, 2.83) Kurtosis(6, 12)								
K = 2								
I: Chisq(3)	10, 10	0.5597	0.3751	0.3769	0.3661	0.2632	0.2975	0.2196
II: Chisq (1)	20,20	0.694	0.4978	0.606	0.6386	0.2657	0.3039	0.202
	30,30	0.7771	0.5796	0.7515	0.7514	0.2744	0.3153	0.1947
Skew(1.63, 2.83) Kurtosis(4, 12)								
K = 3								
I: Chisq(4)	10, 10	0.6551	0.4907	0.5008	0.4499	0.3571	0.3998	0.3071
II: Chisq (1)	20,20	0.8134	0.6564	0.775	0.7579	0.407	0.4505	0.3196
	30,30	0.8805	0.7566	0.8954	0.8617	0.4457	0.4873	0.3212
Skew(1.41, 2.83) Kurtosis(3, 12)								
K = 4								

Table 6

Power Simulation with $\alpha = 0.05$ for Different Distributions

Distribution	n_1, n_2	F	Bonett	WL	Rajić	T3	T6	T9
I: Normal (0,4)	10, 10	0.3709	0.3085	0.2896	0.1203	0.2297	0.2829	0.1906
II: T(4)	20,20	0.5295	0.4741	0.4986	0.3977	0.303	0.3807	0.2573
	30,30	0.6429	0.5878	0.6582	0.5673	0.3788	0.4511	0.3319
Skew(0, 0)								
Kurtosis(0, ∞)								
K = 2								
I: Normal (0,6)	10, 10	0.5652	0.4788	0.4541	0.2378	0.3606	0.4269	0.3209
II: T(4)	20,20	0.7744	0.7163	0.7451	0.6645	0.5237	0.5988	0.4611
	30,30	0.8675	0.8207	0.8871	0.8206	0.6459	0.7047	0.5735
Skew(0, 0)								
Kurtosis(0, ∞)								
K = 3								
I: Normal (0,8)	10, 10	0.6876	0.5967	0.5666	0.3567	0.4597	0.5221	0.4187
II: T(4)	20,20	0.8802	0.8362	0.8664	0.8121	0.6673	0.7257	0.5965
	30,30	0.9429	0.9127	0.9572	0.9208	0.7908	0.8321	0.185
Skew(0, 0)								
Kurtosis(0, ∞)								
K = 4								
I: Normal (0,2)	10, 10	0.3976	0.3221	0.3049	0.1568	0.2494	0.2986	0.2071
II: Exp(1)	20,20	0.5221	0.4507	0.4934	0.402	0.3029	0.3693	0.2495
	30,30	0.6098	0.5295	0.6383	0.5374	0.3523	0.4146	0.2929
Skew(0, 2)								
Kurtosis(0, 6)								
K = 2								
I: Normal (0,3)	10, 10	0.5642	0.4728	0.4514	0.2777	0.3659	0.4229	0.322
II: Exp(1)	20,20	0.7492	0.6743	0.73	0.6417	0.5029	0.5713	0.4307
	30,30	0.8448	0.7714	0.8688	0.7919	0.5974	0.6553	0.5126
Skew(0, 2)								
Kurtosis(0, 6)								
K = 3								
I: Normal (0,4)	10, 10	0.679	0.5815	0.5657	0.3824	0.452	0.5159	0.4037
II: Exp(1)	20,20	0.8639	0.8049	0.8553	0.7884	0.6418	0.7033	0.5556
	30,30	0.9416	0.8947	0.9548	0.9106	0.7486	0.7967	0.652
Skew(0, 2)								
Kurtosis(0, 6)								
K = 4								

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(10,1) II: Normal (0,5) Skew(0.63, 0) Kurtosis(0.6, 0) K = 2	10, 10 20,20 30,30	0.259 0.4208 0.559	0.1848 0.3489 0.489	0.1666 0.3189 0.4431	0.064 0.2954 0.4688	0.1413 0.1895 0.243	0.1886 0.2683 0.3296	0.107 0.1535 0.2074
I: Gamma(15,1) II: Normal (0,5) Skew(0.52, 0) Kurtosis(0.4, 0) K = 3	10, 10 20,20 30,30	0.4572 0.7433 0.8837	0.3572 0.6787 0.853	0.3188 0.6279 0.8089	0.148 0.6002 0.8259	0.2669 0.4494 0.5931	0.3308 0.5487 0.685	0.2207 0.3966 0.5406
I: Gamma(20,1) II: Normal (0,5) Skew(0.45, 0) Kurtosis(0.3, 0) K = 4	10, 10 20,20 30,30	0.6235 0.9001 0.9733	0.5071 0.8567 0.9601	0.4479 0.8145 0.9424	0.2469 0.8052 0.9538	0.3793 0.6399 0.7858	0.4568 0.7192 0.8418	0.3364 0.5853 0.7371
I: Normal (0,4) II: Chisq(1) Skew(0, 2.83) Kurtosis(0, 12) K = 2	10, 10 20,20 30,30	0.4634 0.5612 0.6301	0.3872 0.4827 0.5343	0.4034 0.5933 0.7211	0.242 0.465 0.5697	0.3074 0.351 0.3864	0.353 0.4044 0.4345	0.2629 0.2865 0.3015
I: Normal (0,6) II: Chisq(1) Skew(0, 2.83) Kurtosis(0, 12) K = 3	10, 10 20,20 30,30	0.6007 0.7436 0.8282	0.5178 0.6647 0.7414	0.5337 0.7795 0.897	0.3569 0.6599 0.7828	0.4098 0.5264 0.5825	0.4609 0.5781 0.6284	0.3517 0.4324 0.4742
I: Normal (0,8) II: Chisq(1) Skew(0, 2.83) Kurtosis(0, 12) K = 4	10, 10 20,20 30,30	0.696 0.8475 0.9141	0.6058 0.7739 0.8538	0.623 0.8729 0.9584	0.4515 0.7748 0.8843	0.4872 0.637 0.721	0.5365 0.6862 0.7577	0.4283 0.5352 0.5938

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(1, 1) II: Normal(0, 1) Skew(2, 0) Kurtosis(6, 0) K = 2	10, 10 20,20 30,30	0.2392 0.3894 0.4938	0.0729 0.1408 0.2041	0.0682 0.109 0.1523	0.1143 0.3325 0.4655	0.0619 0.054 0.0459	0.0839 0.0829 0.0739	0.0469 0.0412 0.036
I: Gamma(1, 1) II: Normal(0, 1) Skew(2, 0) Kurtosis(6, 0) K = 3	10, 10 20,20 30,30	0.5128 0.6213 0.7609	0.225 0.3208 0.4759	0.2918 0.2745 0.4002	0.3925 0.5539 0.7318	0.1015 0.1353 0.151	0.1383 0.1781 0.2009	0.0788 0.1089 0.1155
I: Gamma(1, 1) II: Normal(0, 1) Skew(2, 0) Kurtosis(6, 0) K = 4	10, 10 20,20 30,30	0.5108 0.7666 0.892	0.2362 0.4876 0.6816	0.2109 0.4396 0.6204	0.2981 0.7042 0.8774	0.1794 0.2171 0.2599	0.2176 0.2724 0.3135	0.1492 0.1782 0.1983
I: Normal(0, 1) II: Gamma(1, 1) Skew(0, 2) Kurtosis(0, 6) K = 2	10, 10 20,20 30,30	0.3981 0.5177 0.6095	0.3247 0.4505 0.5281	0.3076 0.4949 0.6344	0.1492 0.4072 0.5314	0.2518 0.3081 0.3498	0.3015 0.3703 0.414	0.2123 0.2547 0.2908
I: Normal(0, 1) II: Gamma(1, 1) Skew(0, 2) Kurtosis(0, 6) K = 3	10, 10 20,20 30,30	0.5625 0.74 0.8439	0.4705 0.6672 0.772	0.4539 0.7303 0.8673	0.2747 0.6369 0.7927	0.3658 0.4973 0.5956	0.4251 0.5673 0.6518	0.3229 0.4245 0.5076
I: Normal(0, 1) II: Gamma(1, 1) Skew(0, 2) Kurtosis(0, 6) K = 4	10, 10 20,20 30,30	0.673 0.8576 0.9363	0.5806 0.7958 0.8893	0.5681 0.8519 0.9544	0.3833 0.7799 0.911	0.4575 0.6369 0.7467	0.5094 0.6932 0.7912	0.4083 0.5515 0.6483

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Gamma(2, $\sqrt{3}$)	10, 10	0.3286	0.1866	0.1777	0.1537	0.1422	0.1741	0.114
II: Gamma(3/2, 2)	20, 20	0.4573	0.2838	0.3004	0.3825	0.1428	0.1912	0.1105
	30, 30	0.5494	0.3631	0.4034	0.5118	0.1469	0.1889	0.1152
Skew(1.41, 1.63) Kurtosis(3.5, 4)								
K = 2								
I: Gamma(2, $\sqrt{3}$)	10, 10	0.4871	0.308	0.2915	0.2572	0.2302	0.2704	0.1917
II: Gamma(3/2, 2)	20, 20	0.6826	0.4936	0.5264	0.6026	0.2737	0.3287	0.2256
	30, 30	0.7956	0.629	0.6965	0.7605	0.3153	0.3721	0.248
Skew(1.41, 1.63) Kurtosis(3.5, 4)								
K = 3								

Distribution	n_1, n_2	F	Bonett	W _L	Rajić	T3	T6	T9
I: Gamma(2, $\sqrt{3}$) II: Gamma(3/2, 2) Skew(1.41, 1.63) Kurtosis(3.5, 4) K = 4	10, 10 20,20 30,30	0.6046 0.8134 0.9068	0.4133 0.649 0.7879	0.3933 0.6891 0.853	0.3526 0.7472 0.8829	0.3014 0.3826 0.4502	0.352 0.4402 0.5034	0.2633 0.316 0.3588
I: Gamma(3/2, 2) II: Gamma(2, $\sqrt{3}$) Skew(1.63, 1.41) Kurtosis(4, 3.5) K = 2	10, 10 20,20 30,30	0.3042 0.4465 0.5443	0.1467 0.236 0.313	0.1413 0.2399 0.3206	0.1461 0.3805 0.5126	0.1104 0.1105 0.1086	0.1411 0.1488 0.1472	0.0889 0.0883 0.0841
I: Gamma(3/2, 2) II: Gamma(2, $\sqrt{3}$) Skew(1.63, 1.41) Kurtosis(4, 3.5) K = 3	10, 10 20,20 30,30	0.4601 0.6639 0.7918	0.2577 0.4413 0.4805	0.2431 0.4519 0.6156	0.2493 0.5938 0.7616	0.1898 0.2278 0.2481	0.2289 0.2811 0.302	0.158 0.1828 0.1951
I: Gamma(3/2, 2) II: Gamma(2, $\sqrt{3}$) Skew(1.63, 1.41) Kurtosis(4, 3.5) K = 4	10, 10 20,20 30,30	0.5833 0.8004 0.8998	0.3586 0.6048 0.7562	0.3355 0.6248 0.8006	0.3412 0.7359 0.8797	0.2543 0.3259 0.3765	0.3002 0.3803 0.4344	0.22 0.2676 0.292
I: Normal(0, 1/12) II: Beta(1, 1) K = 2	10, 10 20,20 30,30	0.1881 0.3934 0.5818	0.16 0.3906 0.5979	0.1291 0.2545 0.3726	0.0239 0.2187 0.4436	0.127 0.2119 0.3184	0.1702 0.3167 0.4526	0.0841 0.1508 0.2543
I: Normal(0, 1/12) II: Beta(1, 1) Skew(0, 0) Kurtosis(0, -1.2) K = 3	10, 10 20,20 30,30	0.425 0.7748 0.9261	0.3525 0.7501 0.9245	0.2798 0.577 0.7802	0.0855 0.599 0.8682	0.2569 0.5148 0.7061	0.3308 0.6275 0.8045	0.2086 0.4412 0.6442
I: Normal(0, 1/12) II: Beta(1, 1) Skew(0, 0) Kurtosis(0, -1.2) K = 4	10, 10 20,20 30,30	0.6214 0.9273 0.9887	0.5167 0.9016 0.9872	0.4127 0.7892 0.9374	0.1818 0.8326 0.9757	0.3772 0.6997 0.8732	0.4667 0.7818 0.9216	0.3381 0.6446 0.8244

Distribution	n_1, n_2	F	Bonett	W_L	Rajić	T3	T6	T9
I: Beta(1, 1)	10, 10	0.2398	0.2817	0.2738	0.0194	0.2234	0.2679	0.1546
II: Normal(0,1/12)	20,20	0.4416	0.5647	0.5472	0.1944	0.4631	0.5345	0.293
	30,30	0.6112	0.7356	0.7417	0.4045	0.6151	0.7087	0.4525
Skew(0, 0)								
Kurtosis(-1.2, 0)								
K = 2								
I: Beta(1, 1)	10, 10	0.4897	0.504	0.4702	0.0725	0.4056	0.4777	0.344
II: Normal(0,1/12)	20,20	0.814	0.8635	0.8286	0.5764	0.7832	0.836	0.6746
	30,30	0.948	0.9684	0.9594	0.8649	0.9251	0.9549	0.8689
Skew(0, 0)								
Kurtosis(-1.2, 0)								
K = 3								
I: Beta(1, 1)	10, 10	0.6791	0.6616	0.6141	0.158	0.5558	0.6379	0.5177
II: Normal(0,1/12)	20,20	0.9461	0.959	0.9405	0.8353	0.9091	0.9415	0.868
	30,30	0.99952	0.9975	0.995	0.9804	0.9874	0.9942	0.9761
Skew(0, 0)								
Kurtosis(-1.2, 0)								
K = 4								

APPENDIX B: FORMULAS FOR SKEW AND KURTOSIS APPROXIMATIONS

a. Normal Distribution

Skew: 0

Kurtosis: 0

b. T Distribution

Skew: $\begin{cases} 0, \gamma > 3 \\ \text{UD, otherwise} \end{cases}$

Kurtosis: $\begin{cases} \frac{6}{\gamma-4}, \gamma > 4 \\ \infty, 2 < \gamma \leq 4 \\ \text{UD, otherwise} \end{cases}$

c. Gamma Distribution

Skew: $\frac{2}{\sqrt{\alpha}}$

Kurtosis: $\frac{6}{\alpha}$

d. Weibull Distribution

Skew: $\frac{\Gamma(1+\frac{3}{\alpha})\lambda^3 - 3\mu\sigma^2 - \mu^3}{\sigma^3}$

Kurtosis: $\frac{-6\Gamma(1+\frac{1}{k})^4 + 12\Gamma(1+\frac{1}{k})^2\Gamma(1+\frac{2}{k})^1 - 3\Gamma(1+\frac{2}{k})^2 - 4\Gamma(1+\frac{1}{k})^1\Gamma(1+\frac{3}{k})^1 + \Gamma(1+\frac{4}{k})^1}{\left[\Gamma(1+\frac{2}{k})^1 - \Gamma(1+\frac{1}{k})^2\right]^2}$

e. Beta Distribution

Skew: $\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$

Kurtosis: $\frac{6[(\alpha-\beta)^2(\alpha+\beta+1) - \alpha\beta(\alpha+\beta+2)]}{\alpha\beta(\alpha+\beta+2)(\alpha+\beta+3)}$

f. Chi-Square Distribution

$$\text{Skew: } \sqrt{\frac{8}{k}}$$

$$\text{Kurtosis: } \frac{12}{k}$$

g. LogNormal Distribution

$$\text{Skew: } (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$$

$$\text{Kurtosis: } e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 6$$

APPENDIX C: SIMULATION CODE

#Right Tailed Simulation Script for R

#dist1 = Name of distribution the first dataset is generated from.

#param1 = Vector of parameters for dist1. First value is the shape parameter.

Second value is the scale parameter.

#sample_size1 = The sample size of the first generated dataset.

#dist2 = Name of distribution the second dataset is generated from.

#param2 = Vector of parameters for dist2. First value is the shape parameter.

Second value is the scale parameter.

#sample_size2 = The sample size of the second generated dataset.

#type = Type of hypothesis test.

#alpha = Level of significance.

#n = simulation size.

#K = kvariance1 = variance2.

#seed = 123.

```
simulate <- function(dist1, param1, sample_size1, dist2, param2, sample_size2,  
alpha, n, k, seed)
```

```

{
  set.seed(seed)
  countBonett <- 0
  countRajic <- 0
  countWL <- 0
  countF <- 0
  countT3 <- 0
  countT6 <- 0
  countT9 <- 0
  count = 0
  for(i in 1:n)
  {
    new_count = test(dist1, param1, sample_size1, dist2, param2,
                     sample_size2, type, k)
    if(new_count == 2)
    {
      i = i-1
    }
  }

  print("The Type I Error Rate is:")
  print("F")
  print(countF/n)
  print("Bonett")
  print(countBonett/n)
  print("WL")
  print(countWL/n)
  print("Rajic")
  print(countRajic/n)
  print("T3")

```



```

    print(countT3/n)
    print("T6")
    print(countT6/n)
    print("T9")
    print(countT9/n)
}

```

```

test <- function(dist1, param1, sample_size1, dist2, param2, sample_size2, type,
k)
{
    data_set1 = c()
    data_set2 = c()
    count = 0
    switch(dist1,
        normal =
            {
                data_set1 <- rnorm(sample_size1,
param1[1], sqrt(param1[2]))
            },
        exp =
            {
                data_set1 <- rexp(sample_size1, 1/param1[1])
            },
        gamma =
            {
                data_set1 <- rgamma(sample_size1, param1[1],
scale = param1[2])
            },
        t =
            {

```

```

        data_set1 <-rt(sample_size1, param1[1])
    },
    weibull =
    {
        data_set1<-rweibull(sample_size1, param1[1])
    },
    chisq =
    {
        data_set1<-rchisq(sample_size1, param1[1])
    },
    beta =
    {
        data_set1<-rbeta(sample_size1, param1[1],
param1[2])
    },
    lognorm =
    {
        data_set1<-rlnorm(sample_size1, param1[1],
param1[2])
    }
)
switch(dist2,
    normal =
    {
        data_set2 <- rnorm(sample_size2,
param2[1], sqrt(param2[2]))
    },
    exp =
    {
        data_set2 <- rexp(sample_size2, 1/param2[1])
    },

```

```

gamma =
  {
    data_set2 <- rgamma(sample_size2, param2[1],
    scale = param2[2])
  },
t =
  {
    data_set2 <- rt(sample_size2, param2[1])
  },
weibull =
  {
    data_set2<-rweibull(sample_size1, param2[1])
  },
chisq =
  {
    data_set2<-rchisq(sample_size2, param2[1])
  },
beta =
  {
    data_set2<-rbeta(sample_size2, param2[1],
param2[2])
  },
lognorm =
  {
    data_set2<-rlnorm(sample_size2, param2[1],
param2[2])
  }

)

data_set1 = sqrt(k)*data_set1

```

```

        if(var(data_set1) > var(data_set2))
        {
            countBonett <- countBonett +
            bonett(data_set2, param2, data_set1,
            param1)

            countWL <- countWL + WL(data_set1,
            param1, data_set2, param2)

            countRajic <- countRajic +
            Rajic(data_set1, data_set2, alpha)

            countF <- countF + f(data_set1, param1,
            data_set2, param2)

            countT3 <- T3(data_set1, data_set2, alpha)
            + countT3

            countT6 <- T6(data_set1, data_set2, alpha)
            + countT6

            countT9 <- T9(data_set1, data_set2, alpha)
            + countT9

            count = 1
        }else
        {
            count = 2
        }

    return(count)
}

```

```

bonett <- function(test1, param1, test2, param2)
{
    #Calculation
    data_set1 = c()
    data_set2 = c()
    order = order(test1)
    for(i in 1:length(test1))

```

```

{
    data_set1[i] = test1[order[i]]
}
order = order(test2)
for(i in 1:length(test2))
{
    data_set2[i] = test2[order[i]]
}
sample_mean1 = mean(data_set1)
sample_mean2 = mean(data_set2)
trim1 = 1/(2*((length(data_set1)-4)^(1/2)))
trim2 = 1/(2*((length(data_set2)-4)^(1/2)))
m1 = mean(data_set1, trim = trim1)
m2 = mean(data_set2, trim = trim2)
sum1_m = 0
sum2_m = 0
sum1_mu = 0
sum2_mu = 0
for(i in 1:length(data_set1))
{
    sum1_m = (data_set1[i] - m1)^4 + sum1_m
}

for(i in 1:length(data_set2))
{
    sum2_m = (data_set2[i] - m2)^4 + sum2_m
}

for(i in 1:length(data_set1))
{

```

```

        sum1_mu = (data_set1[i] - sample_mean1)^2 + sum1_mu
    }

    for(i in 1:length(data_set2))
    {
        sum2_mu = (data_set2[i] - sample_mean2)^2 + sum2_mu
    }

    pooled_kurtosis = ((sample_size1 + sample_size2)*(sum1_m +
sum2_m))/((sum1_mu + sum2_mu)^2)
    #Find Critical Value
    upper_critical_value = qnorm(1 - alpha)
    lower_critical_value = qnorm(alpha)
    c = ((sample_size1)/(sample_size1 -
lower_critical_value))/((sample_size2)/(sample_size2 -
lower_critical_value))
    k1 = (sample_size1-3)/(sample_size1)
    k2 = (sample_size2-3)/sample_size2
    top = - log(c*var(data_set1)/var(data_set2))
    bottom = sqrt(((pooled_kurtosis - k1)/(sample_size1 - 1)) +
((pooled_kurtosis - k2)/(sample_size2 - 1)))
    statistic = top/bottom
    if(statistic > upper_critical_value)
    {
        return(1)
    }else
    {
        return(0)
    }
}

```

```

WL <- function(data_set1, param1, data_set2, param2)
{
  z1 = 0
  z1_set = c()
  for(i in 1:length(data_set1))
  {
    temp1 = abs(data_set1[i] - median(data_set1))
    z1_set[i] = temp1
    z1 = temp1 + z1
  }
  z2 = 0
  z2_set = c()
  for(i in 1:length(data_set2))
  {
    temp2 = abs(data_set2[i] - median(data_set2))
    z2_set[i] = temp2
    z2 = temp2 + z2
  }
  z1_bar = z1/length(data_set1)
  z2_bar = z2/length(data_set2)
  z_bar = (z1 + z2)/(length(data_set1) + length(data_set2))
  z_dev1 = 0
  for(i in 1:length(data_set1))
  {
    z_dev1 = (z1_set[i] - z1_bar)^2 + z_dev1
  }
  z_dev2 = 0
  for(j in 1:length(data_set2))
  {
    z_dev2 = (z2_set[j] - z2_bar)^2 + z_dev2
  }
}

```

```

    }
    top = (length(data_set1)*((z1_bar - z_bar)^2)) +
    (length(data_set2)*((z2_bar - z_bar)^2))
    bottom = (z_dev1 + z_dev2)/((length(data_set1) - 1) + (length(data_set2) -
1))

    statistic = sqrt(top/bottom)
    #Find Critical Value
    t_statistic = qt(1-alpha, (length(data_set1) - 1) + (length(data_set2) - 1))
    if(statistic > t_statistic)
    {
        return(1)
    }else
    {
        return(0)
    }
}

Rajic <- function(data_set1, data_set2, alpha)
{
    top_T = (var(data_set1)/var(data_set2)) - ((length(data_set2) +
1)/(length(data_set2) - 1))
    varying = (var(data_set1)/var(data_set2))^2
    second_part_ratio1 = length(data_set1) + length(data_set2) - 2
    second_part_ratio2 = (length(data_set1) - 1)*(length(data_set2)-1)
    var_ratio_s1_s2 = (2*varying*second_part_ratio1)/(second_part_ratio2)
    T = top_T/sqrt(var_ratio_s1_s2)
    M3 = M3(data_set1, data_set2)
    z_alpha = qnorm(1-alpha)
    upper_critical_value1 = (1/3)*(1/sqrt(length(data_set1)))*M3*(T^2)
    upper_critical_value3 = (1/6)*(1/sqrt(length(data_set1)))*M3
    upper_critical_value = T + upper_critical_value1 + upper_critical_value3

```



```

    if(upper_critical_value > z_alpha)
    {
        return(1)
    }else
    {
        return(0)
    }
}
M3 <- function(data_set1, data_set2)
{
    X1i = c()
    X = c()
    V = c()
    part1 = 0
    part2 = 0
    part3 = 0
    sq_sum2 = 0
    for(j in 1:length(data_set2))
    {
        sq_sum2 = (data_set2[j] - mean(data_set2))^2 + sq_sum2
    }
    for(m in 1:length(data_set1))
    {
        s = (data_set1[m] - mean(data_set1))^2
        part1 = s/sq_sum2
        part2 = ((length(data_set1)-
1)*(length(data_set2)+1))/(length(data_set1)*((length(data_set2)-
1)^2))
        V[m] = (part1-part2)^2
    }
}

```

```

        X1i[m] = part1-part2
    }
    for(h in 1:length(X1i))
    {
        X[h]= X1i[h]/sqrt(mean(V))
    }
    M3_values = c()
    for(b in 1:length(X))
    {
        M3_values[b] = (X[b]^3)
    }
    M3 = mean(M3_values)
    return(M3)
}

f <- function(data_set1, param1, data_set2, param2)
{
    f = var(data_set1)/var(data_set2)
    upper_critical_value = qf(1-alpha, length(data_set1)-1, length(data_set2)-
1)
    if(f > upper_critical_value)
    {
        return(1)
    }else
    {
        return(0)
    }
}

T3 <- function(data_set1, data_set2, alpha)
{
    #calculation of sample kurtosis

```

```

kurtosis_sum1 = 0
kurtosis_sum2 = 0
for(j in 1:length(data_set1))
{
    kurtosis_sum1 = (data_set1[j] - mean(data_set1))^4 +
kurtosis_sum1
}
for(j in 1:length(data_set2))
{
    kurtosis_sum2 = (data_set2[j] - mean(data_set2))^4 +
kurtosis_sum2
}
sq_sum1 = 0
sq_sum2 = 0
for(j in 1:length(data_set1))
{
    sq_sum1 = (data_set1[j] - mean(data_set1))^2 + sq_sum1
}
for(j in 1:length(data_set2))
{
    sq_sum2 = (data_set2[j] - mean(data_set2))^2 + sq_sum2
}
first_part1 = (length(data_set1)*(length(data_set1) + 1))
first_part2 = (length(data_set2)*(length(data_set2) + 1))
first_part_kurtosis1 =(first_part1*kurtosis_sum1)-(3*(length(data_set1)-
1)*(sq_sum1^2))
first_part_kurtosis2 =(first_part2*kurtosis_sum2)-(3*(length(data_set2)-
1)*(sq_sum2^2))
second_part_kurtosis1 = (length(data_set1)-1)*(length(data_set1)-
2)*(length(data_set1)-3)
second_part_kurtosis2 = (length(data_set2)-1)*(length(data_set2)-
2)*(length(data_set2)-3)

```



```

if(var_ratio_s1_s2 < 0)
{
    second_part_ratio1 = 0
    var_ratio_s1_s2 = (varying*second_part_ratio1) +
    (2*varying2*(second_part_ratio2))
}
check = second_part_ratio1
T = top_T/sqrt(var_ratio_s1_s2)
M3 = M3(data_set1, data_set2)
samples = c(length(data_set1), length(data_set2))
t_alpha = qt(1-alpha, 2*min(samples) - 2)
upper_critical_value = t_alpha + ((M3/6)*((t_alpha^2) - 1))
if(T > upper_critical_value)
{
    return(1)
} else
{
    return(0)
}
}
T6 <- function(data_set1, data_set2, alpha)
{
    #calculation of sample kurtosis
    kurtosis_sum1 = 0
    kurtosis_sum2 = 0
    for(j in 1:length(data_set1))
    {
        kurtosis_sum1 = (data_set1[j] - mean(data_set1))^4 +
kurtosis_sum1

```

```

    }
    for(j in 1:length(data_set2))
    {
        kurtosis_sum2 = (data_set2[j] - mean(data_set2))^4 +
kurtosis_sum2
    }
    sq_sum1 = 0
    sq_sum2 = 0
    for(j in 1:length(data_set1))
    {
        sq_sum1 = (data_set1[j] - mean(data_set1))^2 + sq_sum1
    }
    for(j in 1:length(data_set2))
    {
        sq_sum2 = (data_set2[j] - mean(data_set2))^2 + sq_sum2
    }
    first_part1 = (length(data_set1)*(length(data_set1) + 1))
    first_part2 = (length(data_set2)*(length(data_set2) + 1))
    first_part_kurtosis1 =(first_part1*kurtosis_sum1)-(3*(length(data_set1)-
1)*(sq_sum1^2))
    first_part_kurtosis2 =(first_part2*kurtosis_sum2)-(3*(length(data_set2)-
1)*(sq_sum2^2))
    second_part_kurtosis1 = (length(data_set1)-1)*(length(data_set1)-
2)*(length(data_set1)-3)
    second_part_kurtosis2 = (length(data_set2)-1)*(length(data_set2)-
2)*(length(data_set2)-3)
    kurtosis1 = first_part_kurtosis1/second_part_kurtosis1
    kurtosis2 = first_part_kurtosis2/second_part_kurtosis2
    top_T =
        (var(data_set1)/var(data_set2)) -
        (((length(data_set2) + 1)/(length(data_set2) -1)) +

```

```

((
    (
        kurtosis2/
        (
            (
                var(data_set1)^2
            )*length(data_set2)
        )
    )
)*
(
    (
        length(data_set1)/(length(data_set1)-1)
    )^2
))
)
varying2 = (var(data_set1)/var(data_set2))
samples = c(length(data_set1), length(data_set2))
second_part_ratio1 = (kurtosis1/length(data_set1)) +
(kurtosis2/length(data_set2))
second_part_ratio2 = (varying2 * (1/(min(samples)-1))) +
1/(min(samples)-1)
comparison_variances = c(var(data_set1), var(data_set2))
varying = (min(comparison_variances))^-2
var_ratio_s1_s2 = (varying*second_part_ratio1) +
((2)*(second_part_ratio2))
if(var_ratio_s1_s2 < 0)
{
    second_part_ratio1 = 0
    var_ratio_s1_s2 = (varying*second_part_ratio1) +
(2*(second_part_ratio2))
}

```

```

    }
    check = second_part_ratio1
    T = top_T/sqrt(var_ratio_s1_s2)
    M3 = M3(data_set1, data_set2)
    samples = c(length(data_set1), length(data_set2))
    t_alpha = qt(1-alpha, 2*min(samples) - 2)
    upper_critical_value = t_alpha + ((M3/6)*((t_alpha^2) - 1))
    if(T > upper_critical_value)
    {
        return(1)
    }else
    {
        return(0)
    }
}
T9 <- function(data_set1, data_set2, alpha)
{
    #calculation of sample kurtosis
    kurtosis_sum1 = 0
    kurtosis_sum2 = 0
    for(j in 1:length(data_set1))
    {
        kurtosis_sum1 = (data_set1[j] - mean(data_set1))^4 +
kurtosis_sum1

    }
    for(j in 1:length(data_set2))
    {
        kurtosis_sum2 = (data_set2[j] - mean(data_set2))^4 +
kurtosis_sum2
    }
}

```



```

}
sq_sum1 = 0
sq_sum2 = 0
for(j in 1:length(data_set1))
{
    sq_sum1 = (data_set1[j] - mean(data_set1))^2 + sq_sum1
}
for(j in 1:length(data_set2))
{
    sq_sum2 = (data_set2[j] - mean(data_set2))^2 + sq_sum2
}
first_part1 = (length(data_set1)*(length(data_set1) + 1))
first_part2 = (length(data_set2)*(length(data_set2) + 1))
first_part_kurtosis1 =(first_part1*kurtosis_sum1)-(3*(length(data_set1)-
1)*(sq_sum1^2))
first_part_kurtosis2 =(first_part2*kurtosis_sum2)-(3*(length(data_set2)-
1)*(sq_sum2^2))
second_part_kurtosis1 = (length(data_set1)-1)*(length(data_set1)-
2)*(length(data_set1)-3)
second_part_kurtosis2 = (length(data_set2)-1)*(length(data_set2)-
2)*(length(data_set2)-3)
kurtosis1 = first_part_kurtosis1/second_part_kurtosis1
kurtosis2 = first_part_kurtosis2/second_part_kurtosis2
top_T =
    (var(data_set1)/var(data_set2)) -
    (((length(data_set2) + 1)/(length(data_set2) - 1)) +
        ((
            (
                kurtosis2/
            (
                (

```

```

var(data_set1)^2
)*length(data_set2)
)
)
)*
(
(
length(data_set1)/(length(data_set1)-1)
)^2
))
)
samples = c(length(data_set1), length(data_set2))
correction1 = (1 + (kurtosis1/(var(data_set1)^2))/2)
correction2 = (1 + (kurtosis2/(var(data_set2)^2))/2)
second_part_ratio1 = (correction1*kurtosis1/length(data_set1)) +
(correction2*kurtosis2/length(data_set2))
second_part_ratio2 = 1/(min(samples)-1) + 1/(min(samples)-1)
comparison_variances = c(var(data_set1), var(data_set2))
varying = (min(comparison_variances))^-2
varying2 = (var(data_set1)/var(data_set2))
var_ratio_s1_s2 = (varying*second_part_ratio1) +
((2*varying2)*(second_part_ratio2))
if(var_ratio_s1_s2 < 0)
{
second_part_ratio1 = 0
var_ratio_s1_s2 = (varying*second_part_ratio1) +
(2*varying2*(second_part_ratio2))
}
check = second_part_ratio1
T = top_T/sqrt(var_ratio_s1_s2)
M3 = M3(data_set1, data_set2)

```

```

samples = c(length(data_set1), length(data_set2))
t_alpha = qt(1-alpha, 2*min(samples) - 2)
upper_critical_value = t_alpha + ((M3/6)*((t_alpha^2) - 1))
if(T > upper_critical_value)
{
    return(1)
}else
{
    return(0)
}
}

```

REFERENCES

- Bonett, Douglas G (2006). "Robust Confidence Interval for a Ratio of Standard Deviations." *Applied Psychological Measurement* 30(5), 432-39.
- Brown, Morton B., and Forsythe, Alan B (1974). "Robust Tests for the Equality of Variances." *Journal of the American Statistical Association* 69(346), 364-67.
- Grossman, M., and Norton, H.W. (1981). "An Approximation of the Minimum-Variance Estimator of Heritability Based On Variance Component Analysis." *Genetics* 98(2), 417-26.
- Hall, Peter. (1983). *Inverting an Edgeworth Expansion*. *The Annals of Statistics*, 11(2), 569-576
- Hall, Peter. (1992) *The Bootstrap and Edgeworth Expansion*. Springer-Verlag. New York, Inc.
- Kendall, Maurice G., Stuart, Alan, Ord, J. K., Arnold, Steven F., and O'Hagan, Anthony (1994). *Kendall's Advanced Theory of Statistics*. London: Edward Arnold
- Lee, S. J., and Sa, Ping (1998). "Testing the Variance of Skewed Distributions." *Communications in Statistics - Simulation and Computation* 27(3), 807-22.
- Ott, Lyman, and Longnecker, Michael (2001). *An Introduction to Statistical Methods and Data Analysis*. Australia: Duxbury,
- Rajić, Vesna Čojbašić, and Stanojević. J (2013). "Confidence Intervals for the Ratio of Two Variances." *Journal of Applied Statistics* 40(10), 2181-187.
- Stuart, A., Ord, J. K., & Arnold, S. F. (1999). *Kendall's advanced theory of statistics*. London: A. Arnold.

VITA

Elizabeth Lesser was born [REDACTED]. Before entering high school, Elizabeth's family moved to Ponte Vedra Beach, FL where she then attended Ponte Vedra High School. Achieving great academic success, Elizabeth was able to enter her Freshman year at the University of North Florida as a Sophomore. From there, Elizabeth studied Computer Science until she saw a new opportunity in data mining and changed her major to Statistics.

Elizabeth not only managed to obtain her Bachelor's degree in just 2.5 years, but also met her soon to be husband as well. Following her Bachelor's degree, Elizabeth was not yet satisfied with her education and returned to back to the University of North Florida to obtain her Master's degree in Statistics.

During this time, Elizabeth not only endured the pressure of a tough curriculum but also maintained a part-time job working as a developer and additionally taught a class at the University. Elizabeth continued her rigorous academia, professional and teaching careers all while maintaining a near perfect GPA. Elizabeth Lesser finished her thesis and Master's degree in Statistics in December of 2016

